Adaptive Quantum Transduction & Quantum State Transfer via Thermal Channel

Liang Jiang

Yale University
AFOSR MURI Review
3/28/2018

Key questions:

- Transfer quantum states without impedance matching?
- Transfer quantum states via thermal channel?
Mode Converters

Quantum Matching Condition

Can we perform QST without quantum matching condition?

\[ \frac{4g^2}{\kappa_1} = \kappa_2 \]

\[ \frac{4g_1^2}{\kappa_1} = \frac{4g_2^2}{\kappa_2} \]

Mode Converter

Temporarily ignore intrinsic loss (will be discussed later)
Direct Quantum Transduction

**Input**

**Ancilla**

**Mode Converter**

**Idler**

**Output**
Adaptive Quantum Transduction– Protocol

Input

Squeezed vacuum

Ancilla

Mode Converter

Idler

Adaptive Control

Homodyne

Displace

Unitary

Output

Output

Liang Jiang

PRL 120, 020502 (2018)

3/28/2018
Adaptive Quantum Transduction – Information Flow

Input

Mode Converter

Idler

Homodyne

[Partial Info] + [Partial noise]

Displace

Unitary

Output

Information

Ancilla

Squeezed vacuum

Partial Info

Partial noise

Adaptive Control

Need LARGE noise in measured quadrature (to ensure no signal leakage)

Measure x (or p) component

Need VANISHING noise in non-measured quadrature (to ensure complete cancelation of noise)

Cancel noise as much as possible

Partial Info

Partial noise

PRL 120, 020502 (2018)
Adaptive Quantum Transduction – Scattering Matrix Formalism

Given symplectic scattering matrix $S$:

$$
\begin{pmatrix}
\hat{y}_b \\
\hat{y}_b' \\
\hat{y}_h \\
\hat{y}_h'
\end{pmatrix} =
\begin{pmatrix}
S_{b,a} & S_{b,a'} & S_{b,z} & S_{b,z'} \\
S_{b',a} & S_{b',a'} & S_{b',z} & S_{b',z'} \\
S_{h,a} & S_{h,a'} & S_{h,z} & S_{h,z'} \\
S_{h',a} & S_{h',a'} & S_{h',z} & S_{h',z'}
\end{pmatrix}
\begin{pmatrix}
\hat{x}_a \\
\hat{x}_a' \\
\hat{x}_z \\
\hat{x}_z'
\end{pmatrix}
$$

Gaussian unitary with scattering matrix

$$
\mathbf{R} = \tilde{S}^{-1}
$$

with symplectic $\tilde{S}$

$$
\tilde{S} =
\begin{pmatrix}
S_{b,a} & S_{b,a'} \\
S_{b',a} & S_{b',a'}
\end{pmatrix} + F
\begin{pmatrix}
S_{h,a} \\
S_{h',a}
\end{pmatrix}
$$

Linear transform to decide displacement

$$
\mathbf{F} = -\begin{pmatrix}
S_{b,z'} (S_{h,z'})^{-1} \\
S_{b',z'} (S_{h,z'})^{-1}
\end{pmatrix}
$$
Adaptive Quantum Transduction – Multiple Modes

Given symplectic scattering matrix $S$:

$$
\begin{pmatrix}
\hat{y}_b \\
\hat{y}_{b'} \\
\hat{y}_h \\
\hat{y}_{h'}
\end{pmatrix} =
\begin{pmatrix}
S_{b,a} & S_{b,a'} & S_{b,z} & S_{b,z'} \\
S_{b',a} & S_{b',a'} & S_{b',z} & S_{b',z'} \\
S_{h,a} & S_{h,a'} & S_{h,z} & S_{h,z'} \\
S_{h',a} & S_{h',a'} & S_{h',z} & S_{h',z'}
\end{pmatrix}
\begin{pmatrix}
\hat{x}_a \\
\hat{x}_{a'} \\
\hat{x}_z \\
\hat{x}_{z'}
\end{pmatrix}
$$

Linear transform to decide displacement

$$
F = -(S_{b,z'}(S_{h,z'})^{-1}) \left( S_{b',z'}(S_{h,z'})^{-1} \right)
$$

Restore Gaussian unitary with scattering matrix

$$
R = \tilde{S}^{-1}
$$

with symplectic $\tilde{S}$

$$
\tilde{S} = \begin{pmatrix}
S_{b,a} & S_{b,a'} \\
S_{b',a} & S_{b',a'}
\end{pmatrix} + F \begin{pmatrix}
S_{h,a} \\
S_{h',a}
\end{pmatrix}
$$
It is generally a multi-mode linear unitary evolution.
- Multi-mode: Both the input and ancilla contain multiple (entangled) modes.
- Linear: Bosonic creation and annihilation operators are evolved to their linear combinations.
- Unitary: Canonical quantization condition is preserved.

Linear unitary \(\Rightarrow\) **Symplectic** Matrix: \( SJS^T = J, \quad J = \begin{pmatrix} 0_N & I_N \\ -I_N & 0_N \end{pmatrix} \)
Adaptive Quantum Transduction – Practical Imperfections

**Imperfect Ancilla Preparation (ν)**

\[ ν = \langle x_z^2 \rangle - \langle x_z \rangle^2 = e^{-2r} (2n_z + 1) \]

Due to finite squeezing & thermal noise

- 15dB optical squeezing (PRL 104, 251102)
- 10dB mw squeezing (NatPhys 4, 929)

**Imperfect Homodyne Measurement (μ)**

\[ μ = \frac{1 - η}{η} (2n_h + 1) \]

Finite efficiency (η) & thermal noise (n_h)

- η ≥96% (μ ≤-15dB) optical homodyne efficiency
- η ≈50% (μ ≈0 dB) mw homodyne (1712.06535)

**Overall Gaussian Quantum Channel:**

**Ideal:**

\[ \hat{x} \rightarrow \hat{x} \]

**Imperfect:**

\[ \hat{x} \rightarrow \hat{x} + \hat{\xi} \]

\[ V_\xi = νBB^T + μ(-SF)(SF)^T \]

**Overall Gaussian Quantum Channel:**

\[ B = -\left( (S^{-1})_{a,h'} \left( (S^{-1})_{z,h'} \right)^{-1} \right) \]

\[ \left( (S^{-1})_{a,h} \left( (S^{-1})_{z,h} \right)^{-1} \right) \]
Due to finite squeezing & thermal noise
- 15dB optical squeezing (PRL 104, 251102)
- -10 dB mw squeezing (NatPhys 4, 929)

Imperfect Homodyne Measurement ($\mu$)

$$\mu = \frac{1 - \eta}{\eta} (2n_h + 1)$$

Finite efficiency ($\eta$) & thermal noise ($n_h$)
- $\eta \geq 96\%$ ($\mu \leq -15\text{dB}$) optical homodyne efficiency
- $\eta \approx 50\%$ ($\mu \approx 0 \text{ dB}$) mw homodyne

Inaccessible Ports (e.g., intrinsic loss)
- Added input noise (c.f. poorly prepared ancilla)
- Limited measurement ports (c.f. $\eta = 0$ msmt)
  which can be included in our framework of AQT.
  However, we still need to design optimal recovery...

Imperfect Ancilla Preparation ($\nu$)

$$\nu = \langle x_z^2 \rangle - \langle x_z \rangle^2 = e^{-2r} (2n_z + 1)$$

Liang Jiang

PRL 120, 020502 (2018)

3/28/2018
Quantum Benchmark #1 – Average Fidelity

**Average Fidelity:** $|\beta\rangle \xrightarrow{\mathcal{E}} \hat{\rho}_\beta^{(E)}$

$$F = \int P(\beta) \langle \beta | \hat{\rho}_\beta | \beta \rangle d\beta$$

Classical Protocol: $|\beta\rangle \xrightarrow{\text{Measure}} \mathcal{C} \xrightarrow{\text{Prepare}} \hat{\rho}_\beta^{(C)}$

Benchmark for Gaussian distributed coherent states $|\beta\rangle$

$$F_{coh}^* (\bar{n}) = \frac{\bar{n} + 1}{2\bar{n} + 1}$$

Uniformly distributed coherent states $\bar{n} \to \infty$

$$F_{coh}^* (\infty) = \frac{1}{2}$$

**No** Classical protocol can go beyond this.

$$p(\beta) = \frac{1}{\pi \bar{n}} e^{-|\beta|^2 / \bar{n}}$$

Quantum Benchmark #1 – Average Fidelity

\[ F = \int P(\beta) \langle \beta | \hat{\rho}_\beta | \beta \rangle d\beta \]

(a) \( T = 0.1 \)

(b) \( T = 0.8 \)

Liag Jiang

PRL 120, 020502 (2018)
Quantum Benchmark #2 – Quantum Channel Capacity

**Quantum channel capacity:** The maximum number of qubits transmitted coherently per channel use

Lower bound \((\tau \neq 1):\) \(Q(\mathcal{G}) \geq \max\{0, \log \left| \frac{\tau}{1-\tau} \right| - g(2\bar{n} + 1)\}\)

Lower bound \((\tau = 1):\) \(Q(\mathcal{G}) \geq \max\{0, \log \left( 1 + \frac{1}{\sqrt{\det Y}} \right) - 1\}\)

\[
\tau := \det X \\
\bar{n} := \begin{cases} \sqrt{\det Y}, & \text{for } \tau = 1 \\ \frac{\sqrt{\det Y}}{2|1-\tau|} - \frac{1}{2}, & \text{for } \tau \neq 1 \end{cases} \\
g(x) := (\frac{x + 1}{2}) \log \left( \frac{x + 1}{2} \right) - (\frac{x - 1}{2}) \log \left( \frac{x - 1}{2} \right) \]
Finite quantum capacity when $\mu \nu < \frac{4}{9(T+T^{-1}-2)}$
Connection with Continuous Variable Quantum Teleportation

EPR pair:
\[ q_1 + q_2 = 0 \]
\[ p_1 - p_2 = 0 \]

Quantum State Transfer

- Transfer quantum state
- Adaptive protocol

Continuous Variable Teleportation

- Transfer quantum state
- Adaptive protocol

**Similarity:**
- Transfer quantum state
- Adaptive protocol

**Distinctions:**
- One ancilla mode
- Only need prior squeezing
- Works for **generic symplectic scattering matrix**
- Two ancilla mode
- Need prior entanglement
- Require 50:50 beam splitter

Extension to Discrete Variable System

Finite phase space $V = \mathbb{Z}_n \times \mathbb{Z}_n$

Phase space $V = \mathbb{R} \times \mathbb{R}$

**Clifford group** $C$

affSp: Symplectic group “+” displacement

**Finite Heisenberg-Weyl group** $H_n$

Heisenberg-Weyl group $H$

**Finite symplectic group** $\text{Sp}(2, \mathbb{F}_n)$

Symplectic group $\text{Sp}(2, \mathbb{R})$

$\text{Sp}(2, \mathbb{F}_n) \cong C/H$

$\text{Sp}(2, \mathbb{R}) \cong \text{affSp}/H$
Quantum Transduction with *Adaptive Control*

**Practical Implication:**
- Enhanced quantum transduction by *adaptive control*
- Pathway to go beyond impedance matching condition
- Generalized theoretical framework (including various practical imperfections)

Zhang, Zou, L.J., PRL 120, 020502 (2018)
(Related to experiment at JILA - arXiv:1712.06535)

Quantum State Transfer (QST) via *Thermal Waveguide*

**Practical Implication:**
- QST via high-Q *thermal* wave guide
- Compatible with bosonic QEC (e.g., binomial code)
- Remote (~100m) mw thermal @4K (Nb) SC waveguide (connecting different dilution fridges across buildings)

Xiang, Zhang, L.J., Rabl, PRX 7, 011035 (2017)
(Related to experiment at Yale - arXiv:1712.05832)
Quantum State Transfer (QST) Protocols in Quantum Networks

Key Idea: Unidirectional QST from atom 1 to atom 2 via shaped photon wave packet

Requirement:
- Zero temperature wave guide \( N_{ch} \ll 1 \)

Questions:
- Can we achieve QST via thermal wave guide \( N_{ch} \gg 1 \)?
- Robust against both excitation loss & thermal noise?

Cirac, Zoller, Kimble, Mabuchi, PRL (1997)
Modified QST Protocols in Quantum Networks

**Key Idea:** Encode into cavity mode before pitch & catch of shaped wave-packet through the waveguide

**Practical Implication:**
- Protocol for quantum state transfer (QST) via high-Q thermal wave guide
- Compatible with bosonic QEC (e.g., binomial code)
- Feasible remote (~100m) mw thermal @4K (Nb) superconducting waveguide (connecting different dilution fridges across buildings)
Reliable QST Protocol via (high-Q) *Thermal* Waveguide

**Practical Implication:**
- QST via high-Q thermal wave guide
- Compatible with bosonic QEC
- Quantum links @4K between fridges

**Recent experimental achievements**
- Pitch & catch with high efficiency
- Entanglement generation
  (Axline, et al., arXiv:1712.05832)
## Comparison between QST Protocols

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Standard CZKM Protocol</th>
<th>Modified Robust Protocol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qubit → Waveguide → Qubit</td>
<td>Oscillator → Waveguide → Oscillator</td>
<td></td>
</tr>
<tr>
<td>Similarity</td>
<td>Shaped pulse for pitch and catch</td>
<td></td>
</tr>
<tr>
<td>Pitch/Catch</td>
<td>Stimulated Raman (assisted by cavity)</td>
<td>Q-switch or Mode conversion between bosonic modes (assisted by Josephson junction)</td>
</tr>
<tr>
<td>Effective Coupling</td>
<td>Nonlinear (spin + boson coupling)</td>
<td>Linear (Effectively, bilinear coupling among bosonic modes)</td>
</tr>
<tr>
<td>Transfer capability</td>
<td>Maximum one excitation</td>
<td>Many excitations</td>
</tr>
<tr>
<td>Thermal waveguide</td>
<td>$N_{th} \ll 1$</td>
<td>$N_{th} \geq 1$</td>
</tr>
<tr>
<td>Bosonic QEC</td>
<td>Limited*</td>
<td>Compatible</td>
</tr>
<tr>
<td>Correct Loss &amp; Gain Noise</td>
<td>Not for single attempt</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Summary

Quantum Transduction with *Adaptive Control*

- Enhanced quantum transduction by *adaptive control*
- Pathway to go beyond impedance matching condition
- Generalized theoretical framework (including various practical imperfections)

*Practical Implication:*

Zhang, Zou, L.J., PRL 120, 020502 (2018)
(Related to experiment at JILA - arXiv:1712.06535)

Quantum State Transfer (QST) via *Thermal Waveguide*

- QST via *high-Q thermal* wave guide
- Compatible with bosonic QEC (e.g., binomial code)
- Remote (~100m) mw thermal @4K (Nb) SC waveguide (connecting different dilution fridges across buildings)

*Practical Implication:*

Xiang, Zhang, L.J., Rabl, PRX 7, 011035 (2017)
(Related to experiment at Yale - arXiv:1712.05832)

Li Sang