ASTR 5110 Atomic and Molecular Processes Fall 2024. Problem Set 4. Due Wed 25 Sep.

The first problem is from the notes; the second is not.

35.1. (10 points) Basic application of the Saha equation. To do this, you will need to figure out the single-particle partition function of a free, nonrelativistic, Boltzmann species, which is an analytically doable integral. All the particles here are spin-1/2, therefore fermions, and therefore (being also massive), have 2 spin states each.

35.4. (20 points) (Not from notes). Longer but useful. Write a computer program to compute the populations of the $n = 1$, 2, and 3 levels of neutral H, and of protons p in thermodynamic equilibrium as a function of temperature T, at a given total hydrogen number density $n_{\rm tot} = n_{\rm H} + n_p = n_1 + n_2 + n_3 + n_p$ (including for simplicity only the $n = 1$, 2, and 3 levels of neutral H; apologies that n for level gets the same symbol as n for number density; it is the standard convention in both cases). Assume all species are Boltzmann, and assume overall charge neutrality $n_e = n_p$. Plot your results for some representative densities, and comment on them. Why is $n_{\rm tot} \approx 10^{30}\,{\rm m}^{-3}$ ($\approx 10^{24}\,{\rm cm}^{-3}$) an interesting choice? [The nth energy level of H has degeneracy $2n^2$, and energy $\epsilon_n = -\chi/(n^2)$ relative to the energy of the just ionized ion, where $\chi = 13.6\,{\rm eV}$ is the ionization energy of H. In your calculation, you will find it necessary to solve a certain quadratic equation. There is a numerically stable and a numerically unstable way to solve the quadratic, and you should of course use the stable solution (if you are puzzled by that statement, ask in Math Methods).]