1. Problem 3.5.14 (A&W)(10 points)

2. Problem 3.5.15 (A&W)(10 points)

3. Find the normal modes of an equilateral triangle molecule (10 points)

4. Consider the $2 \times 2$ matrix define by

$$U = \frac{a_0 + i \sigma \cdot a}{a_0 - i \sigma \cdot a}$$  \hspace{1cm} (1)

where $a_0$ is a real number and $a = \{a_1, a_2, a_3\}$ is a three dimensional vector with real components.

(a) Prove that $U$ is unitary and unimodular ($|\det U| = 1$)(3 points)

(b) In general a $2 \times 2$ unitary unimodular matrix represents a rotation in three dimensions. Find the angle of rotation appropriate for $U$ in terms of $a_0, a_1, a_2, a_3$ (7 points)

5. Consider a particle of mass $m$ subject to a one dimensional potential of the form (10 points)

$$U = \begin{cases} 
1/2kx^2 & \text{for } x \geq 0 \\
\infty & \text{for } x < 0 
\end{cases}$$  \hspace{1cm} (2)

(a) What is the ground state energy? What is its corresponding wave function?(2 points)

(b) Solve numerically the Schrödinger equation and confirm your answer above(4 points)

(c) Plot the three lowest eigenmodes using your numerical solution(4 points)