Composite quantum particles at the interface with general relativity

Magdalena Zych
Stockholm University

ICOLS, 29 June 2023

Knut and Alice Wallenberg Foundation

Composite quantum systems in curved space time – foundations and applications
Wallenberg Academy Fellow

Since February 2023 @ Stockholm Uni

Looking for postdoc!

Current Members	Past members
Germain Tobar	Dr Carolyn Wood
Evan Gale	Dr Laura Henderson
Dr Rodrigo Bruni	Dr Joshua Foo*
	Rebecca Haustein
	Ahmad Mohit

* co-sup with Tim Ralph

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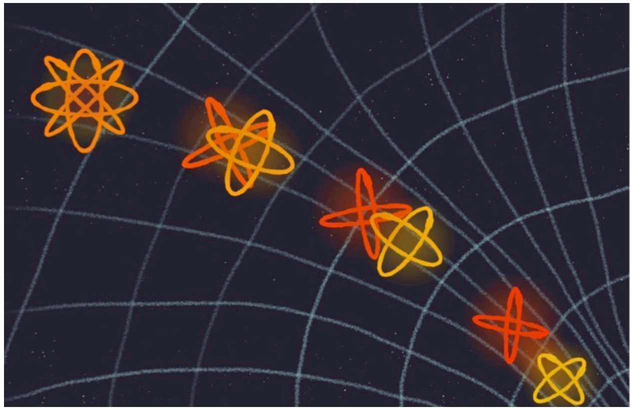
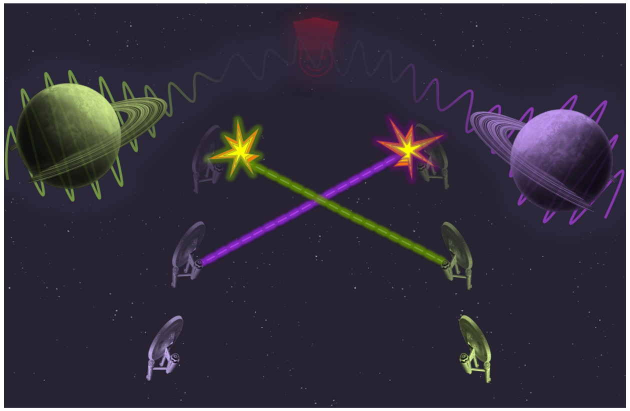
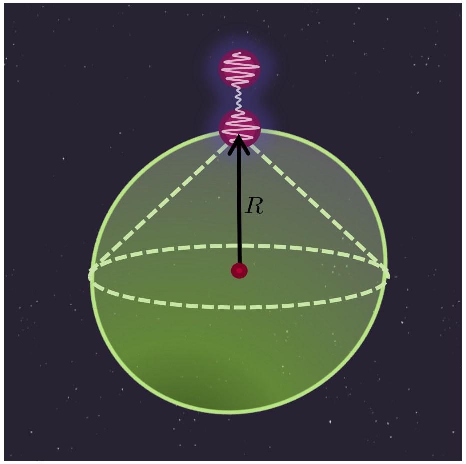
Theory and tests of QM+GR



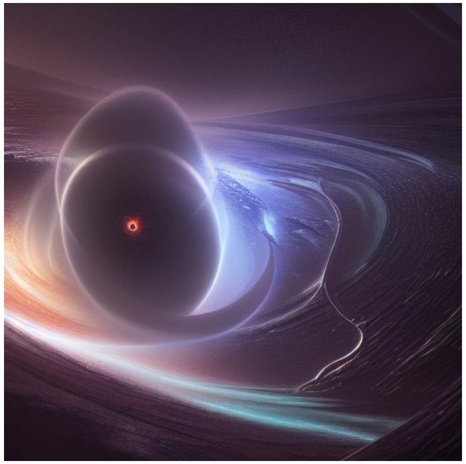
Quantum causal order



Tests of alternative theories
(collapse models, classical-channel gravity)



QFT in classical & quantum curved
spacetimes, link to quantum thermo



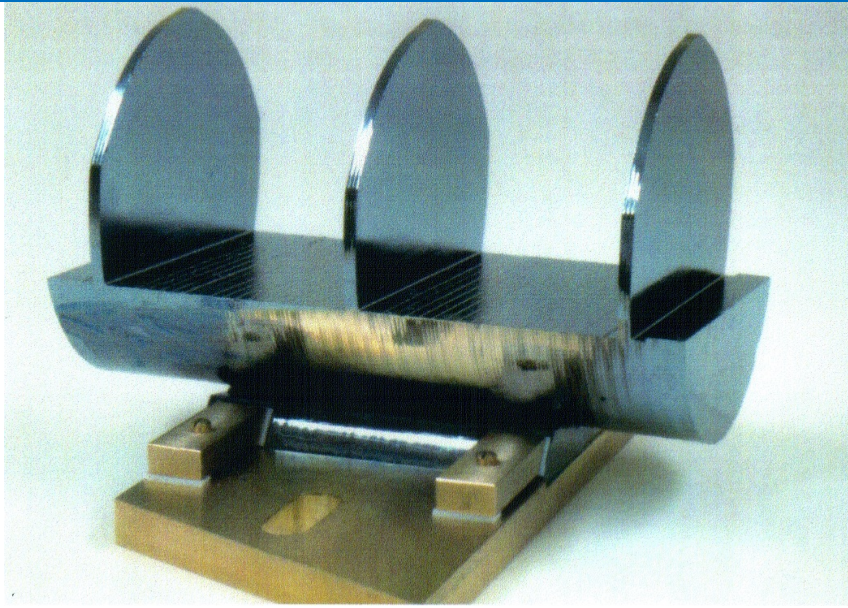
Gravitational phase shift (COW)

gravitational potential

$$H = H_{int} + \frac{p^2}{2m} + mgh$$

$$\frac{1}{\sqrt{2}}\left(e^{-i\Delta\phi}|\gamma_1\rangle + |\gamma_2\rangle\right)$$

$$\Delta\phi = mght/\hbar$$



detection probability: $P_{\pm} = \frac{1}{2} \pm \frac{1}{2}\cos(\Delta\phi)$

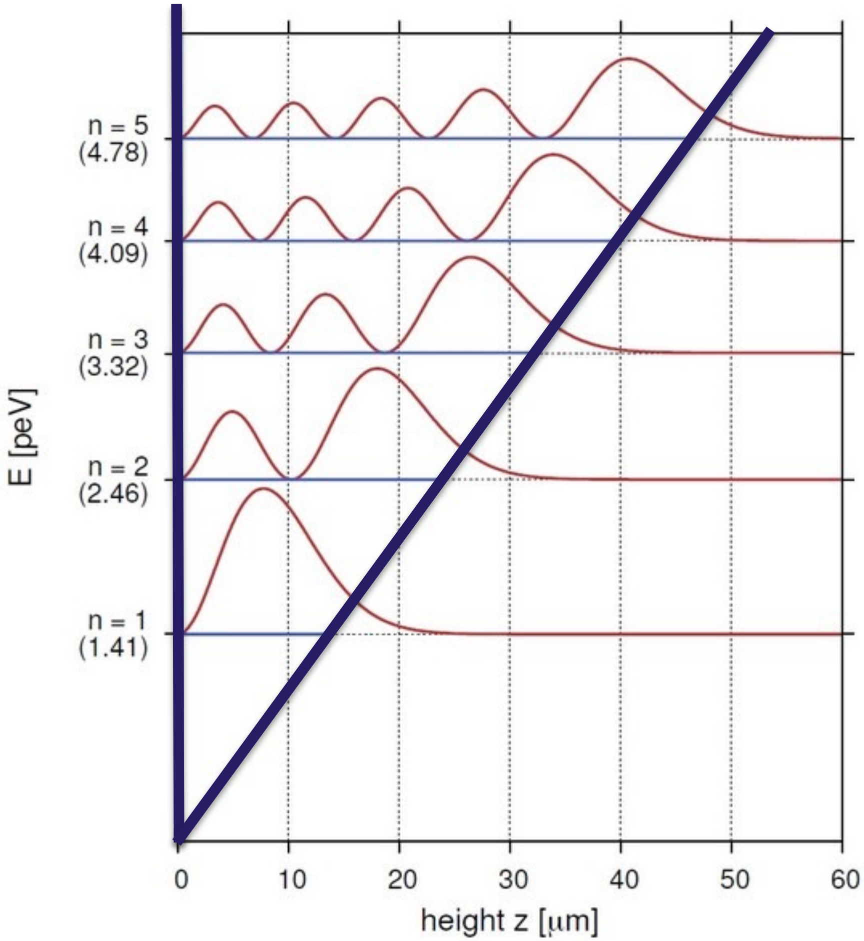
- Colella, Overhauser, Werner (COW) *PRL* **34**, 1472– 1474 (1975)
- Müller, Peters, Chu, *Nature* **463**, 926-929 (2010)
- Rosi et al, *Nature* **510** 518 (2014)
- Asenbaum et al, *PRL* **118**, 123602 (2017) → first time beyond const. gravitational acceleration!

Gravitationally trapped neutrons (q-Bounce)

gravitational potential

$$H = H_{int} + \frac{p^2}{2m} + mgh$$

potential well → quantised bound states



Nesvizhevsky, Boerner, Petukhov

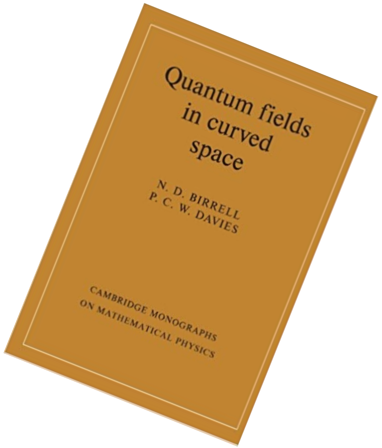
source: cerncourier.com

Nesvizhevsky, et al *Nature* 415 6869 (2002)

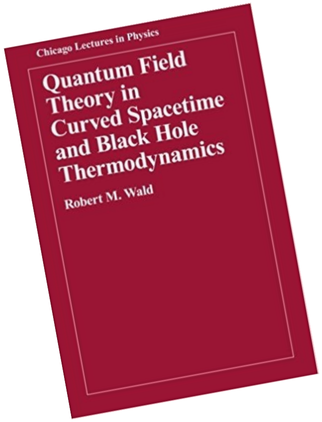
Abele et al, *Nature Phys.* 7 468 (2011)

Theory at the fundamental level

N.D. Birrel and P.C.W Davies
"Quantum fields in curved space"
Cambridge Monographs on Mathematical Physics (1984)



R.M. Wald
"Quantum Field Theory in Curved Spacetime
and Black Hole Thermodynamics"
Chicago Lectures in Physics (1994)

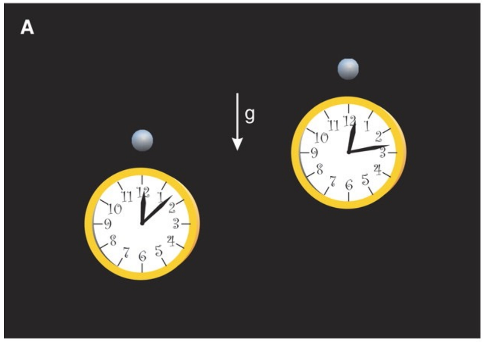


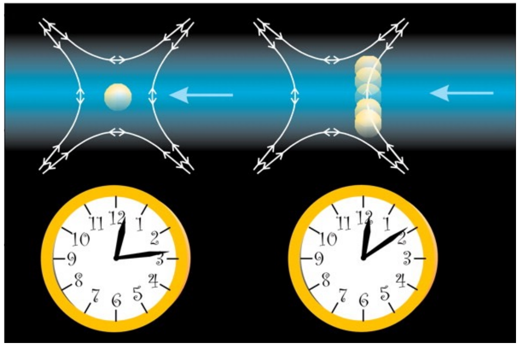
- Directly describes: elementary particles
- New effects: high energies, strong gravity, …

Strategy here: Include internal DOFs

Time dilation or redshift of internal states can be measured very precisely!

Chou at al *Science* **329** 1630–1633 (2010)





Δh = -17, +33cm

ΔV = 10÷35 m/s

Composite particles in GR

N-interacting particles

MZ, Rudnicki, Pikovski, PRD 99, 104029 (2019)
Sonnleitner, Barnett, PRA 98, 042106 (2018)
Schwartz, Giulini, Class. Quant. Grav. 36, 095016 (2019)
Schwartz, Giulini, PRA 100, 052116 (2019)

$$S = -\sum \left(m_i c^2 \int d\tau + q_i \int A_\mu (x_i) dx_i^\mu \right)$$

QFT in curved spacetime

MZ *Quantum Systems Under Gravitational Time Dilation* (Springer, 2017),
MZ, Brukner, Nat. Phys 14, 1027–1031 (2018)
Roura PRX 10 (2), 021014 (2020)

$$S = \int d^4x \sqrt{-g} \left(\sum_J g^{\mu\nu} \partial_\mu \varphi_J \partial_\nu \varphi_J + \sum_{J,K} M^2_{JK} c^2 \varphi_J \varphi_K \right)$$

Give rise to the same Hamiltonian at low energies

Composite particles in GR

Pointlike particle in static symmetric spacetime

Hamiltonian $$\hat{H} = \sqrt{-g_{00}\left(c^2 \hat{P}_i \hat{P}^i + \boxed{\hat{M}^2 c^4}\right)}$$ rest energy

@ low energies $$\hat{M}c^2 + \frac{\hat{p}^2}{2\hat{M}} + \hat{M}\Phi(\hat{x})$$

$$\hat{M} = \boxed{m}\hat{\mathbb{1}}_{int} + \frac{\boxed{\hat{H}_{int}}}{c^2}$$ dynamical part → internal Hamiltonian

static part → effective mass parameter

grav. potential

$$H = H_{cm} + H_{int}\left(1 + \frac{\boxed{\Phi(x)}}{c^2} - \frac{p^2}{2m^2c^2}\right)$$

$$H_{cm} = mc^2 + \frac{p^2}{2m} + m\Phi(x)$$

Terms p^4/m^3c^2 or $p^2\Phi/mc^2$ are higher order for slow/cold particles & Earth's gravity

Composite particles in GR

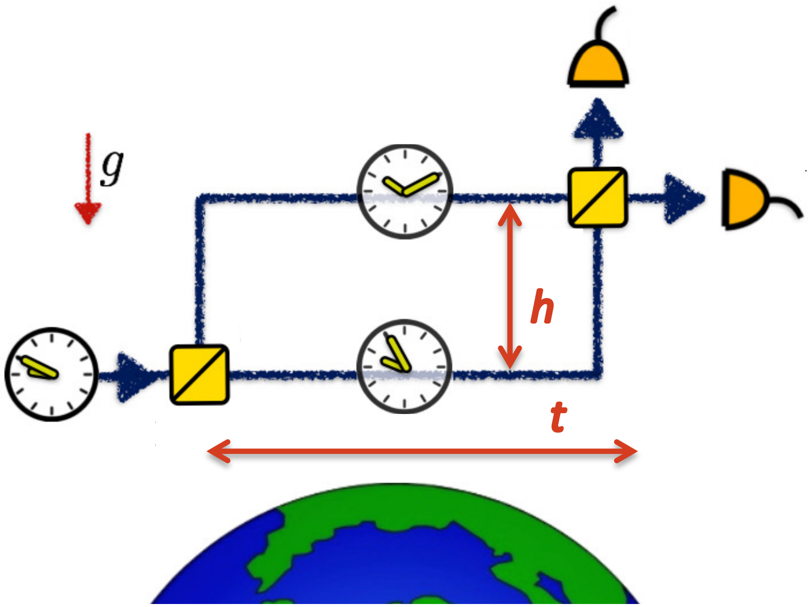
So far effectively classical

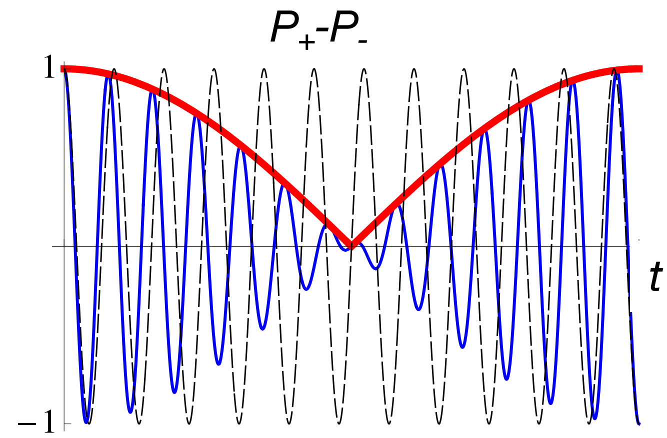
$$H = H_{cm} + \hat{H}_{int}\left(1 + \frac{\Phi(x)}{c^2} - \frac{p^2}{2m^2c^2}\right)$$

Gravitational time dilation

Special relativistic time dilation

Composite particles as clocks





- - Newtonian/GR with no “clock”

– GR with “clock”

– **Visibility**

time dilation $\Delta\tau \approx \frac{ght}{c^2}$

two-level clock, frequency ω

$$P_{\pm} = \frac{1}{2} \pm \frac{1}{2}\left|\langle\tau_1|\tau_2\rangle\right|\cos(\Delta\phi)$$

$$\left|\cos\left(\frac{\omega g h t}{2c^2}\right)\right|$$

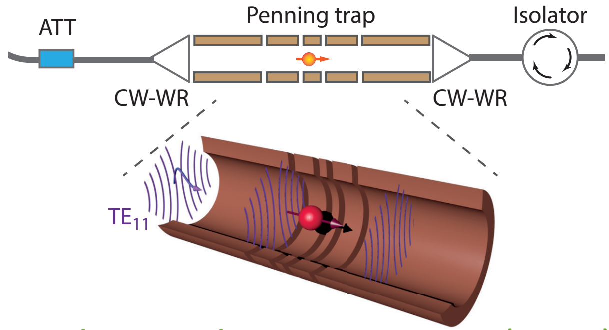
To observe the effect: *ω* = *10*^{15} Hz → ht = 10 m • s

Interference of “clocks”: can we do it?

Atoms: VLBI @ Leibniz University Hannover

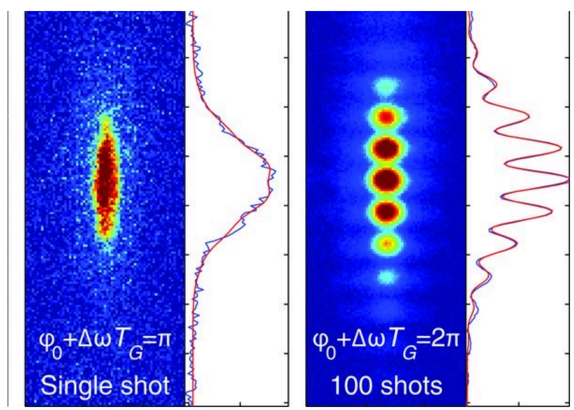
Loriani et al *Science Advances* **5**: eaax8966, (2019)
Roura *PRX* **10** (2), 021014 (2020)
Roura, Schubert, Schlippert, Rasel *PRD* **104**, 084001 (2021)

Electrons: Penning trap Saarbrücken/Jülich

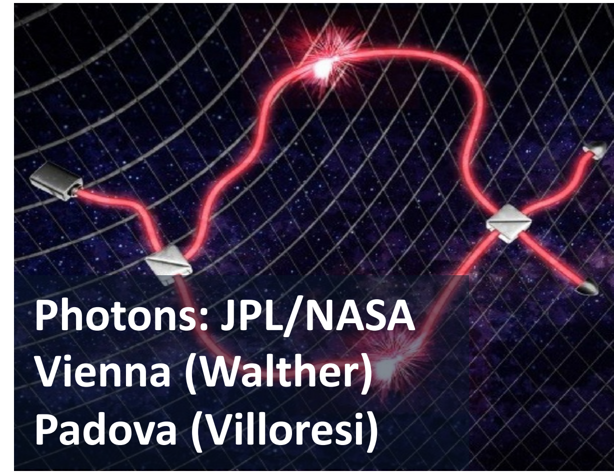


Bushev et al *NJP* **18** 093050 (2016)
Editorial by JD Franson, NJP ***18*** *101001 (2016)*

Analog time dilation in spin precession –> magnetic field



½ “clock” period | *full “clock” period*



MZ et al *CQG*. **29**, 224010 (2012)
Brodutch et al *PRD* **91**, 064041 (2015)
Hillweg et al *NJP* **19** 033028 (2017)
Beig et al *CQG*. **35** 244001 (2018)
Calderaro et al QST **4** 015012 (2019)
Terno et al *PRD* **101** 104052 (2020)


In Science M
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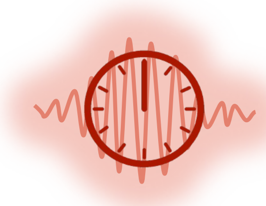
In Science Magazine
PERSPECTIVE
QUANTUM MECHANICS
Interference of atomic clocks
Markus Arndt, Christian Brand
Science 11 September 2015: 1168–1169.

Composite quantum particles

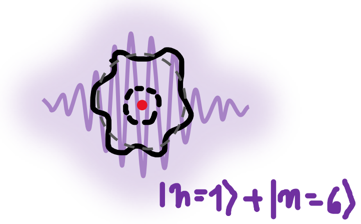
- Evolving internal states → **Quantum clocks**

  MZ, Costa, Pikovski, Brukner Nature Commun. 2, 505 (2011)
  Bushev, Cole, Sholokhov, Kukharczyk, MZ , NJP 18 093050 (2016)
  Loriani et al Science Advances 5: eaax8966, (2019)



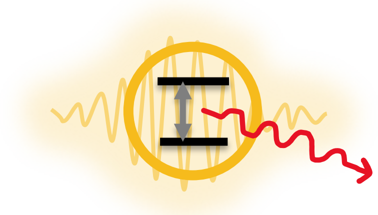
- Internal energy quantized → **Quantum test masses**

  MZ, Brukner, Nat Phys 14 1027–1031 (2018)
  Rosi et al Nat. Comm. 8, 15529 (2017)
  Tobar, Heine, Costa, MZ PRA PRA 106 (5), (2022)



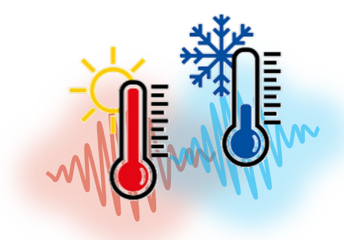
- Coupled to scalar field → **model of atom-light interaction**

  Foo, Onoe, MZ PRD 102, 085013 (2020)
  Henderson et al. PRL 25 (13), 131602 (2020)
  Foo, Arabaci, MZ, Mann PRL 129 (18), 181301 (2022)
  Gale, MZ PRD (2023)

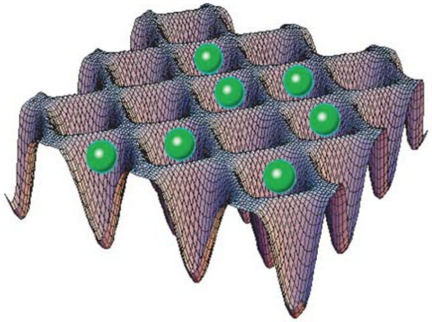


- Coupled to termal bath(s) → **quantum thermometers**

  Foo, Mann, Zych CQG 38 (11), 115010 (2021)
  Wood, Verma, Costa, Zych arXiv:2112.07860 (2021)



Quantized mass-energy in trapped particles



Electrons:

Bushev et al *NJP* **18** 093050 (2016)

→ *Editorial by JD Franson, NJP* ***18*** *101001 (2016)*

Neutrons:

Tobar, Heine, Costa, MZ PRA PRA 106 (5), (2022)

$$\hat{H} = \hat{M}c^2 + \frac{\hat{p}^2}{2\hat{M}} + \hat{M}\Phi(\hat{x}) + V(\hat{x})$$

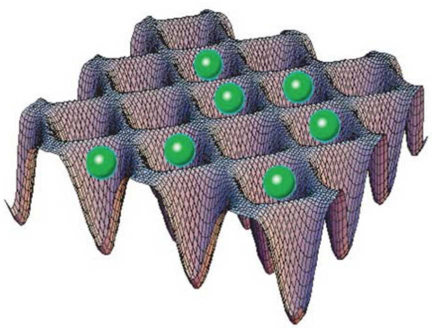
Atomic clocks:

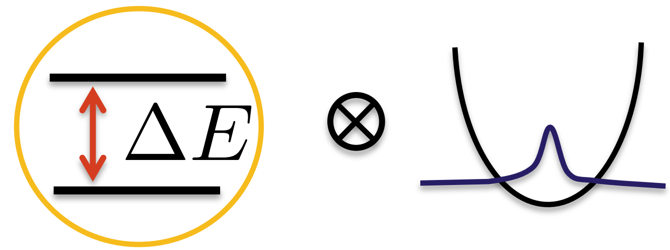
Yudin, Taichenachev *Laser Phys. Lett*. **15** 035703 (2018)

Haustein, Milburn, MZ arXiv:1906.03980 (2019)

Martinez-Lahuerta et al *PRA* **106**, 032803 (2022)

Quantized mass-energy in trapped particles





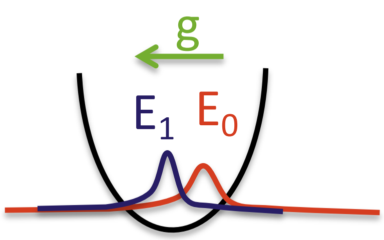
$$\hbar\omega = \Delta E$$

$$\hat{H} = \hat{M}c^2 + \frac{\hat{p}^2}{2\hat{M}} + \hat{M}g\hat{x} + \frac{1}{2}k\hat{x}^2$$

$$\hat{a}_M = \sqrt{\frac{\hat{M}\hat{\omega}}{2\hbar}}(\hat{x}_M + \frac{i\,\hat{p}}{\hat{M}\hat{\omega}}) \qquad \hat{\omega} = \sqrt{\frac{k}{\hat{M}}} \qquad \hat{x} + \frac{g}{\hat{\omega}^2}$$

Quantized mass-energy in trapped particles

correlated internal & CM modes



$$\hat{a}_i = \cosh(r_i)\hat{a}_0 - \sinh(r_i)\hat{a}_0^\dagger + \alpha_{gi}$$

$$\frac{1}{2}\left(\frac{M_0}{M_i}\right)^{\frac{1}{4}} \pm \frac{1}{2}\left(\frac{M_i}{M_0}\right)^{\frac{1}{4}} \qquad \frac{g\Delta M_i}{\sqrt{2\hbar M_i \omega_i^3}}$$

Energy difference between $|E_0, n_0\rangle$, $|E_i, n_i\rangle$

$$E_i\left(1 - \frac{g^2}{\omega_0^2 c^2} - \frac{\hbar\omega_0}{2M_0c^2}\left(n + \frac{1}{2}\right)\right)$$

time dilation shift $M_0=10^{-26}$kg, ω_0=1MHz –> 10^{-20} – 10^{-19}

minimum frequency shift *10* $^{-22}$ at ω_0 = 4kHz $\delta_{min} = -\frac{3}{2\sqrt[3]{2}}\left(\frac{\hbar g(n+\frac{1}{2})}{c^3 M_0}\right)^{\frac{2}{3}}$

Bonus 1: Gravitational mass of a composite system

the retardation terms must be removed from (4·1). The mass is then

$$M = \Sigma_i m_i + \tfrac{3}{2}\Sigma_i m_i v_i^2 - \Sigma_i \Sigma_j \frac{m_i m_j}{\Delta_{ij}}, \qquad (4\cdot5)$$

and the formal difficulty, caused by the divergence of (4·3) as $r \to \infty$, is avoided.

Let T, V be the kinetic and potential energies of the system. We have

$$T = \tfrac{1}{2}\Sigma_i m_i v_i^2, \quad V = -\tfrac{1}{2}\Sigma_i \Sigma_j \frac{m_i m_j}{\Delta_{ij}}.$$

Hence (4·5) becomes $\quad M = \Sigma m_i + 3T + 2V. \qquad (4\cdot6)$

Eddington, Clark, *Proc. R. Soc. Lond. A*166, 465 (1938)

$$m_G = \sum m_i + \frac{1}{c^2}(R + 3T + 2V)$$

→ Universal violation of EP

→ EP holds only on average (virial theorem)

K. Nordtvedt, *Int. J. Theor. Phys.* 3, 133-139 (1970)

E. Fishbach et al., *Phys. Rev. D.* 23, 2157-2180 (1981)

S. Carlip, *Am. J. Phys.* 66, 409-413 (1998)

A. G. Lebed, *Cent. Eur. J. Phys.* 11, 969–976 (2013)

MZ, Rudnicki, Pikovski PRD 99, 104029 (2019)

$$V_{local} = \sum_{i,j=1}^{N} \frac{q_i q_j}{r_{ij}^{local}} = V\left(1 + \frac{\phi(r)}{c^2}\right)$$

$$T_{local} = \sum_{i}^{N} \frac{1}{2} m_i v_i^{local\,2} = T\left(1 + 2\frac{\phi(r)}{c^2}\right)$$

$$m_G = \sum m_i + \frac{1}{c^2}(R + T_{local} + V_{local})$$

→ Same expression in different coordinates

→ T and V included *redshifted* quantities

→ Correct description: different coordinates for COM and for the internal DOF

Bonus 2: semiclassical states?

Gaussian = semi classical in phase space $\Delta x \Delta p \geq \hbar/2$

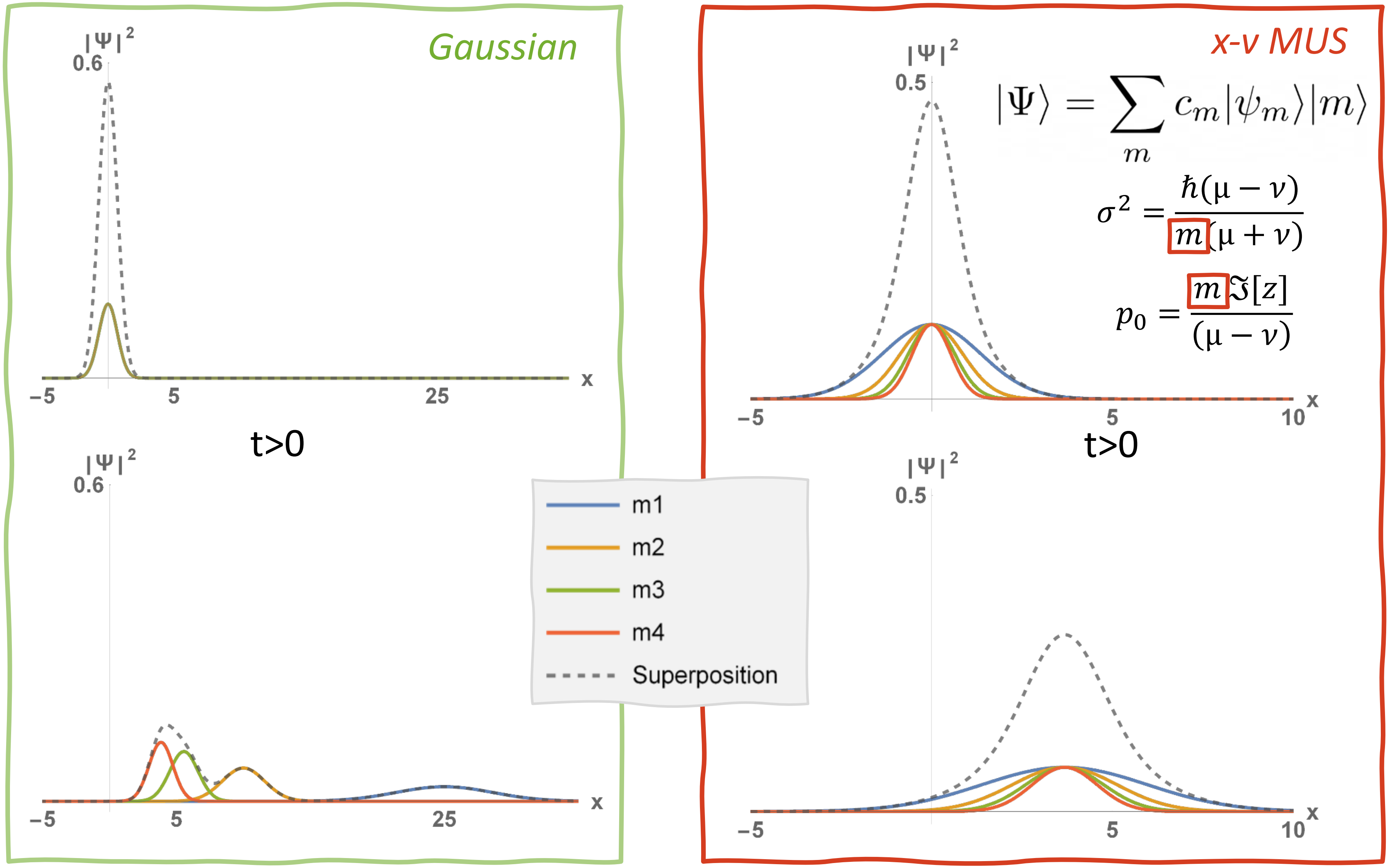
Semi classical in spacetime → minimize uncertainty for (x,v)

S-R Uncertainty $(\Delta x)^2(\Delta v)^2 - (\Delta xv)^2 \geq \frac{1}{4}\left|\langle[\hat{x},\hat{v}]\rangle\right|^2$

Velocity operator: $\hat{v} := -\frac{i}{\hbar}[\hat{x},\hat{H}] = \frac{\hat{p}c^2}{\sqrt{\hat{p}^2c^2+\hat{M}^2c^4}}$

Wood, MZ *Phys. Rev. Research*, 3, 013049 (2021)

x-v Minimum Uncertainty States – low energies



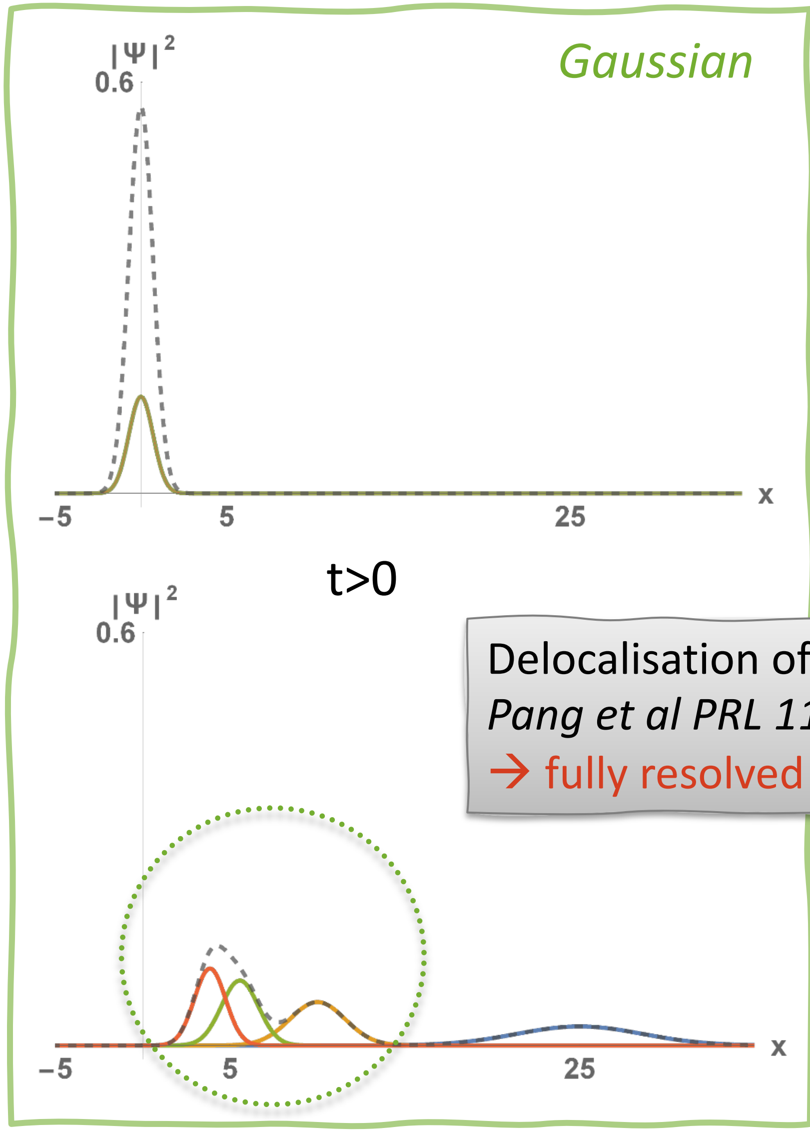
$$|\Psi\rangle = \sum_m c_m |\psi_m\rangle |m\rangle$$

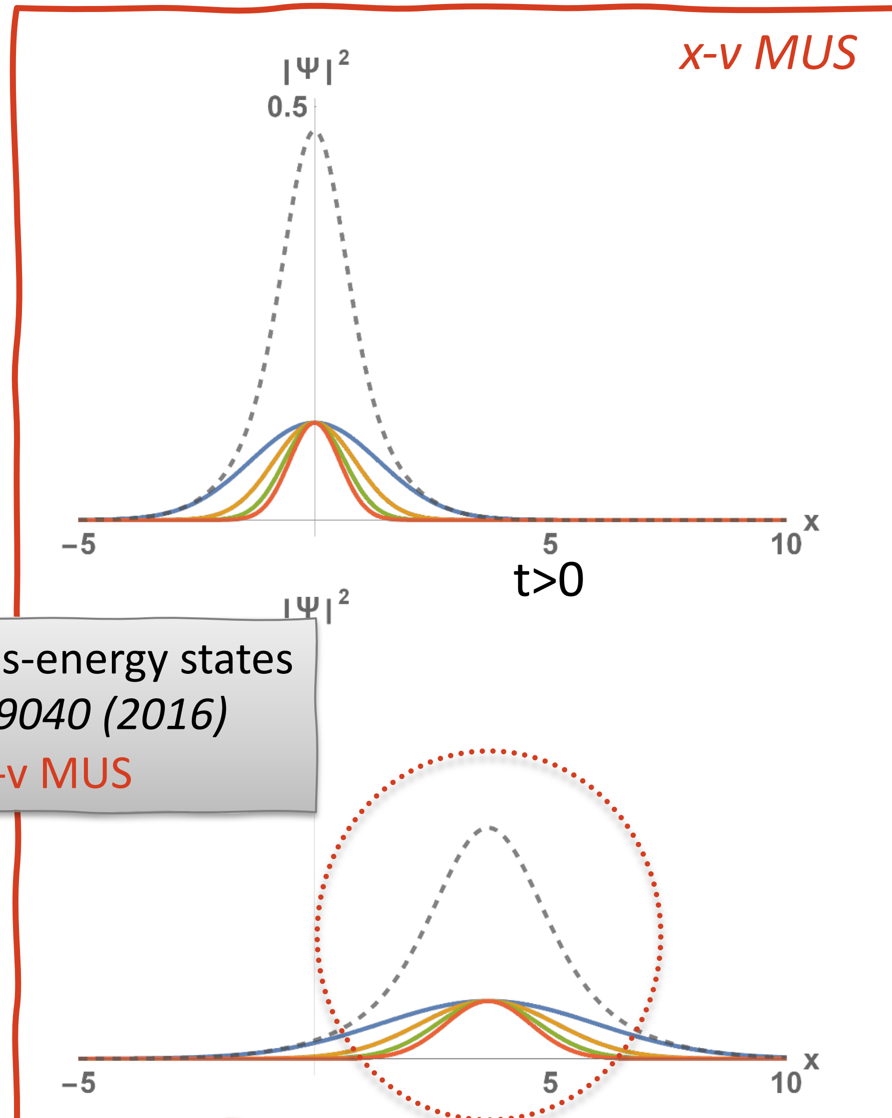
$$\sigma^2 = \frac{\hbar(\mu - \nu)}{m(\mu + \nu)}$$

$$p_0 = \frac{m\,\Im[z]}{(\mu - \nu)}$$

Wood, MZ *Phys. Rev. Research*, 3, 013049 (2021)

x-v Minimum Uncertainty States – low energies





Delocalisation of mass-energy states
Pang et al PRL 117, 09040 (2016)
→ fully resolved by x-v MUS

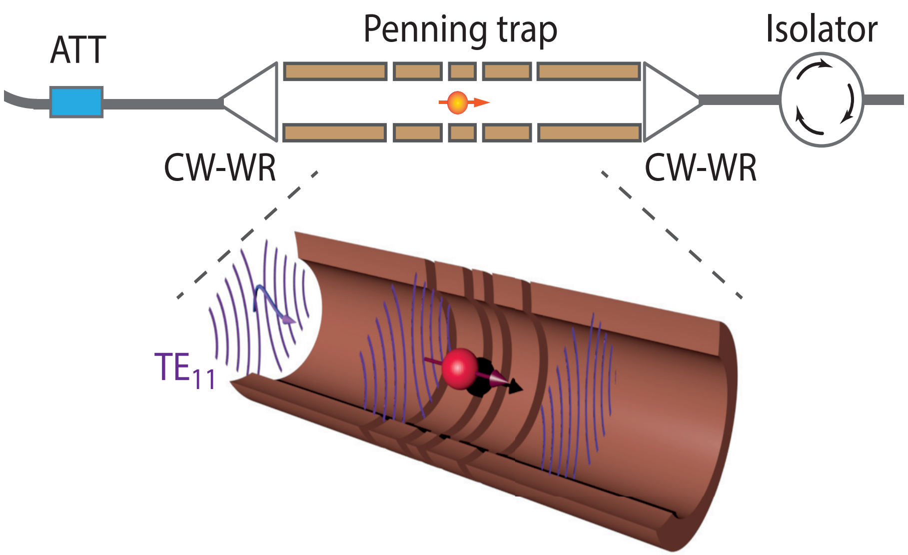
Delocalisation of mass-states of neutrinos="oscillation" length

Summary

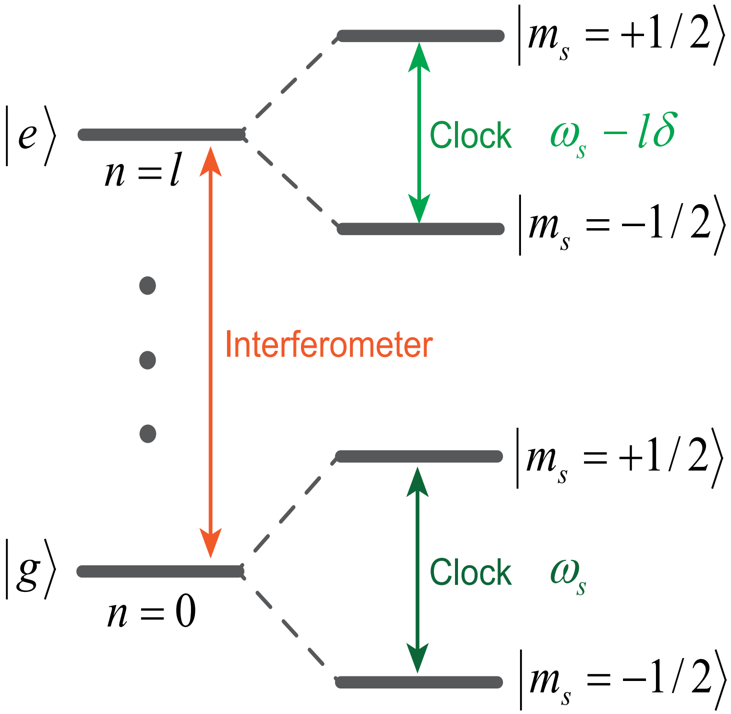
- Composite quantum particles = ideal quantum clocks, masses, detectors, thermometers
- Relativity → interaction between CoM and internal DoF
- For any potential – CoM energies & states depend on internal DoF
- Resulting effects relevant in high-precision experiments

Thank You

SR effects only: electron in a Penning trap



harmonic trap states = paths
spin = clock



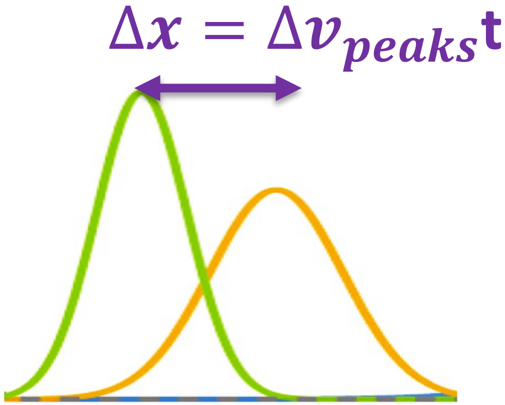
"Single electron relativistic clocks interferometer" ArXiv 1604.06217

P. Bushev, J. H. Cole, D. Sholokhov, N. Kukharchyk, M.Z.

MUS properties: longitudinal coherence

Gaussian state m_g to m_e separation:

$$\Delta v_{peaks} = v_g - v_e \approx v_g \left(\frac{\hbar\omega}{m_g c^2}\right)$$



Wood, MZ *Phys. Rev. Research*, 3, 013049 (2021)

Example: Sr atoms

$m_g = 10^{-25}$kg; $\omega = 10^{15}$Hz [1], T~800K [2]

$$v_g = \sqrt{\frac{2k_B T}{m_g}}$$

➔ $\Delta v_{peaks} = 10^{-9}$m/s

➔ Sr deBroglie wavelength: $\sim 10^{-12}$m

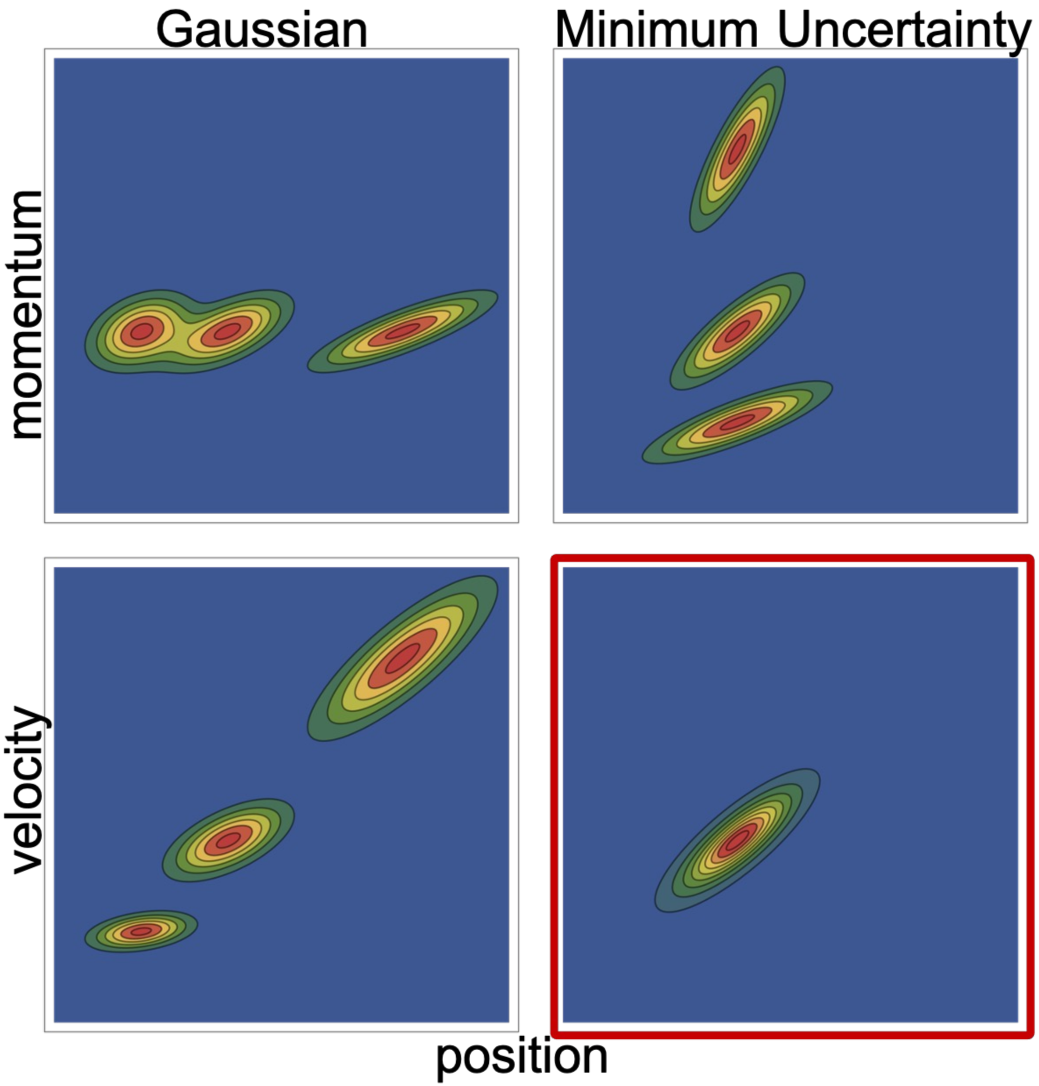
Longitudinal coherence suppressed after one millisecond

[1] Bober, et al. (2015) *Meas Sci Technol*, **26**, 075201

[2] Poli, et al. (2005) *PRA*, **71**, 061403

MUS may be relevant for tests of QM in macroscopic limit

MUS properties: covariance



Wood, MZ *Phys. Rev. Research*, 3, 013049 (2021)

$$\psi_{MUS}(x,t) = \frac{e^{-\left[\frac{m\Omega}{2\hbar}\frac{e^{-2r}(x+vt)^2}{(1+e^{-4r}t^2\Omega^2)}+\frac{r}{2}+\frac{imc^2t}{\hbar}\mathcal{F}(\Omega,r,v,t)\right]}}{\sqrt[4]{\frac{\pi\hbar}{m\Omega}}\sqrt{1+ie^{-2r}t\Omega}}$$

For highly localised state (large r)

$$\approx 1+\frac{vx}{c^2t}+\frac{v^2}{2c^2}$$

$$\frac{t+\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lorentz transformation to $O(1/c^2)$

MUS are covariant – the correct time dilation factor!

Is mass delocalization = time-dilation decoherence?

Wood, MZ *Phys. Rev. Research*, 3, 013049 (2021)

Two separately verifiable effects:

→ from time dilation

→ from CoM delocalisation