ASTR 5110 Atomic and Molecular Processes Fall 2024. Problem Set 7. Due Wed 23 Oct.

1. Pink and Green Auroral lines of O I

(a) (6 points) Consider a 3-level atom in which the only important processes connecting the three levels are spontaneous radiative decay, and collisional excitation and deexcitation by a Maxwellian distribution of electrons. Show that the steady state populations of the 3 levels are in the ratios

$$n_3 : n_2 : n_1 = R_{12}R_{23}+R_{21}R_{13}+R_{13}R_{23} : R_{31}R_{12}+R_{13}R_{32}+R_{32}R_{12} : R_{23}R_{31}+R_{32}R_{21}+R_{21}R_{31} \ , \tag{1}$$

where

$$R_{ij} = n_e C_{ij} \qquad (i < j) \ , \tag{2a}$$

$$R_{ji} = A_{ji} + n_e C_{ji} \qquad (i < j) \ , \tag{2b}$$

with A_{ji} the Einstein coefficient for spontaneous emission, and C_{ij} and C_{ji} the rate coefficients for collisional excitation and deexcitation.

(b) (6 points) Define the departure coefficient b_{ji} to be the ratio of n_j/n_i to its value n^*_{ji} in thermodynamic equilibrium at temperature T,

$$b_{ji} \equiv \frac{n_j/n_i}{n^*_{ji}} \ , \tag{3}$$

$$n^*_{ji} \equiv \left.\frac{n_j}{n_i}\right|_{\text{TE}} = \frac{g_j}{g_i} e^{-E_{ji}/kT} \ , \tag{4}$$

with $E_{ji} \equiv E_j - E_i$. Show that the departure coefficient b_{32} of the upper 2 levels can be expressed in the form

$$b_{32} = \frac{n_a + n_e}{n_b + n_e} \ , \tag{5}$$

where the critical densities n_a and n_b are

$$n_a \equiv \frac{C_{31}A_{21}}{C_{21}C_{31} + C_{21}C_{32} + C_{31}C_{32}n^*_{32}} \ , \tag{6a}$$

$$n_b \equiv \frac{C_{21}A_{31} + C_{21}A_{32} + C_{31}A_{32}n^*_{32}}{C_{21}C_{31} + C_{21}C_{32} + C_{31}C_{32}n^*_{32}} \ . \tag{6b}$$

Hint: Notice that $b_{32} \to 1$ as n_e becomes large, which says that the relative number densities of the levels tend to relative thermodynamic equilibrium at high electron density n_e, which is as it should be. Detailed balance implies that the ratio of collisional excitation to de-excitation rates is

$$\frac{C_{ij}}{C_{ji}} = n^*_{ji} \ . \tag{7}$$

(c) (6 points) Consider the 3 levels under consideration to be the 3 LS terms (ignore fine structure) of the ground electronic configuration of O I. For definiteness, let the order of energies be $E_3 > E_2 > E_1$. For this case, it turns out that the critical densities n_a and n_b in equation (5) are

$$n_a = 3 \times 10^8 \, \mathrm{m}^{-3} \, , \tag{8a}$$
$$n_b = 1 \times 10^{11} \, \mathrm{m}^{-3} \, , \tag{8b}$$

at electron temperatures in the vicinity of 10^4 K. Plot a graph of b_{32} against n_e.

(d) (6 points) Astronomers express line strengths in a variety of units; at visible wavelengths astronomers commonly express line strengths j_{UL} in terms of photon energy (rather than photon number), so that $j_{UL} \propto n_U A_{UL} E_{UL}$. Write down an expression for the ratio of [O I] emission line strengths,

$$\frac{j(557.7 \, \mathrm{nm})}{j(630.0, 636.4 \, \mathrm{nm})} \, , \tag{9}$$

in terms of b_{32} and electron temperature. Look up the relevant energy levels and A-values of the transitions in the NIST Atomic Spectra Database.

(e) (6 points) Explore on the internet about the pink and green auroral lines. Can you connect your calculations to what you find on the internet?