NOTE: Be sure to show your work and explain what you are doing. (Correct answers, for which we cannot follow the work, are worth no credit).

1. You have a function \( y(x) \) tabulated at equally spaced values of the argument \( y_n = y(x_n) \) with \( x_n = x + nh \), show that the linear combination \( \frac{1}{12h}(-y_2 + 8y_1 - 8y_{-1} + y_{-2}) \) yields \( y^{(1)}_0 = \frac{h^4}{30}y^{(5)}_0 + \ldots \). Here \( y^{(n)}_0 \) is the \( n^{th} \) order derivative: \( y^{(n)}_0 = \frac{d^n y}{dx^n} \rvert_{x=x_0} \). Hence this linear combination yields \( y^{(1)}_0 \) if \( \frac{h^4}{30}y^{(5)}_0 \) and higher powers of \( h \) and higher derivatives of \( y(x) \) are negligible (10 points)

2. Develop a series expansion of \( y = \sinh^{-1} x \) (that is \( \sinh y = x \)) in powers of \( x \) by
   (a) inversion of the series \( \sinh y \) (5 points)
   (b) a direct Maclaurin expansion (5 points)

Include terms up to the \( 7^{th} \) order.

3. The Bloch-Gruneissen approximation for the resistance in a monovalent metal is

\[
\rho = C\frac{T^5}{\Theta^6} \int_0^{\Theta/T} \frac{x^5}{(e^x - 1)(1 - e^{-x})} dx
\]

where \( \Theta \) is the Debye temperature characteristic of the metal

(a) For \( T \to \infty \), show that \( \rho \approx \frac{CT^5}{3\Theta^6} \) (5 points)

(b) For \( T \to 0 \), show that \( \rho \approx (5!)\zeta(5)\frac{CT^5}{3\Theta^6} \) with \( \zeta(n) \) the Riemann Zeta function (5 points)

4. Determine the limit to which \( \Pi_{n=2}^{\infty} \left( 1 + \frac{(-1)^n}{n} \right) \) converges (10 points)

5. Evaluate \( \sum_{n=-\infty}^{\infty} \left( \frac{(-1)^n}{n^2 + \pi^2} \right) \) (10 points)