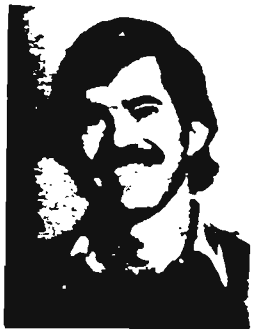

Rotation, Gyroscopes and General Relativity

by Dana Z. Anderson
Department of Physics and Joint
Institute for Laboratory Astrophysics
University of Colorado,
Boulder, CO

What, if anything, could be remarkable about an everyday experience such as rotation? An innocent inquiry into this seemingly mundane subject plows a path leading directly to the depths of Albert Einstein's theory of General Relativity. How can I tell, when the world spins around me, whether I am spinning or the world is rotating around me? It is a deceptively simple and disarming question. General Relativity makes very specific predictions about the behavior and effects of rotating bodies. One of those predictions that at once seems unreasonable will later seem perfectly natural, even intuitive. It is this: If an observer positions himself at rest near a massive spinning body (such as a rotating black hole, for example) he will see the entire universe rotate about him. The same phenomena applied to our own rotating earth says that the rotation rate we measure for the earth depends upon whether it is measured by observing the rate at which the universe apparently rotates about the earth or measured with a gyroscope located at the surface of the earth. The two measurements will differ. The difference is very slight because it depends upon the mass of the spinning object, and the earth, on a cosmic scale, is a featherweight.

If the rotation rate measured by the gyroscope is subtracted from the rate measured by observing the stars, one is left with the rate at which the universe rotates about the earth. Well, perhaps the universe having some net rotation rate is not too unreasonable. But then two observers at different locations on the earth will get different answers. In fact an observer stationed at the South Pole will conclude that the universe is rotating in one direction, an observer at the equator will say it's rotating the other. Such is the prediction of Einstein's theory. General Relativity, though, suffers from the embarrassment that this prediction has not been tested. This is because the experiment to measure this effect is enormously difficult. The evolution of laser gyroscope technology may make feasible an earth-based experiment which could make this unique test of General Relativity possible.

The prediction that the spinning earth mass causes an observer to conclude that the universe is rotating about him is indeed pretty strange; especially since the details depend where on earth the observer happens to be. One should be suspicious of a theory which predicts strange things. So that I can convince you that this prediction is in fact quite reasonable, consider the innocent question posed above. If I twirl my body, I don't need to open my eyes to know that I'm spinning: in the first place, I feel dizzy. The dizziness is a result of the motion of little hairs suspended in a fluid in my inner ear. They communicate to the brain that I'm spinning. Similarly, an inertial guidance system uses gyroscopes to sense rotation. Both the gyroscope and my inner ear rely on the *inertia* of the components involved. Inertia is the tendency of a body to resist a change in its velocity; mass is the quantity which measures it. The inertia of the fluid and of the tiny hairs in my ear are responsible for the detection of the rotational motion when I twirl around. In a mechanical gyroscope it is the inertia of a spinning wheel that causes the gyroscope to resist a tilt and thereby indicate a rotation. Apparently, neither device (my ear or the gyro) needs any external stimulus (such as light) in order to detect rotation. Only the property of inertia is needed.

Gyroscope Needs No External Stimulus

It is a remarkable fact of nature that a gyroscope needs no external stimulus to sense rotation of a body. Contrast this with two bodies, one moving at constant velocity with respect to the other. Can an observer on one of the bodies distinguish whether she is moving while the other body is at rest or vice versa, or whether both bodies are moving? The answer is no, there is no way for her to tell. There is nothing equivalent to a gyroscope that can measure a constant velocity. Another way of putting it is that there exists in nature no absolute reference frame against which the (constant) motion of a body may be compared. Any choice of such a frame would be perfectly arbitrary. The motion between the two bodies is totally relative.

This comparison leads to a lot of questions: Why then can a gyroscope sense whether or not it is being rotated? What absolute frame of reference does it measure rotation against? Imagine a universe completely devoid of anything except for a single observer. Could that observer determine whether or not he was rotating? Conversely, if our entire universe is rotating, could we know it?

To contemplate these questions leads one to Mach's principle. Ernst Mach, in his book *The Science of Mechanics* printed near the turn of the century, condemns the notion of motion with respect to an absolute space. He introduced the idea that one should consider, for example, accelerations relative to the distant stars. The distant stars somehow influence the behavior, that is, the inertia of bodies here on earth. As Misner Thorne and Wheeler in their book *Gravitation* put it, "matter there influences inertia here".

Notice that Mach's idea has the right flavor: first, it does nothing to upset the concept that motion for bodies traveling with a *constant* velocity is relative. For the distant stars affect the inertia of a body which concerns bodies undergoing a *change* in their velocity. Second, it does define a reference frame against which one can measure accelerations and rotations which do involve inertia and changes in velocity. The reference frame is determined by some sort of averaging over all the stars (really, over all masses). Thus, if the entire universe is rotating, our frame of reference would be rotating too, so we wouldn't be able to tell it was rotating. One might say that it is meaningless to talk of the rotation of the entire universe; at least Mach probably would have said so.

Influenced Einstein

Mach's work greatly influenced Einstein during the time he was formulating his general theory of relativity. With this theory, Einstein generalized the relativity that referred to bodies undergoing constant motion to bodies moving under the influence of a gravitational field. Einstein's theory lends rigor to the ideas conceived by Mach. His remarkable insight led him to prove that gravitation is the mechanism by which matter there can influence inertia here. As an illuminating example consider a massive hollow shell (Fig. 1). Inside of that shell is a second, smaller mass upon which an observer sits. If the shell is accelerated, the inner mass will also be accelerated; as though it was being dragged along by the shell. This is what one could say viewing the scene from a distance. The distant observer would also see that the acceleration of the smaller body was less than that of the shell: the dragging of the inner mass by the outer one is not perfect. However philosophically satisfying Mach's principle may be, from a scientific point of view it lacks potency. From it one cannot properly predict how much one massive body affects the inertia of another. General Relativity gives an unambiguous and unique prediction.

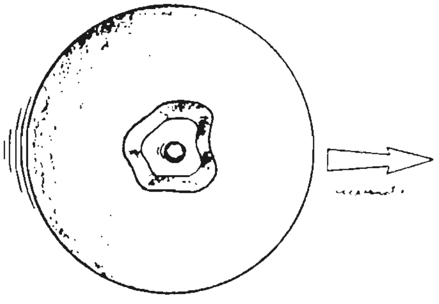

Figure 1

Now consider the point of view of the observer on the inner mass. If he has an inertial navigation instrument, when the shell is accelerated his instruments will *not* indicate an acceleration. He could observe, say by using a telescope, that the outer shell is accelerating but his measurement would yield an acceleration smaller (by the amount his mass is being dragged) than the distant observer outside the shell. Furthermore, if he could look outside the shell, he would observe the entire universe accelerating in a direction opposite the acceleration of the shell (at the rate the distant observer sees the inner mass is being dragged along).

To put these observations in the context of Mach's principle, one can say the following: The reference with respect to which accelerations can be measured is determined by taking some sort of average over all the masses in the universe weighted inversely as the distance from a given mass to the point at which acceleration is taking place. Thus all masses contribute to the determination of the reference frame. Nearby massive bodies contribute more than far away light ones. Since the massive shell is so close to the body inside, it significantly influences the local reference frame for the latter. This also means that the frame against which one may measure acceleration (or rotation) depends on where one is. Empirically such a frame is easy to determine: when an inertial navigation system (full of accelerometers and gyroscopes) indicates no reading (no forces) it is in an inertial frame.

What, then, is the effect of a rotating massive body on a nearby object? By analogy with the case of acceleration, one might draw on Mach's principle again and conclude that the rotational motion of all masses in the universe must somehow determine the nonrotating frame at a given location. Thus, one should suspect that a small mass near a larger rotating one might be dragged along. This is in fact what happens according to General Relativity. Calculating the direction and magnitude of the drag on the test body is not simple. Nature however has provided a very good analogy in the motion of electric charges. When an electrically charged body rotates, a magnetic field is generated. The magnetic field induces a test charge to undergo circular motion. The magnetic field is represented by field lines which indicate the direction of the circular motion at a given point. The plane of the circular motion is perpendicular to the field line and the direction of rotation for a test charge having the same sign of charge as the spinning charge is given by the right hand rule: point your thumb along the direction of the field lines and your fingers will curl in the direction of the rotation. Figure 2 shows the magnetic field lines for a spinning charge sphere. The field lines look something like the field lines for a bar magnet. In very much the same way, a spinning "mass charge" (instead of electric charge) will give rise to what is appropriately called a gravitomagnetic field. The difference between electric charge and mass charge is that the former comes in two flavors, positive and negative charge, while the latter comes in only one. Furthermore, two electric charges attract or repel depending on whether they have the opposite or same sign whereas mass charges always attract. Nevertheless, figure 2 is also the correct representation of the gravitomagnetic field of a spinning mass.

Now we're back to where we were at the beginning of this article. An observer near the North or South pole sees the universe rotating in the opposite direction as his spinning neighbor, whereas an observer at the Equator sees it spinning in the same direction. It is as though the earth were immersed in a viscous fluid with tiny cylinders suspended throughout. The cylinders represent local frames of reference. Near the poles these cylinders are dragged along with the spinning earth; at the equator they are pushed like gears in the opposite direction.

Gravitomagnetic Field is Small

This gravitomagnetic field induced drag due to the earth is incredibly small. If only the gravitomagnetic field was present, at the surface of the earth an observer would need to wait about thirty million years to see the universe rotate once around him (due only to the gravitomagnetic field). The smallness of the effect makes its detection an experimenter's nightmare. So far, only one effort to measure the effect is well under way. Two others have only recently been proposed and are under study. An experiment at Standford University has been under development since 1963. The experiment involves having a gyroscope and telescope aboard a satellite. The telescope observes the distant stars while the gyroscope monitors the reference frame of the satellite. The difficult task the scientists face is constructing a gyroscope sensitive enough to measure the dragging which is also compact enough to fly on a satellite. It looks as though the apparatus may fly in the next decade. Another experiment, proposed only this past year involves very precisely measuring the trajectory of two satellites in special earth orbits. In this case one measures the difference of the effect on the two satellites. The nice thing about this experiment is that the satellites do not need to be terribly sophisticated and it does not require a telescope to monitor the distant stars.

With the advance of laser gyroscope technology, a ground-based experiment may also be feasible. Repeatability is an attractive feature of doing things directly on earth; low cost is the other. This experiment requires a very sensitive laser gyroscope to measure the earth's rotation rate at a given point on its surface. It must also be tied to a telescope which determines the rate from the fixed stars. The gyroscope and telescope must be at the same location because even small tilts from tides and weather changes would mask the General Relativistic effect if the two devices were not at a single location. The ground-based experiment allows the experimenter to choose the location of the experiment quite freely. By making measurements at several locations a map of the gravitomagnetic field can be made. The map serves as a consistency check for the measurements.

The sensitivity of a ring laser gyroscope to rotation rate detection is fundamentally limited by quantum mechanics. In determining a rotation rate, the accuracy of the measurement improves as the square-root of time. For example, if one measures a rotation rate to an accuracy of 1 percent in 10 seconds, it would take 40 seconds to make the same measurement to 0.5 percent. The sensitivity, measured in terms of a signal-to-noise ratio, increases directly with the area of the ring. If more sensitivity is desired, make the ring bigger. After a point, though, the ring becomes unwieldy.

On the other hand, measuring the earth's rotation via the telescope has its limitations as well. The earth's atmosphere through which starlight must travel in order to reach the telescope behaves as an ever-changing lens. As a result, when looking through a telescope, the stars are not very stationary: they can appear to wiggle around a lot as the atmosphere above the telescope changes. This wiggling must be averaged over time so that its effects on the measurement of the rotation rate of the earth diminish. In order to make a measurement of the drag, it will take about two months of averaging time on the telescope. There is not much point in building a gyroscope which can measure the rotation rate to the required accuracy faster than the telescope

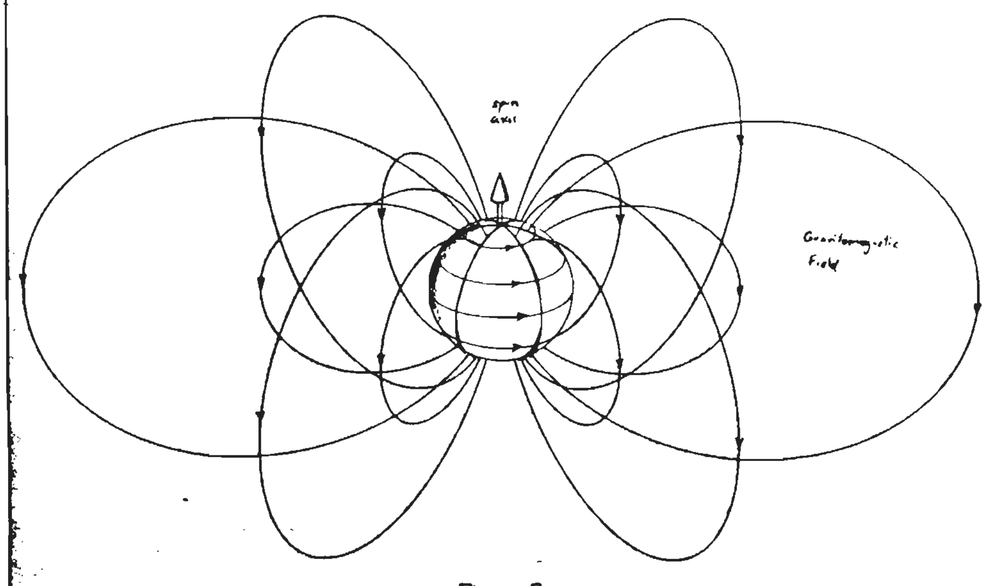

Figure 2

can make its measurements; at least, not if by doing so one significantly sacrifices ease in building the gyroscope. So, one designs a gyroscope matched to the telescope. Matching to the 60 days of averaging time for the telescope requires a ring laser gyroscope which is about 4 meters on a side! Already, that's pretty unwieldy. Needless to say, this size for ring laser gyroscope will entail techniques which are quite different than the state of the art 30 centimeter or so ring laser gyroscopes available commercially. Nevertheless, such a gyroscope is conceivable and can probably be built to serve its intended purpose.

If the experiments are so difficult, why go to all that trouble? It has often been said that the theory of General Relativity is so mathematically beautiful and philosophically satisfying that it must be a correct description of nature. It is very easy to slip into the intuitive comfort of a pleasing painting of why things are the way they seem to be and leave unquestioned the reality of the portrait. But the issues here are so fundamental to our understanding of nature that we must feel compelled to test the validity of the theory if it seems at all feasible to do so. The doorway to this probe into rotation and the effects of inertia was an innocent question: it has often been the case that an innocent inquiry or a small anomaly in a test of a theory has led to a gaping inadequacy in our understanding. This is what led us from the genius of Newton and his theory of gravity to the genius of Einstein and his theory of General Relativity.

How can I tell as the world spins about me whether I am spinning or the universe is spinning around me? General Relativity provides an answer: I cannot.