ASTR 5110 Atomic and Molecular Processes Fall 2023. Problem Set 9. Due Wed 8 Nov.

1. Zeeman Splitting of the 12.32 μm Mg I Solar Emission Line

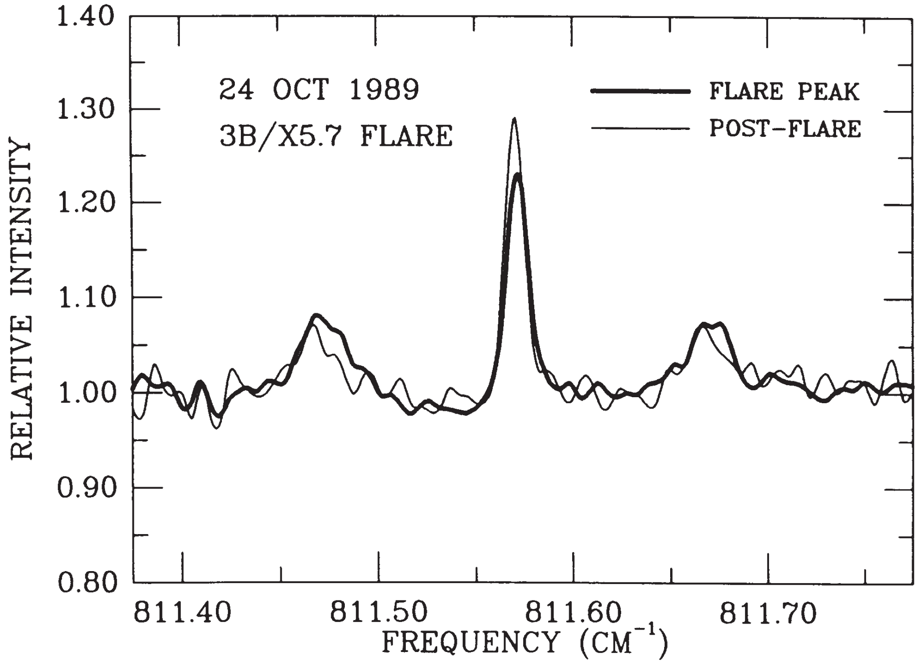

Figure 1: This spectrum of a part of the limb of the Sun, just above its surface, shows the three Zeeman-split components of the Mg I $6h - 7i$ 12.32 μm emission line observed during and shortly after a major solar flare (Deming et al. 1990, ApJ 364, L49). Deming et al. argue that the lines are formed in the upper photosphere (low chromosphere), where these upper levels of Mg I are in LTE (local thermodynamic equilibrium) with the dominant ion of magnesium, Mg II.

(a) The Mg I $6h - 7i$ 12.32 μm spectral line shown lies close in frequency to the corresponding spectral line of atomic hydrogen, H (6 − 7). Use the hydrogenic formula to calculate the wavelength, in μm, and frequency, in cm^{-1}, of the line of atomic hydrogen. Why is the Mg I line close to that of H I? Is the Mg I line higher or lower in frequency than the H line? Why? [Hint: As regards the last part of the question, I'm looking for brief physical arguments, not detailed mathematics. There are three effects, (i) shielding of nucleus, (ii) polarization of the inner electrons by the valence electron, (iii) reduced mass, all of which tend to shift the frequency in the same direction.]

(b) The spectrum appears to show three lines. Why three, not more or less? Illustrate your answer with a sketched (not necessarily comprehensive) Grotrian diagram of the relevant energy levels. [Hint: In this case the Zeeman energy perturbation is much larger than the fine-structure energy perturbation of states, so the relevant formula is (3.14) in the notes on

Spin in Atoms. Which possible transitions are permitted by the selection rules for electric dipole transitions? You should find that, for the $6h$ lower level, the energy levels of the spin-singlet term ^1H (which has $L = 5$ and $S = 0$) go from $M_L + 2M_S = -5$ to 5, while the energy levels of the spin-triplet term ^3H (which has $L = 5$ and $S = 1$) go from $M_L + 2M_S = -7$ to 7; and something similar for the $7i$ upper level.]

(c) Use the Zeeman splitting formula to measure the magnetic field implied by the spectrum shown.

(d) The strength of the central line is somewhat greater than the strengths of either of the two outer lines. Use this information to infer the angle θ between the direction of the magnetic field $\boldsymbol{B}$ and the line of sight. [Hint: $\Delta m = \pm 1$ transitions emit a dipole emission pattern with number of photons proportional to $1 + \cos^2\theta$, while $\Delta m = 0$ transitions emit a dipole emission pattern with number of photons proportional to $1 - \cos^2\theta$. Firstly, what must be the relative constants of proportionality in these two patterns, given that the $\Delta m = -1, 0, 1$ transitions must be of equal strength when each is averaged over all directions? You can check your answer to this from the fact that the sum of the strengths of the three Δm components must be isotropic. Hence derive θ from the observed spectrum.]

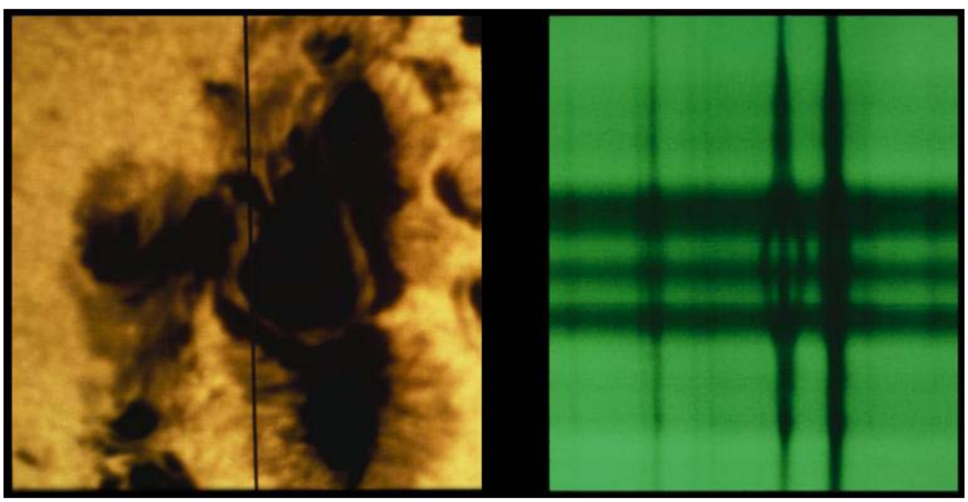

Figure 2: Zeeman splitting of absorption lines across a sunspot, from Solanki (2009) "Magnetic Fields in the Atmospheres of the Sun and Stars" Saas Fee 39 https://www2.mps.mpg.de/homes/solanki/saas_fee_39/SaasFee39_Handout_L1.pdf. Solanki does not say what line this is.