Direct Link between Microwave and Optical Frequencies with a 300 THz Femtosecond Laser Comb

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We demonstrate a great simplification in the long-standing problem of measuring optical frequencies in terms of the cesium primary standard. An air-silica microstructure optical fiber broadens the frequency comb of a femtosecond laser to span the optical octave from 1064 to 532 nm, enabling us to measure the 282 THz frequency of an iodine-stabilized Nd:YAG laser directly in terms of the microwave frequency that controls the comb spacing. Additional measurements of established optical frequencies at 633 and 778 nm using the same femtosecond comb confirm the accepted uncertainties for these standards.

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Optical frequencies are attractive for precision measurements in fundamental physics in comparison to microwave counterparts since the linewidth limiting processes in both domains are similar, but the optical carrier can be $\sim 5 \times 10^4$ higher in frequency. This large frequency increase enhances the Q of suitable resonances in the optical domain, such that one has the capability to reach the 10^{-15} stability domain with only seconds of averaging. However, a long-standing obstacle to popularizing precision optical frequency standards as general laboratory tools is the difficulty of generating highly stable yet widely tunable optical oscillators and determining their absolute frequencies with respect to the cesium microwave standard. Significant investment at national research facilities was generally required to overcome the challenges associated with the many intermediate oscillators required to bridge the vast gap from the 9.193 GHz cesium frequency to the optical domain [1–4].

In this Letter we report an enormous simplification in the connection of the microwave and optical domains by using a broadband (>300 THz) optical frequency comb generated with femtosecond (fs) laser technology and novel optical fiber [5]. The octave-spanning comb permits us to measure optical frequencies relative to the microwave standard when we phase-coherently bridge the gap between the fundamental and its second harmonic. In doing so, we demonstrate the possibility of an accurate frequency grid, with an even 100 MHz spacing, that spans the entire near-infrared and visible spectrum. Established optical frequency standards at 778 and 633 nm are measured in a straightforward manner and their reproducibilities confirmed. These measurements open the field of direct phase-coherent microwave to optical frequency comparison using a single optical frequency comb, and they illustrate a simple technique for precision frequency synthesis over the entire optical spectrum.

Recent experiments have shown that the frequency-domain mode comb of a fs laser is controllable and strictly uniform, making it a valuable precision "frequency scale" to measure across gaps of many tens of THz [6]. Indeed, the pulses from a mode-locked fs laser are produced in a periodic train; therefore, the broad spectrum of the laser is composed of a vast array, or comb, of distinct frequency modes spaced by the cavity repetition rate. Following the concept of the frequency division technique introduced by Telle *et al.* [7], a 44 THz femtosecond comb was incorporated into a chain to measure the $1s$-$2s$ transition in hydrogen [8]. In similar fashion, spectral broadening of the output of a 10-fs laser in standard silica fiber permitted us to measure the 104 THz gap between two cw stabilized lasers at 778 and 1064 nm [9]. Moreover, the usable bandwidth in that experiment was more than 150 THz, such that with a single bisector [7] one could expect to readily connect optical and microwave frequencies. However, the need for even the single divider stage has recently been obviated with the introduction of special air-silica microstructure optical fibers [5] which broaden the fs laser frequency comb to span an entire optical octave. As diagramed in Fig. 1(a), if a stable frequency comb can be made to span the distance between an optical frequency (f_{1064} in this case) and its second harmonic, then the fundamental can be expressed as an integer multiple of the comb spacing Δ plus measured frequency offsets $\delta_{1,2}$. Once f_{1064} has been determined in this fashion, any other frequency (e.g., f_{778} or f_{633}) that falls within the bandwidth of the comb can be measured with respect to f_{1064} in terms of Δ.

Our experiments employ a Kerr-lens mode-locked Ti:sapphire laser spectrally centered near 800 nm with

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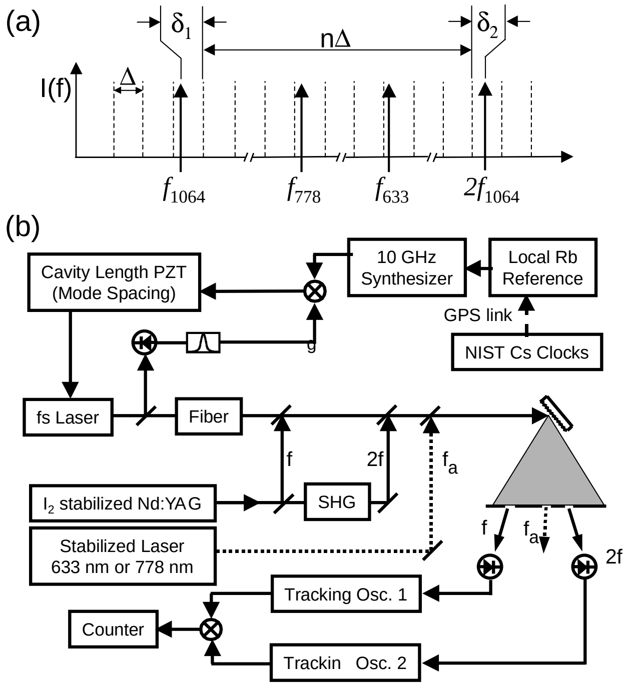

FIG. 1. (a) The femtosecond laser comb spans an octave such that the optical frequency f_{1064} can be expressed as an integer multiple of the comb spacing Δ plus small measured offsets δ_1 and δ_2. (b) Apparatus used for measurement of the optical frequencies in terms of the cesium microwave standard.

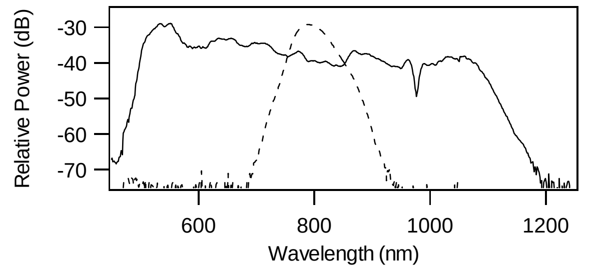

FIG. 2. Output spectrum from the silica microstructure fiber (solid line). The spectrum directly from the laser is indicated by the dashed line.

a bandwidth sufficiently wide to generate pulses on the order of 10 fs [10,11]. The 100 MHz repetition rate Δ, which is equivalent to the frequency-domain comb spacing of the emitted pulse train [6], is determined by the cavity length L and the group velocity v_g of the intracavity pulse, such that $\Delta = v_g/2L$. To control Δ, a portion of the pulse train is detected with a fast photodiode and the 100th harmonic of the repetition rate is phase locked to a stable microwave source at 10 GHz by controlling the laser cavity length with a piezoelectric transducer (PZT) [Fig. 1(b)]. The internal clock of the microwave generator is referenced to a local rubidium microwave standard, which has its average frequency offset measured against the NIST ensemble of cesium clocks via common view Global Positioning System (GPS) reception [12].

Approximately 40 mW of the fs laser light is coupled into a 5 or 10 cm piece of an air-silica microstructure fiber with an $\sim$1.7 μm core diameter. The unique waveguide properties of the air-silica microstructure fiber provide guidance in a single spatial mode and zero group velocity dispersion near 800 nm [5]. Because temporal spreading of the pulse is minimized, peak intensities in the range of hundreds of GW/cm^2 are maintained over a significant propagation length, thus providing enhanced spectral broadening due to self-phase modulation. As seen in Fig. 2, the spectral output of the fiber covers more than an octave in the optical domain. The 100 MHz periodicity of the driving field is transferred to the new spectral components via four-wave mixing among the stable components of the preexisting comb emitted by the laser. The output of the fiber is combined with both the fundamental (f_{1064}) and the second harmonic ($2f_{1064}$) of a Nd:YAG laser that is locked to the a_{10} component of the R(56) 32-0 transition in $^{127}I_2$ [13]. The mode-matched beams are dispersed with a grating and two photodiodes are used to measure the rf beat frequencies (δ_1 and δ_2) between the cw fields and the adjacent fs modes at both 1064 and 532 nm. In the actual experiment, the sign of the frequency differences between the two cw lasers and their respective fs comb lines is initially ambiguous; therefore, we must consider four possibilities in the determination of f_{1064}:

$$f_{1064} = n\Delta \pm (\delta_1 \pm \delta_2). \qquad (1)$$

With only the mode spacing of the fs comb fixed, the variations of the beats at 1064 and 532 nm are correlated as the comb position shifts. This correlated noise is removed before counting by preparing either the difference or the sum of the two beats with a balanced mixer—thus eliminating two of the possible solutions to Eq. (1). Because the frequency f_{1064} is already known at a level (20 kHz) much less than the magnitude of the rf beats $\delta_{1,2}$, the remaining ambiguous sign of Eq. (1) is removed by incrementing or decrementing n. The signal to noise ratio (S/N) of the heterodyne signals at both the green and infrared portions of the spectrum is typically 15–20 dB in a 100 kHz bandwidth, which is insufficient for accurate mixing and counting. We therefore use tracking oscillators in each channel to lock a voltage controlled oscillator to the relatively weak beat signal. The sine-wave outputs of the tracking oscillators are then mixed and counted. Figure 3(a) shows a representative time record of the counted sum frequency between the rf heterodyne beats at 1064 nm (δ_1) and 532 nm (δ_2). The uncertainty for a 1 s measurement is 1.8 kHz. The noise processes are predominantly Gaussian and average down with the $\tau^{-1/2}$ dependence seen in the Allan deviation of Fig. 3(b). From this 2500 second measurement, the 282 THz optical frequency can be determined with a statistical uncertainty of about 50 Hz, or 2×10^{-13}. The 1064 nm laser is certainly not the source of this uncertainty as its noise level is about 100-fold lower as shown by the dashed line in

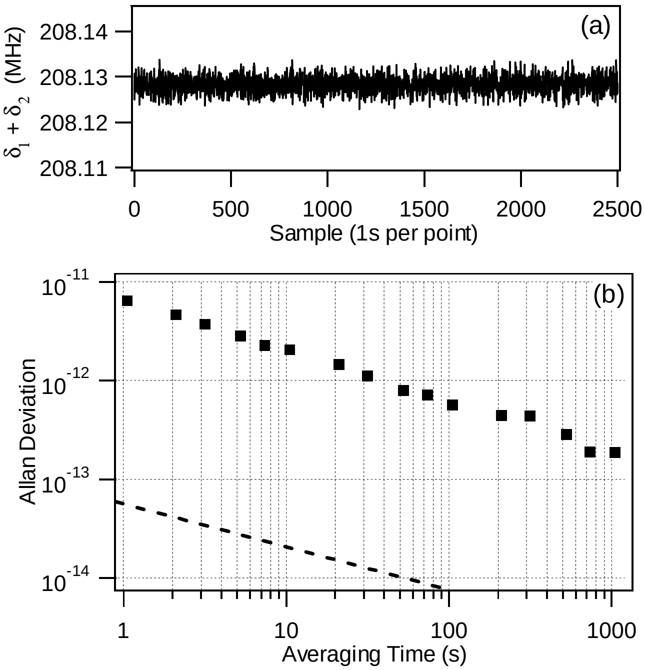

FIG. 3. (a) Time record of the counted sum of the heterodyne beats at 1064 and 532 nm, which represents a measurement of the 282 THz optical frequency of f_{1064}. (b) The squares are the Allan deviation of the optical frequency computed from the above time record. For comparison, the dashed line is the measured Allan deviation between "JILA East" and "JILA West" Nd:YAG/I_2 systems.

Fig. 3(b) [13]. It is believed that the microwave oscillator controlling Δ provides the current noise limitation, as the observed $6 \times 10^{-12}/\sqrt{\tau}$ instability is nearly equal to that specified for our microwave standard.

Data recorded over nearly a two month period are shown in Fig. 4, where we plot our measurements with respect to the Comité International des Poids et Mesures (CIPM) recommendation for one-half the a_{10} frequency. As seen, our measurements with the femtosecond laser comb yield

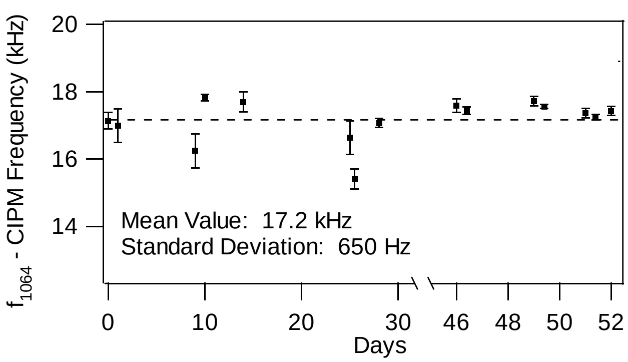

FIG. 4. Summary of measurement of one-half the a_{10} frequency (f_{1064}) plotted with respect to the CIPM recommendation of 281 630 111 740 kHz. The +17.2 kHz average of all measurements is indicated by the dashed line.

a mean offset of +17.2 kHz for the infrared frequency. While the standard deviation from the mean is 680 Hz, the statistical uncertainty for a specific measurement (typically 20 min) can be a factor of 10 lower—as exemplified in Fig. 3. The scatter of the points over the first 30 days is believed to be predominantly the result of tracking and cycle slips, which were minimized at later dates. The GPS-based comparison can permit the calibration of the average daily frequency of our rubidium standard with an inaccuracy of 6×10^{-13}; however, the S/N over a few hours can only provide calibration of the local rubidium clock at the level of 4×10^{-12}, corresponding to an uncertainty of 1.1 kHz in the optical frequency. Thus, it is not possible to completely rule out environmentally driven variations of the local rubidium microwave standard that occur over the several hour measurement period. On a much smaller scale are the day-to-day variations in the locking conditions of the "JILA West" Nd:YAG/I_2 system at the level of 5×10^{-13}. A more important limitation for an ultimate determination of the a_{10} frequency is the -2 kHz frequency offset (infrared) now observed for our second independent "JILA East" Nd:YAG/I_2 system. Therefore, accounting for this uncertainty in the realization of the a_{10} frequency, we report a measured frequency of 563 260 223 514 $\pm$ 5 kHz. This is in comparison to the recommended values of 563 260 223 480 $\pm$ 40 kHz [14].

Confidence in this fs comb approach to optical frequency metrology can be enhanced by measuring other "known" optical frequency standards. With the value of f_{1064} just determined, we then used the same broadened fs comb to measure the gap between f_{1064} and optical standards in our lab at 633 nm (HeNe/I_2 [14]) and 778 nm (Rb 2-photon [4]). This is diagramed in Fig. 1, where f_a is the frequency of the 633 or 778 nm light. Note that only the stability of the Nd:YAG/I_2 system, and not its larger realization uncertainty, is relevant in the subsequent determination of f_{778} and f_{633}.

Two 633 nm standards in our lab are commercial helium-neon (He-Ne) lasers with intracavity iodine cells. The lasers can be stabilized to component a_{13} of the transition 11-5, R(127) of $^{127}I_2$, and when operated under prescribed conditions [14], the CIPM-adopted value for this frequency is 473 612 214 705 $\pm$ 12 kHz. In the JILA-BIPM (Bureau International des Poids et Mesures) intercomparison of September 1998, our local reference laser (SN-126) was determined to be offset +1.1 kHz with respect to the BIPM-4 standard. Currently, our second HeNe/I_2 system (SN-145) differs from the local reference laser by +5 kHz, which is comparable to the +3.7 kHz difference that was measured between SN-145 and SN-126 in the same 1998 intercomparison. Using the femtosecond comb to span from f_{1064} to f_{633}, we have measured for our reference laser the frequency of peak "i" (a_{13}) of the transition 11-5, R(127) to be 473 612 214 714.2 $\pm$ 2.1 kHz. This is +9.2 kHz with respect to the CIPM recommendation, but within the accepted uncertainty. Our determination was made by measuring the a_{13} component directly as well as measuring the frequency of the nearby a_{16} (peak "f") and subsequently measuring the 138 892 kHz gap between a_{13} and a_{16}.

The 778 nm frequency was realized with two independent rubidium two-photon spectrometers: One utilizes a Ti:sapphire laser and a second is based on an external cavity diode laser. Both systems employ buildup cavities around heated rubidium cells (90 °C) and detect the 420 nm fluorescence for locking purposes. When both systems are locked to the $5S_{1/2}$ $(F = 3) \rightarrow$ $5D_{5/2}$ $(F = 5)$ two-photon transition in ^{85}Rb and with power shifts measured and removed, the diode-based system is -2.5 kHz with respect to the Ti:sapphire-based system. Using the femtosecond comb, the optical frequency of the Ti:sapphire laser locked to the rubidium two-photon transition (corrected to zero power) is measured to be 385 285 142 374.8 $\pm$ 3.0 kHz, which is -3.2 kHz with respect to the CIPM recommendation of 385 285 142 378 kHz, but well within the ± 5.0 kHz accepted uncertainty.

With the tremendous intensities ($\sim$250 GW/cm^2) used in the nonlinear generation of the octave-spanning spectrum, questions about the spectral properties of the comb should be taken seriously. Considering the results of Fig. 4, a realistic upper bound for the comb uniformity is $\sim 3 \times 10^{-12}$. We note that these data were acquired with different lengths of fiber, differing input powers, and polarizations, as well as different choices of Δ. We have further established that the phases of the comb separation frequency Δ at the green and infrared ends of the spectrum are mutually stable to $<$30 mrad over a minute time scale. This places a 0.1 mHz upper limit on the variation of Δ in the two spectral regions. If the mode spacing were assumed to vary linearly, this 0.1 mHz difference would accumulate to a frequency offset of $<$300 Hz across the 282 THz gap. Another model would consider that at some region the comb could be discontinuous, perhaps restarting after Raman scattering to a new spectral region. This could result in a frequency offset between the two extremes that differs from an integer multiple of Δ. A test of this hypothesis requires an independent interval divider where a laser is stabilized at the 709 nm bisector of f_{1064} and $2f_{1064}$ [6,7]. A simplified variation of this experiment could be carried out with no external cw lasers by measuring the heterodyne beat between the second harmonic of the 709 portion of the comb and the sum of the 1064 and 532 nm portions. A zero offset in the beat would confirm no discontinuity between the extremes of the comb. This experiment is in preparation.

In the measurements described above we did not stabilize the frequency offset of the fs comb, but rather used it as a "floating" scale to measure between fixed cw oscillators. However, in separate experiments we have fixed the comb offset by frequency locking an element of the comb to the iodine-stabilized Nd:YAG laser [9]. Most recently, deviations of the comb offset as low as 50 Hz in a 1 s counting time have been measured at 1064 nm. In a different approach, we have frequency doubled a 10-nm infrared portion of the fs comb and heterodyned it with the existing visible portion of the comb, resulting in a beat frequency that itself is equal to the offset of the fs comb from harmonics of the repetition rate. By fixing this offset to a chosen fraction of Δ, we have thus controlled the pulse-to-pulse offset between the time-domain carrier and envelope [15]. For metrology, setting the value of the offset to zero is attractive as it makes the fs optical comb fall exactly on the harmonics of the 100 MHz repetition rate. The end result is a self-referenced optical frequency sythesizer with output directly linked to the cesium standard that controls Δ.

In summary, we have used a 300 THz fs laser comb to directly measure optical frequencies across the visible and near-infrared spectrum in terms of the cesium microwave standard. The small scale of the straightforward techniques described here, in comparison to previous absolute frequency chains, should make them accessible and valuable for a wide range of optical domain experiments in time and frequency metrology, precision spectroscopy, and fundamental physics.

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