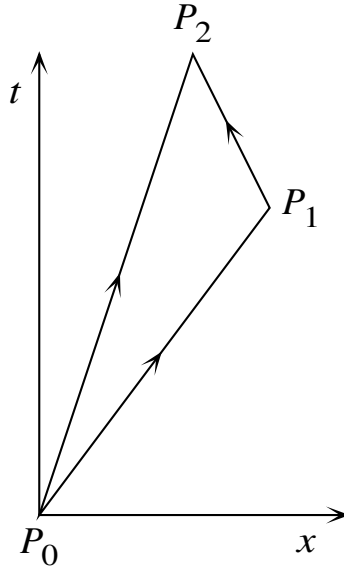# ASTR 3740 Relativity & Cosmology Spring 2023. Problem Set 3.
Due Wed Feb 22

1. The Longest Proper Time between Two Events is a Straight Line

Consider a person whose worldline goes from spacetime event P_0 to spacetime event P_1 at velocity v_1 relative to some inertial frame, and then from P_1 to spacetime event P_2 at velocity v_2, as illustrated in the Figure. Assume for simplicity that the velocities are both in the (positive or negative) x-direction. Show that the proper time along a straight line from P_0 to P_2 is always greater than or equal to the sum of the proper times along the two straight lines from P_0 to P_1 followed by P_1 to P_2. Hence conclude that the longest proper time between two events is a straight line. What does this imply about the twin paradox? [Hint: It is simplest to use rapidities α rather than velocities. Let the segment from P_0 to P_1 be $\{t_1, x_1\} = \tau_1\{\cosh\alpha_1, \sinh\alpha_1\}$, and the segment from P_1 to P_2 be $\{t_2, x_2\} = \tau_2\{\cosh\alpha_2, \sinh\alpha_2\}$. The segment from P_0 to P_2 is the sum of these, $\{t, x\} = \{t_1+t_2, x_1+x_2\}$. Show that

$$\tau^2 - (\tau_1 + \tau_2)^2 = 4\tau_1\tau_2 \sinh^2\left(\frac{\alpha_2 - \alpha_1}{2}\right) , \tag{1.1}$$

which is a minimum for $\alpha_2 = \alpha_1$.]

2. Falling into a Black Hole

Explore the "Falling into a Black Hole" and "Inside Black Holes" websites at

http://jila.colorado.edu/~ajsh/bh/

and

http://jila.colorado.edu/~ajsh/insidebh/

Formulate a quiz question on black holes *different* from any of those at

http://jila.colorado.edu/~ajsh/bh/quiz.html

and answer it. 80% of your grade on this problem will be on your quiz question, and 20% on your answer to it. So be sure to state the question clearly and precisely, in a way that anyone browsing the web site would understand unambiguously. You may collaborate with others, but you should write your quiz question and answer by yourself.

The (anonymized) questions will be posted on the course website, and you will vote for the best questions.