Tying it all together

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Georges Friedel had his nematic threads, Paul Dirac had his monopole, and Alexei Abrikosov had his flux line. Very different systems and very different scientists, but deep down they all captured the same idea: Integers cannot vary continuously. The number of times a closed curve winds around a point under smooth evolution in time and space—both position and momentum space—must be invariant.

Integers arise in physics through degrees of freedom that take values on circles, tori, annuli, and any number of manifolds that are not simply connected—in other words, manifolds with holes or handles around which a path can wind. The mathematical way to study the paths falls under the field of topology. Some say that in topol-
ogy, a coffee cup and a doughnut are the same because one can be distorted into the other. But that picturesque trope can be stated from the point of view of the surface as well. A doughnut has closed paths that go around its hole just as a coffee cup has closed paths that go around its handle. If we lived on a doughnut, we could go around the hole some integer number of times and return to our starting point. Moreover, we cannot change the net number of times we go around (going around one way is the negative of going around the other way). As the doughnut deforms into a coffee cup, the winding number does not change—it cannot unless we tear the doughnut and reconnect it.

When everything changes smoothly, the winding number must change smoothly as well. But how can an integer change smoothly? It cannot. That is the essence of topology. Because an integer cannot relax smoothly, it must remain constant, even as the surface is smoothly distorted. The winding is a feature not just of doughnut handles but of degrees of freedom in ordered media.

For instance, in the two-dimensional model of two-component unit vectors—the XY model—the angle that each spin makes with the x-axis is defined only up to 360°. Thus the angle can wind as the path moves around a particular point, a defect. More generally, the Nambu–Goldstone modes that map out the degeneracies of a broken symmetry state are coordinates for the space of equivalent ground states, the ground-state manifold (think of the wine-bottle potential).

Continuous distortion of one function into another goes by the mathematical term homotopy. In 1958 Charles Frank applied homotopy theory to the phase changes of liquid crystals. In the early 1970s, Maurice Kléman and Gérard Toulouse, and Dominik Rogula abstracted Frank’s innovation and showed how homotopy theory could be extended from spheres and other simple surfaces to ground-state manifolds. The result was a coherent framework for studying defects, not just in liquid crystals but also in superconductors, superfluids, and other systems. Those singularities are seen as topological defects.

In 1979 N. David Mermin penned a classic, pedagogical article in Reviews of Modern Physics (RMP) on topological defects from which many practitioners learned homotopy theory. Together with Louis Michel’s RMP contribution and Sidney Coleman’s Erice lectures, a new understanding emerged. The application of the powerful body of modern mathematical results—in this case algebraic topology and homotopy theory—cleared up in one fell swoop what had been a mélange of isolated results, cobbled together by intuitive but unsystematic, error-prone methods.

Viewed from that perspective, the flux line and the nematic defects are all the same; they are characterized by the first homotopy group of a circle. But the mathematical framework extends to hedgehogs in ferromagnets and nematics through the second homotopy group of the sphere. The seminal work of Tony Skyrme on meson theory takes advantage of the third homotopy group of the sphere. The Dirac monopole, also characterized by the first homotopy group of a circle, is the first application of those ideas to Yang–Mills fields in general, themselves characterized by their associated Lie groups. The unified language led to unanticipated insights into quantum field theory, high-energy physics, and condensed matter.

Indeed, although crystals and other ordered media offer a natural arena for observing topological defects, defects are not always visible. As Michael Berry observed, sometimes they are in more abstract configuration spaces. For instance, when viewed appropriately, a winding and its associated “defects” let researchers understand anomalies in quantum field theory as obstructions to defining a basis in Hilbert space. Ideas from homotopy theory lend themselves to quantum computing, topological materials, and entangled polymer loops. Most recently, they illuminated particle-vortex duality, which transmutes bosonic and fermionic statistics.