COSMOLOGY

"Cosmologists: often in error but never in doubt"
— Lev Landau

Key Observational Evidences favoring the standard Hot Big Bang Cosmology

★★★★★ 1. The Expansion of the Universe
★★★★★ 2. The 2.725K Cosmic Microwave Background
★★★ 3. Primordial Nucleosynthesis of Light Elements
★★★ 4. Large Scale Galaxy Distribution
★★★ 5. Age of Oldest Stars.
★★★★★ 6. Hubble diagram of Type Ia Supenovae (1998)

Key Theoretical Concepts

1. Geometry $\longleftrightarrow$ Mass-energy-momentum content
$\longleftrightarrow$ Fate
2. Early Universe was in Thermodynamic Equilibrium.
3. Structure in the Universe has grown by gravity.

Standard Model of Cosmology (1998)

Dates to discovery, from Hubble diagram of high redshift supernovae, that Universe is ACCELERATING.
Discovery, announced in Jan 1998, is confirmed by many other evidences:

1. Cosmic Microwave Background (CMB).
   Power spectrum is powerful tool for measuring cosmological parameters precisely.
   Demonstrates Universe is flat to within 1%.
2. Hubble diagram of Type Ia supernovae
   Demonstrates Universe is accelerating.
3. Primordial nucleosynthesis of light elements H, D, ^3He, ^4He, Li.
   Agreement between predicted vs. observed abundances requires $\Omega_b \simeq 0.05$
   in baryons
4. Galaxy distribution.
   Power spectrum consistent with CMB.
   Galaxy velocities (and power spectrum) indicate
   $\Omega_m \approx .2 - .3$
   matter, including Dark Matter and baryons.

In 1998 two teams, the Supernova Cosmology Project (Perlmutter, 1999), and the High-z Supernova Search n (Riess and Filippenko, 1998), precipitated the revolution that led to the Standard Model of Cosmology. They reported that observations of Type Ia supernova at high redshift indicated that the Universe is not only expanding, but also accelerating. The acceleration requires the mass-energy density of the Universe to be dominated at the present time by a gravitationally repulsive component, such as a cosmological constant Λ.

In the Hubble diagram of Type Ia supernova shown in Figure 10.1, the fitted curve is a best-fit flat cosmological model containing a cosmological constant and matter.

10.1.3 The Cosmic Microwave Background (CMB)

The single most powerful observational constraints on the Universe come from the Cosmic Microwave Background (CMB). Modern observations of the CMB have ushered in an era of precision cosmology, where key cosmological parameters are being measured with percent level uncertainties.

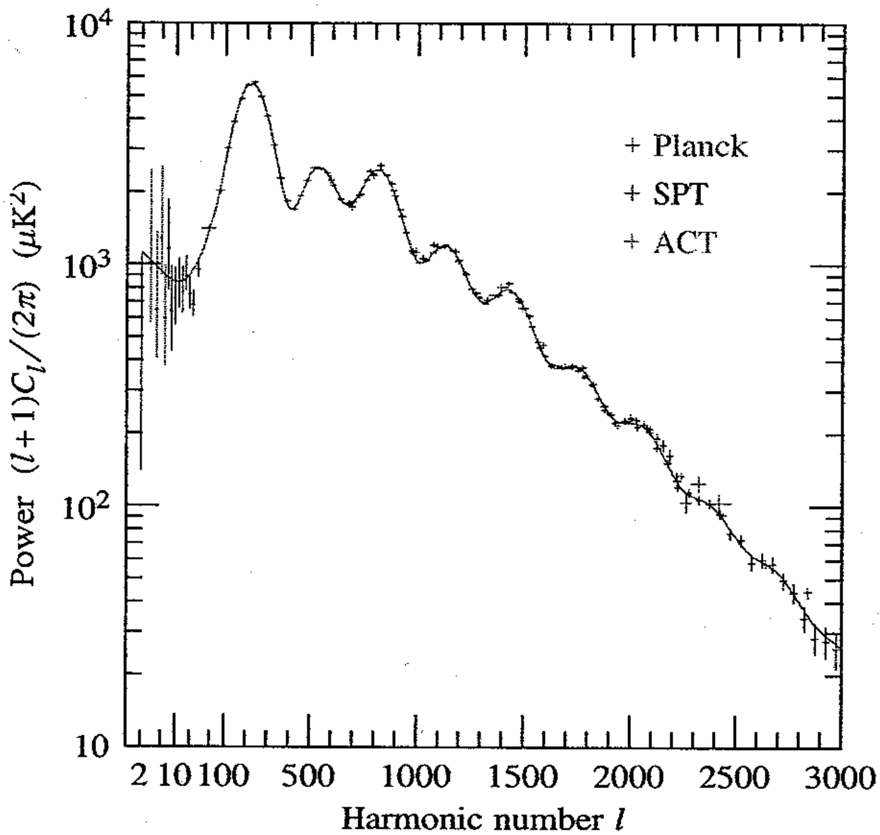


Figure 10.2 Power spectrum of fluctuations in the CMB from observations with the Planck satellite (Ade, 2013), the South Pole Telescope (Keisler and Reichardt, 2011), and the Atacama Cosmology Telescope (Das and Sherwin, 2011). The plot is logarithmic in harmonic number l up to 100, linear thereafter. The fit is a best-fit flat ΛCDM model.

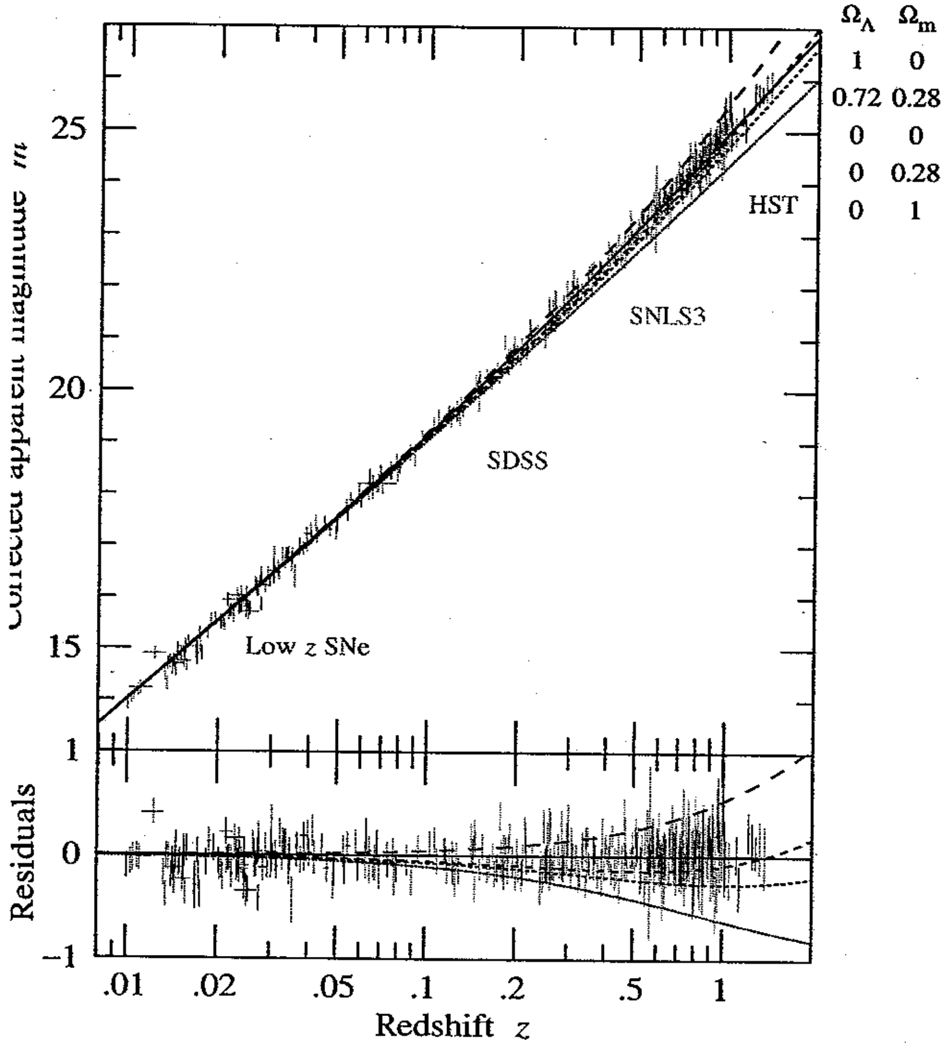
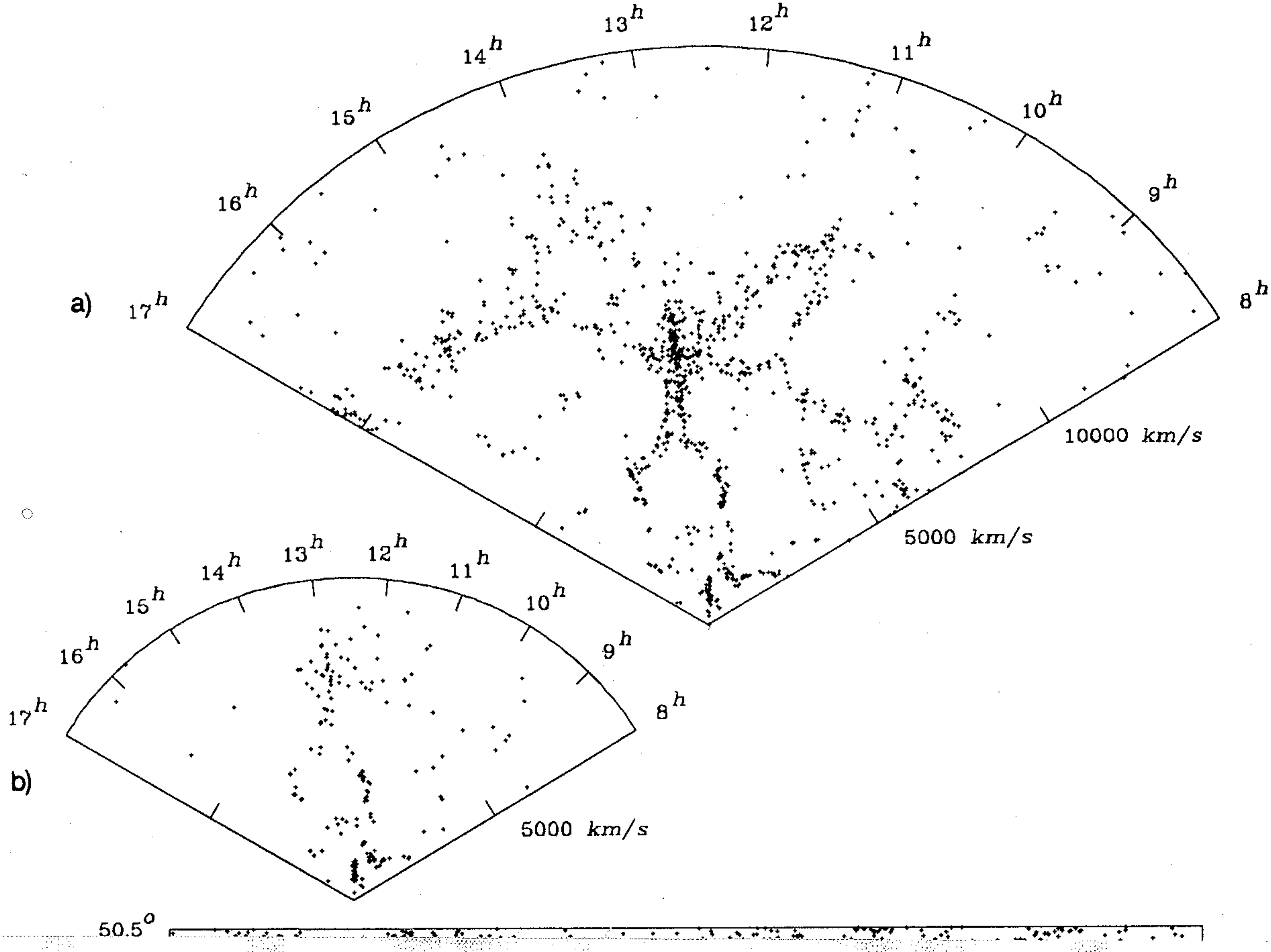


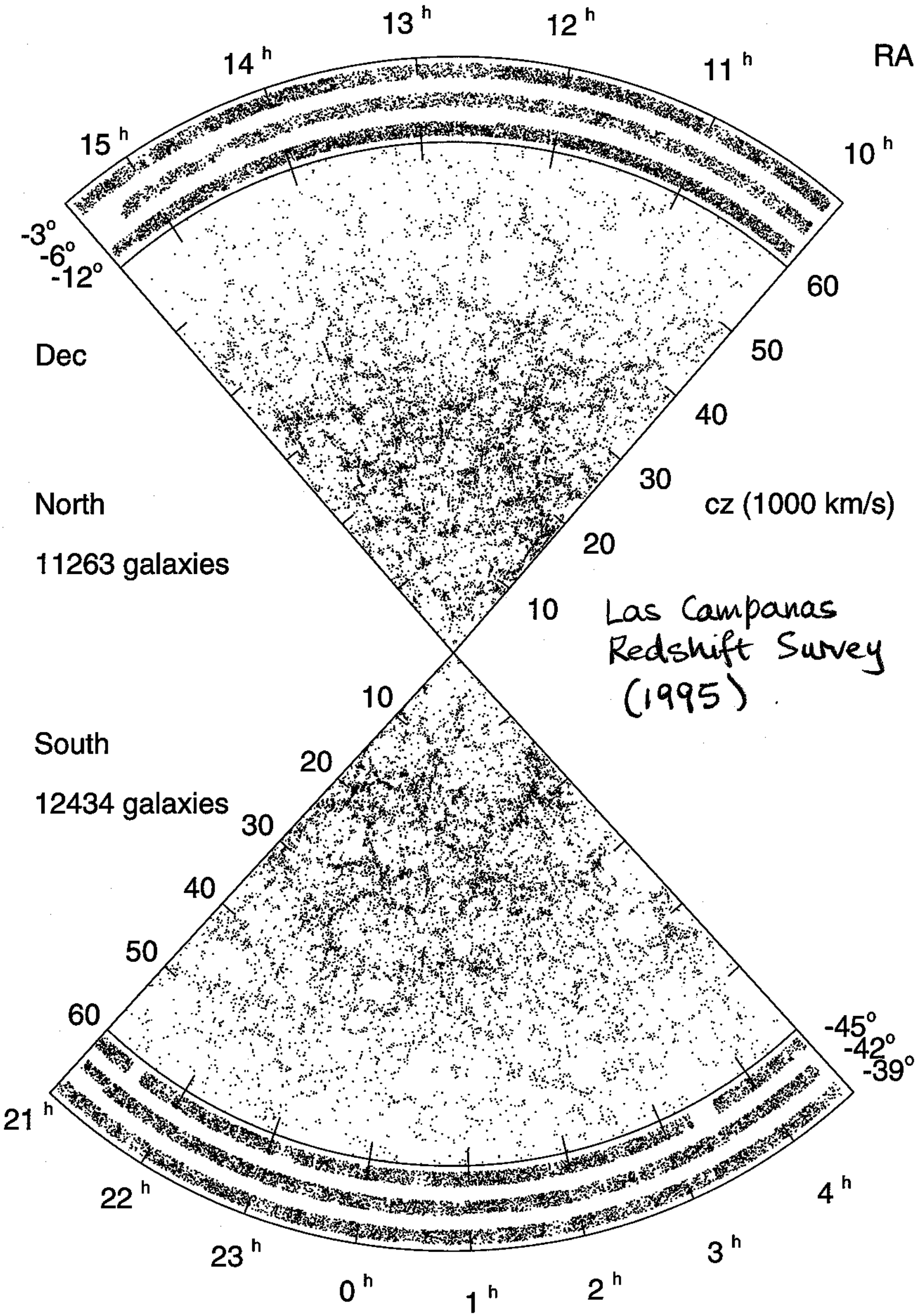
Figure 10.1 Hubble diagram of 472 Type Ia supernovae from a compilation of surveys, adapted from Conley et al. (2011). The vertical axis is the apparent magnitude m (= apparent brightness) of the supernova, with an empirical correction based on the shape of its lightcurve. The various smooth curves are 5 theoretical model Hubble diagrams, with parameters from top to bottom as indicated at top right. The solid line is a flat ΛCDM model with $\Omega_\Lambda = 0.72$ and $\Omega_m = 0.28$. The bottom panel shows residuals.

^{56}Ni synthesized in the explosion, and which can be corrected at least in part through an empirical relation between luminosity and how rapidly the lightcurve decays (higher luminosity supernovae decay more slowly).

10.1.2 The acceleration of the Universe

Since light takes time to travel from distant parts of the Universe to astronomers here on Earth, the higher the redshift of an object, the further back in time astronomers are seeing.

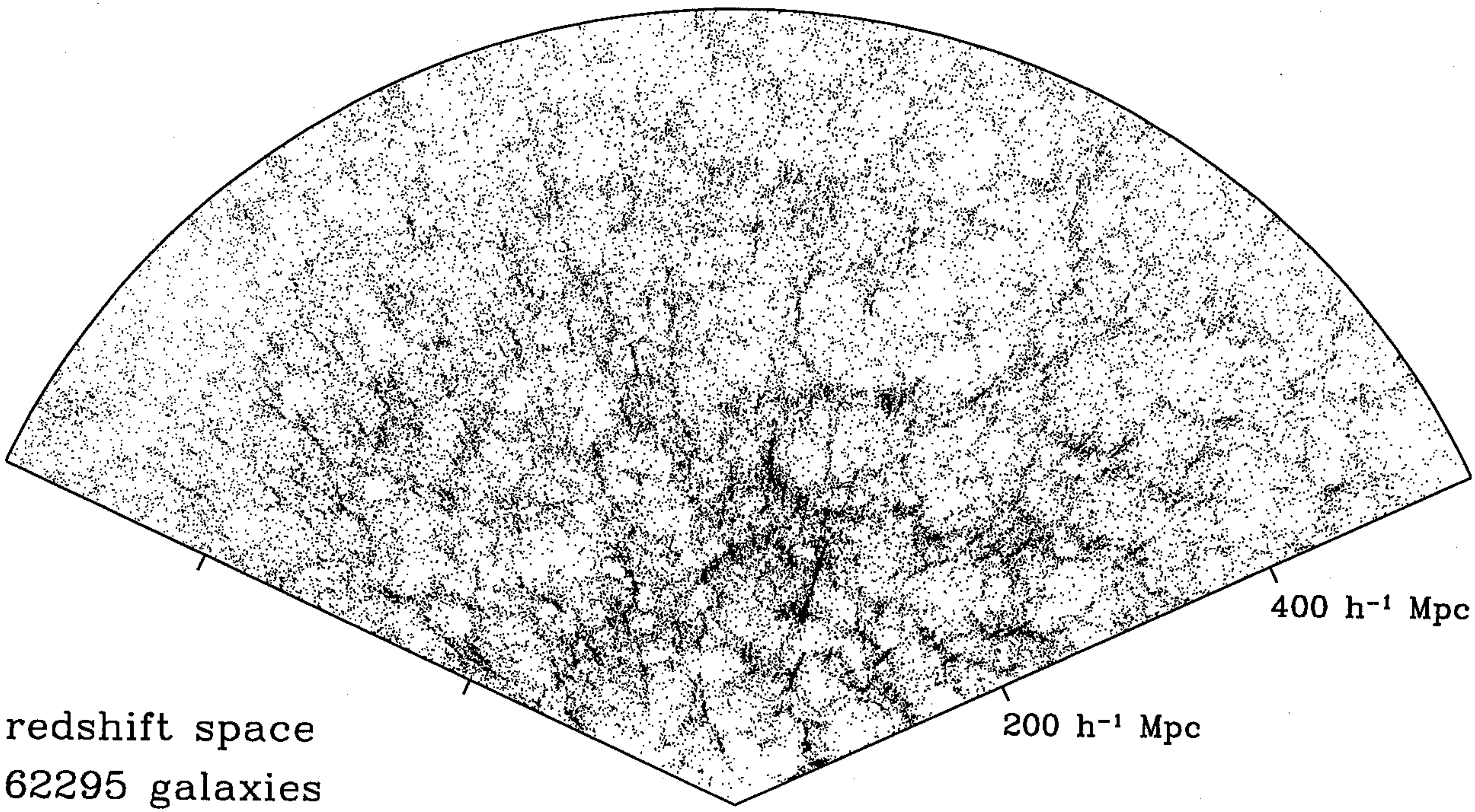

a)
17^h
16^h
15^h
14^h
13^h
12^h
11^h
10^h
9^h
8^h
10000 km/s
5000 km/s
b)
17^h
16^h
15^h
14^h
13^h
12^h
11^h
10^h
9^h
8^h
5000 km/s
50.5^o

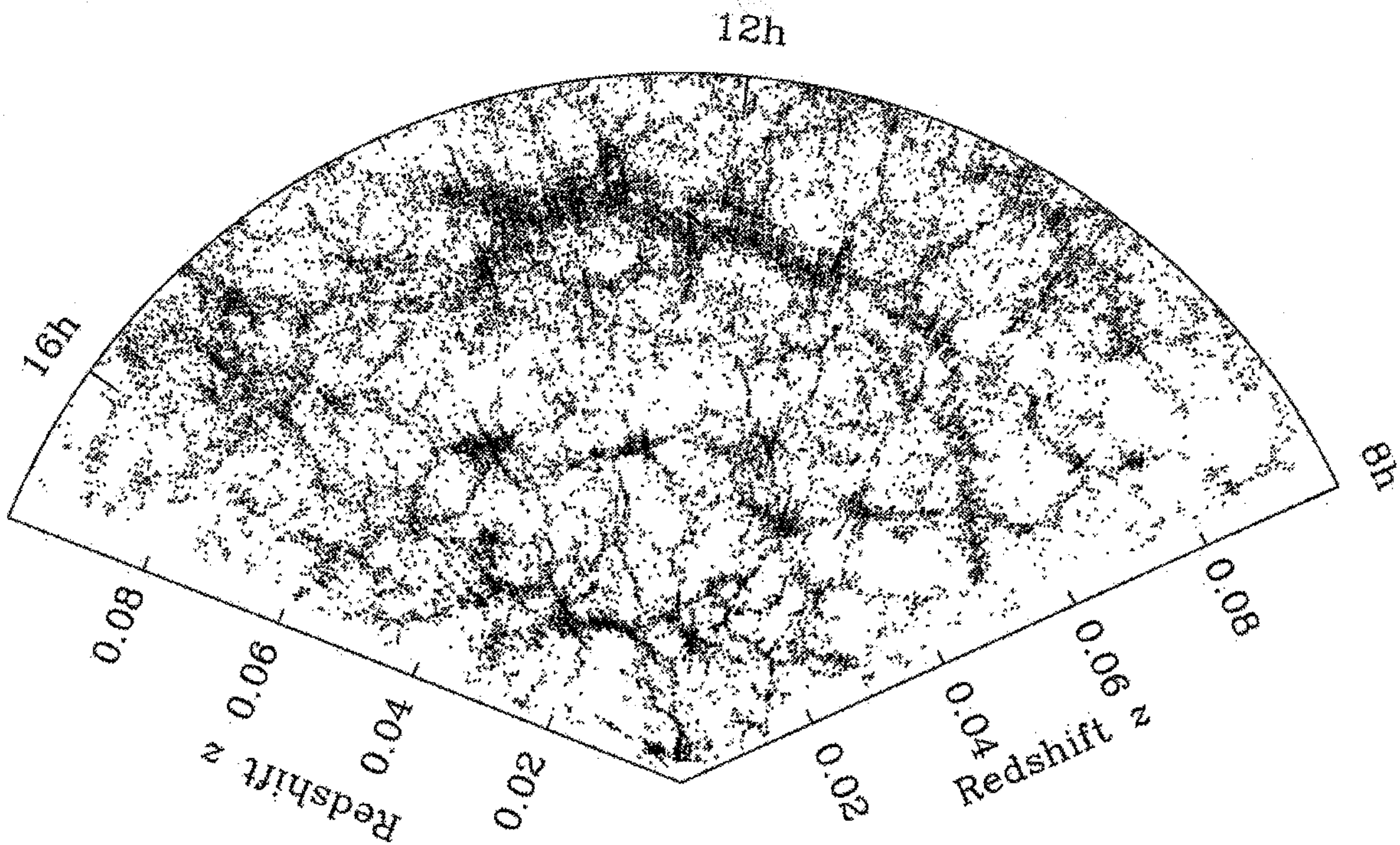


Las Campanas Redshift Survey (1995)

Weinberg (1993) Sloan Simulation (in Black Book)

$r' < 17.55$, $d > 2''$, 6°slice





4° Slice of SDSS North - Actuall Data
Sloan Digital Sky Survey

Other observational evidence

0. The Universe is expanding (Hubble 1929).

5. Age of oldest stars.
6. Abundance of galaxy clusters
   at high redshift.
7. Ubiquitous evidence for Dark Matter
   - rotation curves of spiral galaxies
   - velocity dispersion of elliptical galaxies
   - gravitational lensing.

Expansion of the Universe

Hubble (1929) law

subscript 0 denotes present

$$\boxed{\begin{array}{ccccc} v & = & H_0 & & d \\ \text{Recession velocity} & & \text{Hubble constant} & \times & \text{distance} \end{array}}$$

Applies (not exactly) to galaxies beyond the Local Group of ~30 galaxies to which our Milky Way Galaxy belongs.

Compare to

$$v = \frac{d}{t} \Rightarrow H_0 \approx \frac{1}{t}$$

velocity $\quad \dfrac{\text{distance}}{\text{time}}$

not exact because of deceleration.

- Does Universe have a center?
- What is Universe expanding into?

 M.Rowan-Robinson (1985)

3.5

Edwin Hubble, 1889–1953 (Courtesy of Mount Wilson and Las Campanas observatories.)

Kapteyn and Shapley distance scales, the former being raised upward by a factor of 3 and the latter downward by a similar factor. Finally, in 1935 Hubble demonstrated that van Maanen's controversial apparent rotations did not exist and that they must have been some artifact of van Maanen's plate-measuring machine.

In the same years that the enormous scale of the universe was first being realized, the universe was also being shown to be expanding. In 1912 Vesto Slipher announced the results of his study at the Lowell Observatory of a sample of 12 spiral galaxies. Measuring their radial velocities from the Doppler shifts of their spectral lines, he had found that most showed large recession velocities. By 1925 he had measured the radial velocities of 40 galaxies, almost all of which were positive. In 1929 Hubble was able to compare these with his distance estimates for 18 galaxies, and he showed that there existed a clear correlation between the apparent velocity of recession, inferred from the red-shifting of the spectral lines, and the distance (Fig. 1.5). The possibility of an expanding universe had already been proposed by

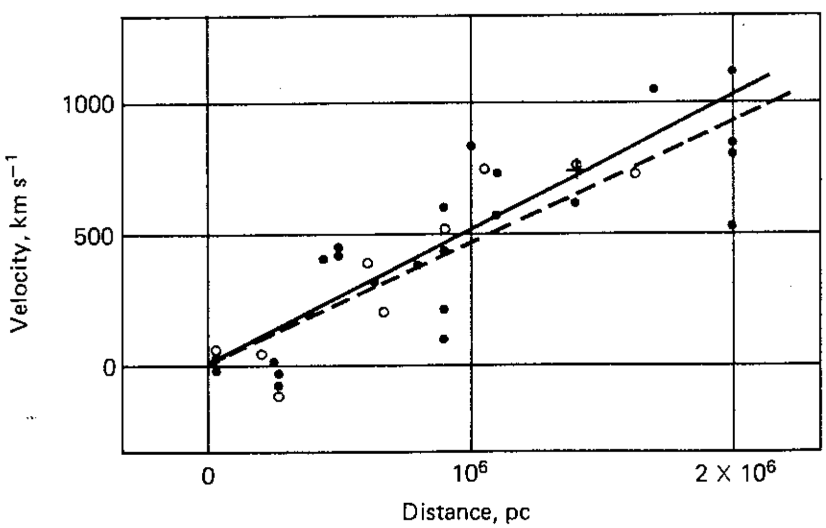


FIGURE 1.5

Hubble's velocity–distance law. The recession velocity of the galaxies is plotted versus their distance. (From [H1].)

26-3 The Hubble law states that the redshifts of remote galaxies are proportional to their distances from Earth

Whenever an astronomer finds an object in the sky that can be seen or photographed, the natural inclination is to attach a spectrograph to a telescope and record the spectrum. As long ago as 1914, V. M. Slipher, working at the Lowell Observatory in Arizona, began taking spectra of "spiral nebulae." He was surprised to discover that of the 15 spiral nebulae he studied, the spectral lines of 11 were shifted toward the red end of the spectrum, indicating that they were all moving away from Earth. This marked dominance of redshifts was presented by Curtis in the Shapley–Curtis debate as evidence that these spiral nebulae could not be ordinary nebulae in our Milky Way Galaxy.

During the 1920s, Edwin Hubble and Milton Humason photographed the spectra of many galaxies with the 100-inch telescope on Mount Wilson. Five representative elliptical galaxies and their spectra are shown in Figure 26-15. As

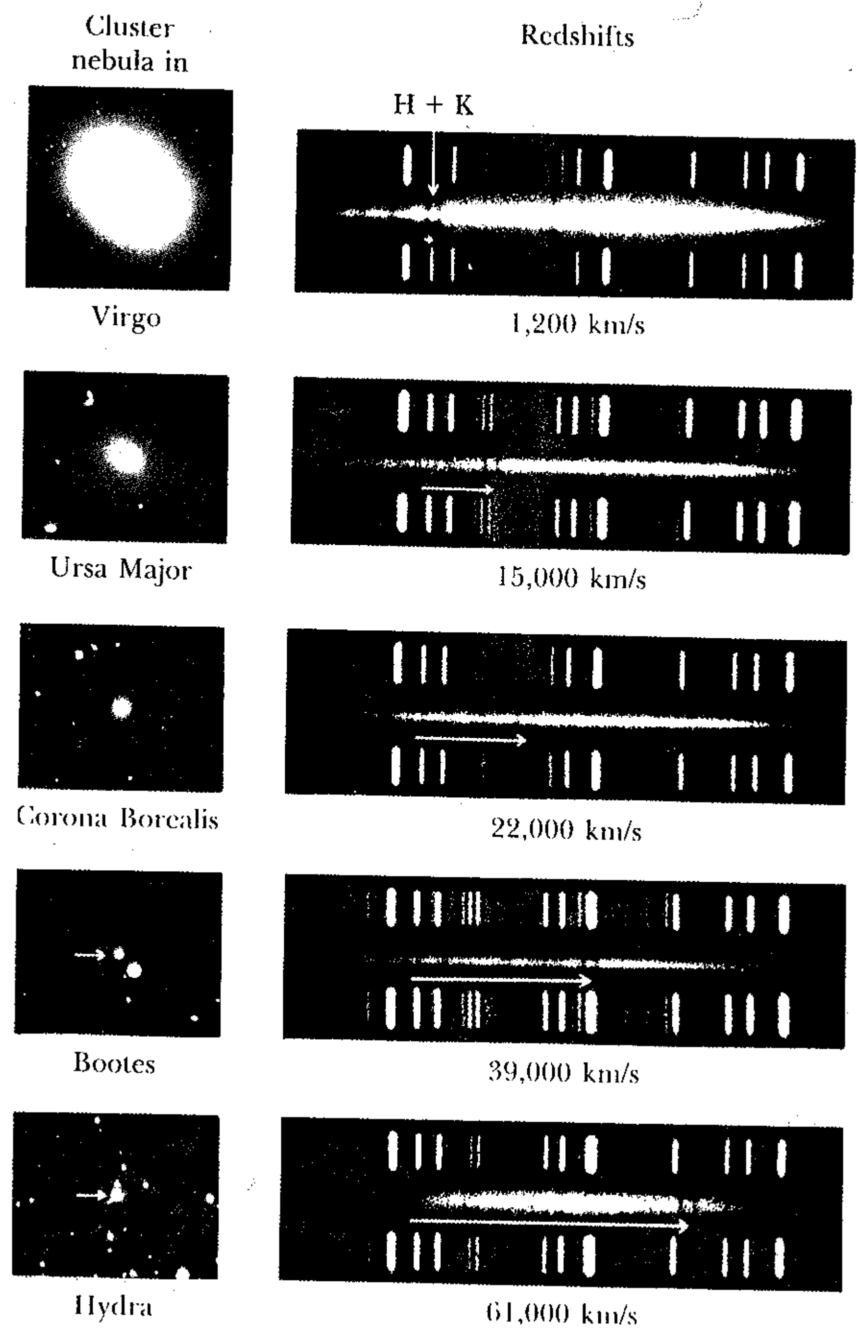
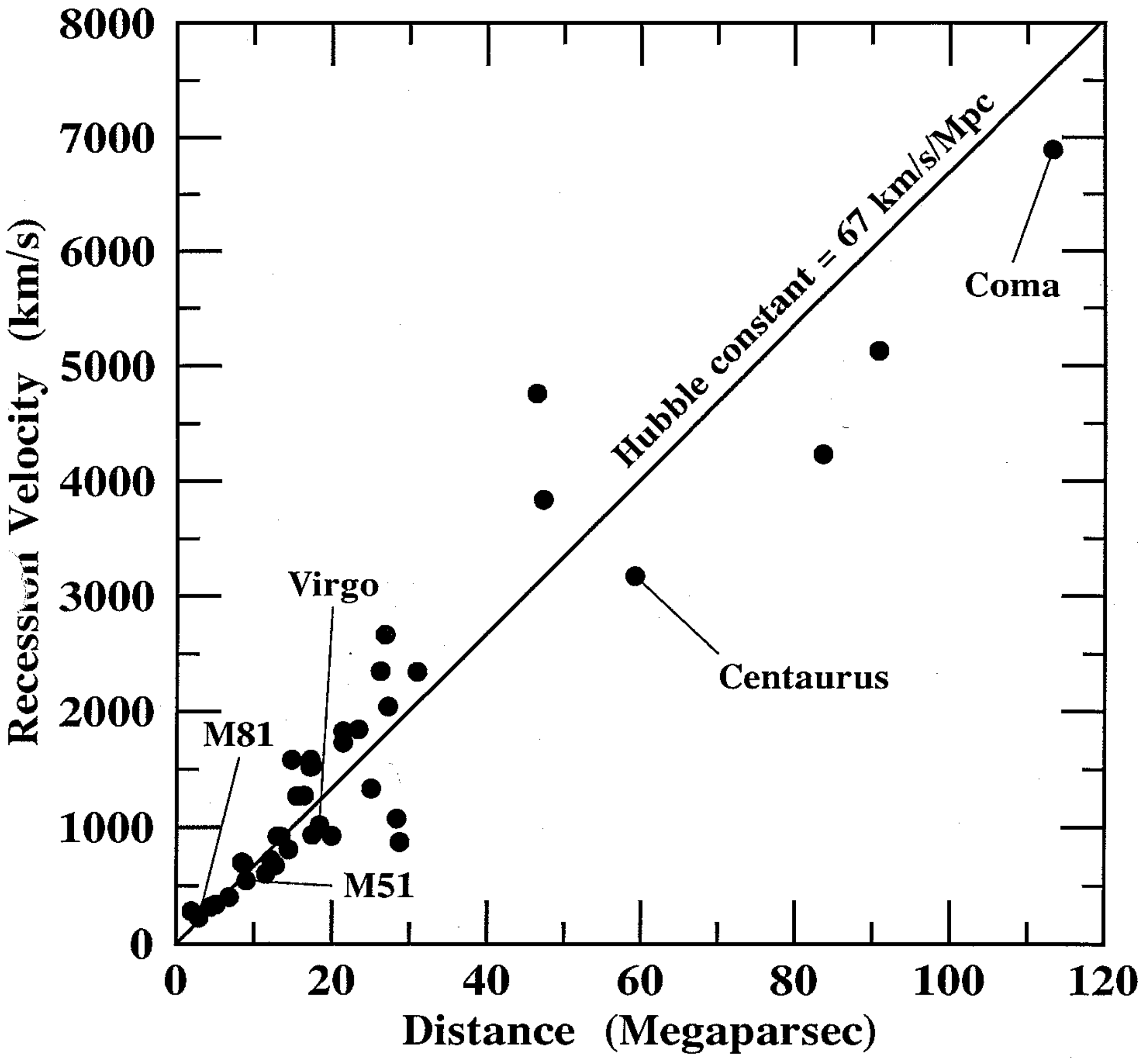


FIGURE 26-15 Five Galaxies and Their Spectra The photographs of these five elliptical galaxies all have the same magnification. They are arranged, from top to bottom, in order of increasing distance from Earth. The spectrum of each galaxy is the hazy band between the comparison spectra. In all five cases, the so-called H and K lines of singly ionized calcium can be seen. The recessional velocity, calculated from the Doppler shifts of the H and K lines, is given below each spectrum. Note that the more distant a galaxy is, the greater its redshift. (The Carnegie Observatories)

From compilation of distances by
M. Rowan-Robinson (1985) "The Cosmological
Distance Ladder"

Hubble Diagram for Clusters of Galaxies



To measure H_0, need to measure

1. <u>Recession velocity v</u>

From redshift z of spectral lines of galaxy

$$z \equiv \frac{\Delta\lambda}{\lambda_{emit}} = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}},$$

Conventionally convert z to v using nonrelativistic Doppler shift formula

$$v = cz.$$

Note special relativistic Doppler shift formula does NOT apply to Universe (not inertial frame).

2. <u>Distance d</u>

Tricky, tricky.
Measure using step-by-step process called "Cosmological Distance Ladder".

Final rung invariably involves "standard candles"
= objects whose intrinsic luminosity you think you know.

Two of the most important standard candles:

a. Cepheid variables

Pulsating yellow supergiant stars.
Leavitt (1907) discovered

$$\text{Luminosity} \propto \text{Period}$$

b. Type Ia Supernova

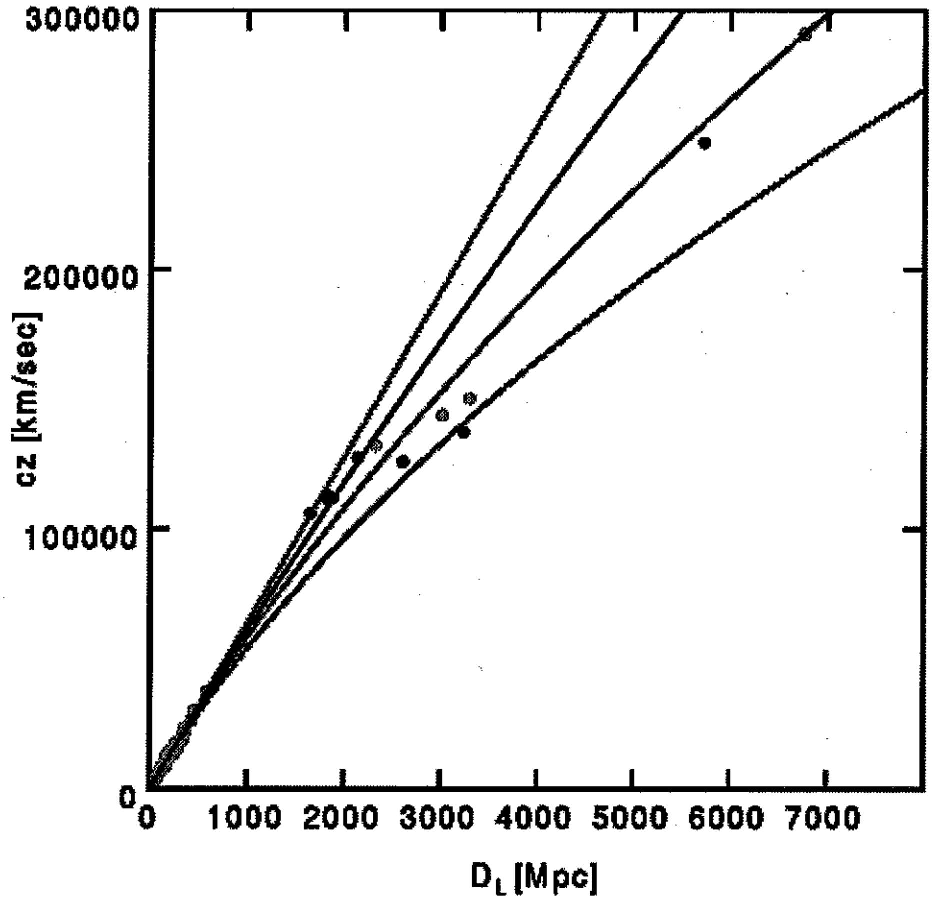
Thermonuclear explosion of white dwarf accreting in a binary system.

http://www.astro.ucla.edu/~wright/sne_cosmology.html

3-9

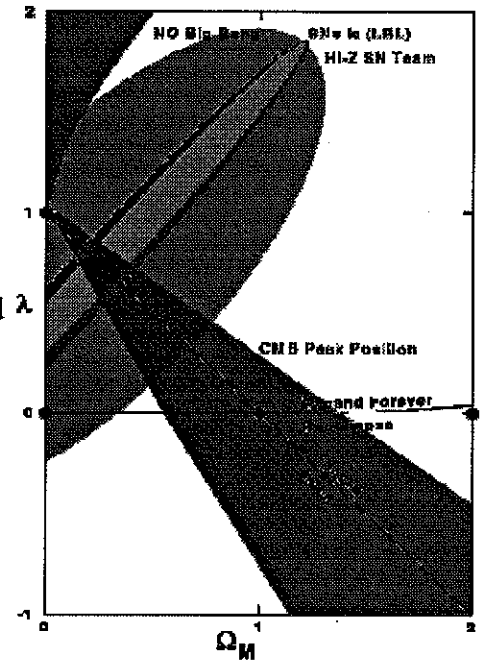
Measuring the Curvature of the Universe by Measuring the Curvature of the Hubble Diagram

Two groups are measuring distant supernovae with the goal of determining whether the Universe is open or closed by measuring the curvature in the Hubble diagram. The figure below shows both data sets: Perlmutter *et al.* (1997) and (1998) in black and Garnavich *et al.* (1997) in orange (which looks more like green in the image below).



The curves show a closed Universe (Omega = 2) in red, the critical density Universe (Omega = 1) in black, the empty Universe (Omega = 0) in green, and the steady state model in blue. The supernova luminosity calibration used to compute the luminosity distance D_L is from Riess, Press and Kirshner (1996) which gives H_0 = 65 km/sec/Mpc. The data show a larger scatter than that expected based on the accuracy of the low redshift sample, but Omega appears to be in the range 0 to 1.

Both groups have been the subject of news articles in Science, on 30 Jan 1998 and 27 Feb 1998. I have combined their two error ellipses along with another constraint from the location of the Doppler peak in the angular power spectrum of the CMB anisotropy. The two SNe groups give very similar error ellipses, and the combined CMB-SNe fit indicates that a flat Universe with a cosmological constant is preferred. But the systematic errors on the SNe data, shown as the large pink ellipse, could allow for a vanishing cosmological constant lambda. The red, black, green and blue circles on the Figure to the right are keyed to the colors of the curves on the Figure shown above.



Ned Wright's Home Page

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Holy Grail Numbers of Cosmology

1. The Hubble constant H_0

Why? Because $1/H_0$ is the age of the Universe.

Comment: Actually the age is only approximately $1/H_0$, because of the deceleration of the Universe. For example, if $\Omega = 1$ (flat, matter-dominated Universe) then age $= \frac{2}{3H_0}$.

2. The cosmological density parameter Ω (Omega)

$$\Omega \equiv \frac{\text{actual density of Universe}}{\text{critical density needed to make it flat}}$$

Why? Because Ω tells us the geometry & fate of the Universe

2'. Various contributions to Ω

2'. Various contributions to Ω

- baryonic matter Ω_b
- radiation (CMB) Ω_{rad}
- Cold Dark Matter (CDM) Ω_{CDM}
- Hot Dark Matter = neutrinos Ω_ν
- Cosmological constant = vacuum energy Ω_Λ

Total: Ω

Less likely:
- cosmic strings 1D

or other "topological defects"
(monopoles 0D ; domain walls 2D ; textures 3D).

Best current observations (Planck 2013)

$$H_0 = 68 \pm 1 \text{ km/s/Mpc}$$

$\Omega_{rad} = 5 \times 10^{-4}$ CMB

$\Omega_b = 0.05$ CMB or Primordial Nucleosynthesis

$\Omega_{CDM} = 0.26$ CMB or dynamics of galaxy clusters

$\Omega_\nu \lesssim 0.01$ "

$\Omega_\Lambda = 0.69$ CMB, SNIa

$\Omega_{tot} = 1.00 \pm .007$ from CMB (Planck)

Horizon of the Universe

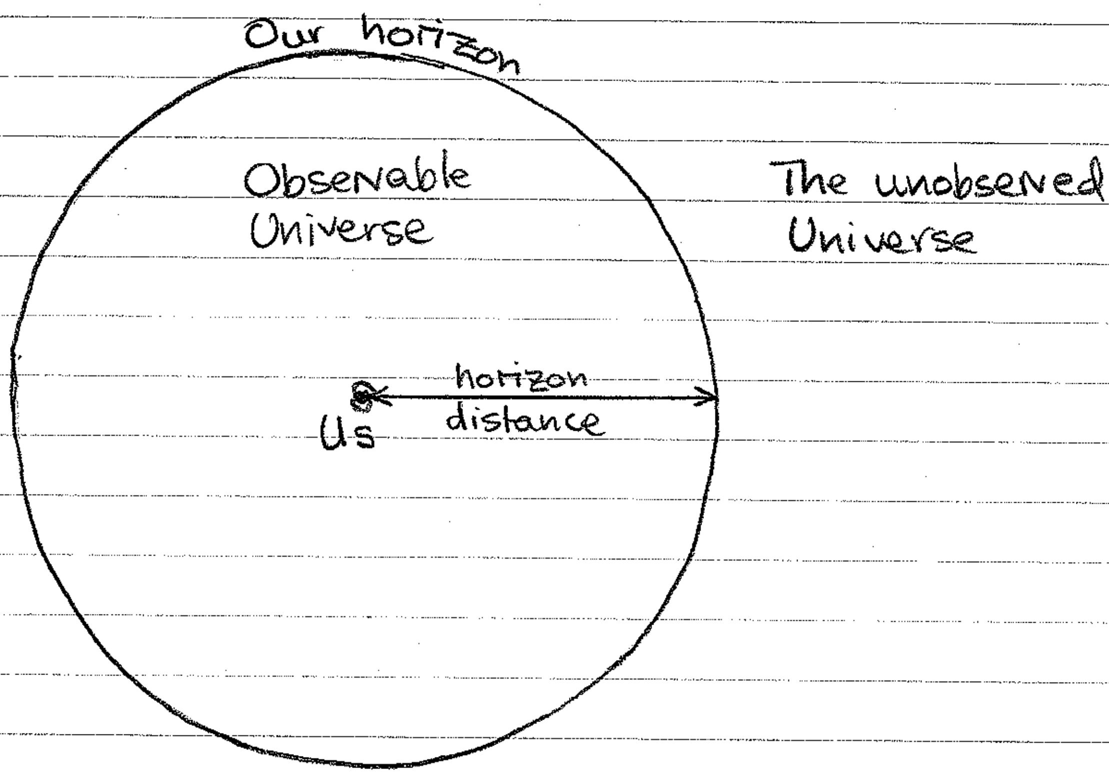
As we look deeper in space
we see further back in time.

Universe is $t \approx 1.4 \times 10^{10}$ years old
so we see no further than

$$ct \approx 1.4 \times 10^{10} \text{ lightyears away}$$

Actually horizon distance is somewhat further than this, because of deceleration of Universe. E.g. for Standard Model

$$\text{Horizon distance} \approx 3ct$$



Some Problems of the Universe

1. <u>Horizon Problem</u>

   The horizon size at Recombination (where the CMB comes from) corresponds to 2° on the CMB observed today.
   Problem: How could causally disconnected regions at the time of Recombination (those more than 2° apart today) know to have the same temperature, as is observed to high precision?

   Possible solution: Inflation

2. <u>Expansion Problem</u>

   Why is the Universe expanding, if gravity is always attractive?

   Possible solution: Infation

3. <u>Flatness Problem</u>

   Why is the density of the Universe so close to the critical density?

   Possible solution: Inflation

4. <u>Isotropy Problem</u>

How is it possible to grow observed present day structure — galaxy clusters, galaxies, stars, us — from fluctuations of a few 10^{-5} at Recombination in "only" the age of the Universe?

as observed in CMB

Possible solution: <u>Non-baryonic Dark Matter</u>

$\underbrace{\text{Non-baryonic}}$ this just means ordinary matter. Baryon (Greek baryos = heavy) is another word for nucleon (like proton or neutron).

5. <u>Age Problem</u>

Age of the oldest stars — those in globular clusters — seems to exceed the age of the Universe inferred from the Hubble constant.

Possible solutions:

- ~~Star ages wrong~~
- ~~H_0 wrong~~
- Cosmological constant ✓
- ~~helps if Ω is low~~

Cosmological Principle

Asserts that the Universe is
- homogeneous — same at all positions
- isotropic — same in all directions

Observational evidence:

★★★★★ Cosmic Microwave Background

★★ Large scale galaxy distribution.

Note CP allows Universe to change in time.

FRW Friedmann - Robertson - Walker metric

describes the geometry of a Universe satisfying the CP.

$$\boxed{ds^2 = \underbrace{-dt^2}_{\text{time part}} + a(t)^2 \left[\underbrace{\frac{dx^2}{1-\kappa x^2}}_{\text{radial part}} + x^2 \underbrace{\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)}_{\text{spherically symmetric angular part}} \right]}$$

t ≡ proper cosmic time experienced by "comoving" observers in expanding Universe.

$a(t)$ ≡ cosmic scale factor.

x ≡ comoving radial coordinate, defined so proper circumference is $2\pi a x$.

κ ≡ curvature constant.

FRW

1. Spatial part of FRW metric

Cosmological Principle $\Longrightarrow$
spatial part of FRW geometry is a
$\boxed{\text{3D hypersphere}}$

Imagine embedding 3D hypersphere in 4D space.
3D hypersphere is set of points $(\underbrace{x, y, z}_{\text{3 physical dimensions}}, w)$
satisfying

extra spatial dimension

$$x^2 + y^2 + z^2 + w^2 = R^2 = \text{constant}$$

Define $r^2 \equiv x^2 + y^2 + z^2$.
r is circumferential radius:
proper circumference of circle at r is $2\pi r$.

Define $r_{/\!/}$ to be geodesic radius
= proper distance to radius r along spatial geodesic.

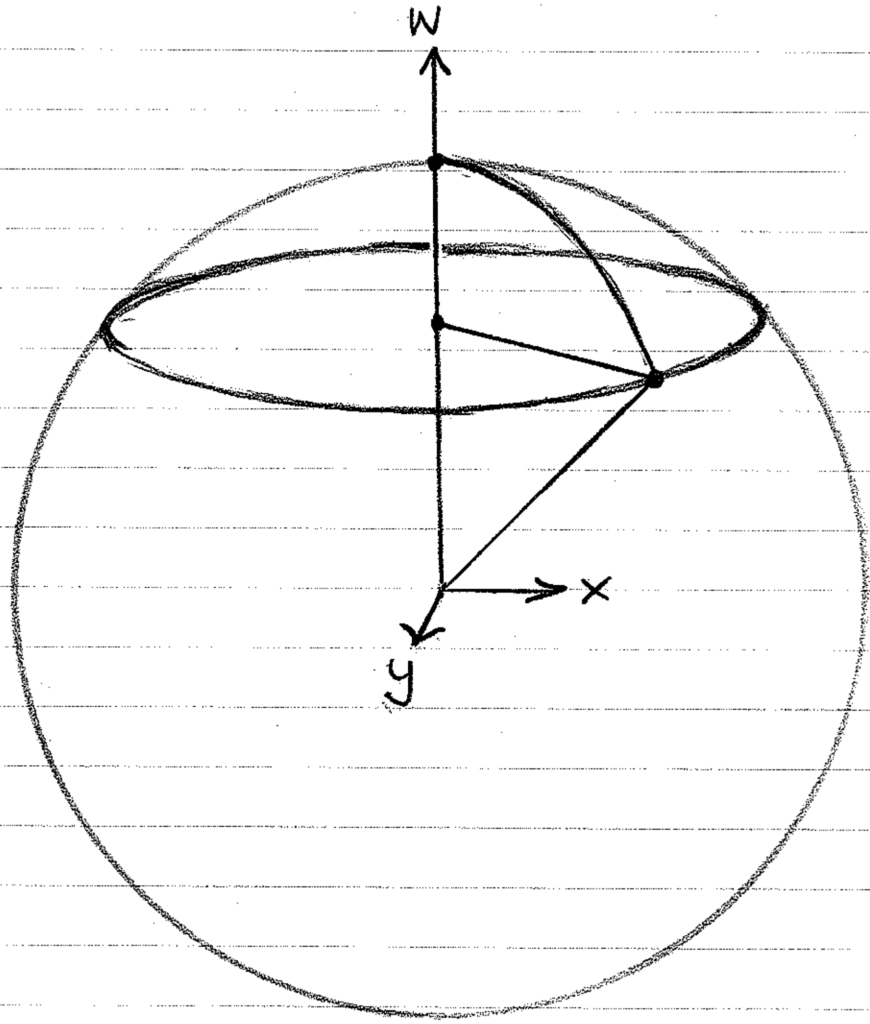
Spatial FRW metric is

$$dl^2 = dr_{/\!/}^2 + r^2 \underbrace{\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)}_{\substack{\text{angular part of} \\ \text{metric is spherically} \\ \text{symmetric}}}$$

RW

2. Embedding diagram of FRW geometry

Following embedding diagram shows spatial FRW geometry at a fixed instant of cosmic time t.



Introduce angle χ

Have (from diagram – but more rigorous proofs exist)

$$r = R\sin\chi$$
$$r_{//} = R\chi \, .$$

So

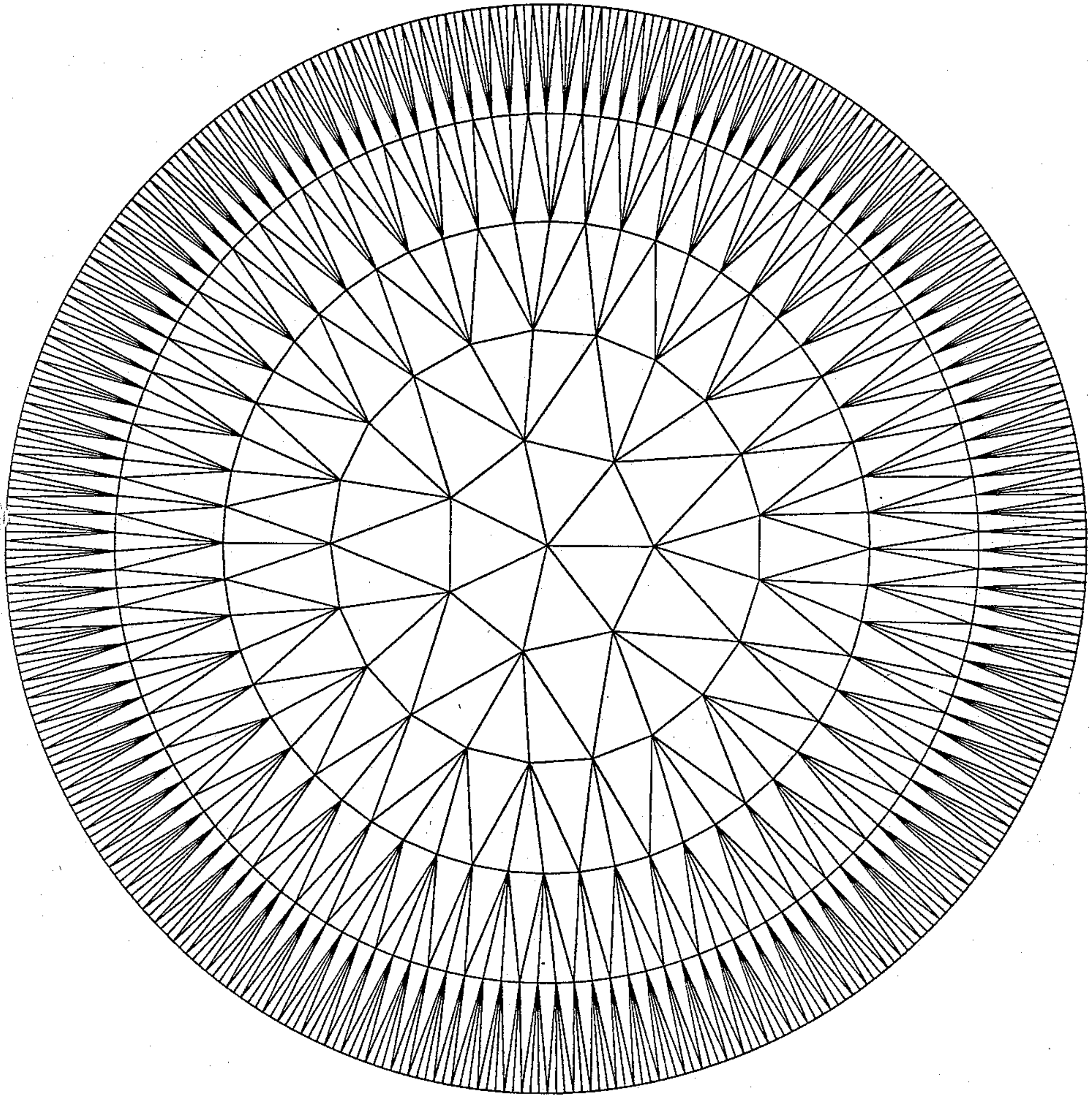
$$dr_{//} = R\, d\chi$$
$$= R\, \frac{d\sin\chi}{\cos\chi}$$
$$= \frac{dr}{\sqrt{1 - \frac{r^2}{R^2}}}$$
$$= \frac{dr}{\sqrt{1 - kr^2}} \, .$$

where $k \equiv \frac{1}{R^2}$ is <u>curvature</u>

Hence spatial FRW metric is

$$dl^2 = \frac{dr^2}{1 - kr^2} + r^2 \underbrace{d\Omega^2}_{\text{shorthand for } d\theta^2 + \sin^2\theta\, d\phi^2} \, .$$

	$>$	0	closed
k	$=$	0	flat
	$<$	0	open



"Think comoving".
— summarizes first 3rd of Hamilton's
rKW graduate course in Cosmology.

3. Comoving coordinates

Expansion of Universe
$\Rightarrow$ "radius of Universe" $R(t)$ changes with time.

In closed case, $(k > 0)$, spatial part of FRW metric is

$$dl^2 = R^2 \left[\frac{d(r/R)^2}{1 - (r/R)^2} + \left(\frac{r}{R}\right)^2 do^2 \right]$$

More generally, spatial FRW metric is

$$dl^2 = a^2 \left[\frac{dx^2}{1 - \kappa x^2} + x^2 do^2 \right]$$

where $\boxed{a(t) \text{ is Cosmic Scale Factor}}$
and

$x \equiv \frac{r}{a}$ is comoving circumferential radial coordinate

and $\kappa \equiv k a^2$ is the curvature constant.

By construction:

- objects expanding with the Universe are fixed in comoving coordinates.
- $a(t)$ is the expansion factor of the Universe; can choose overall scale of $a(t)$ arbitrary.

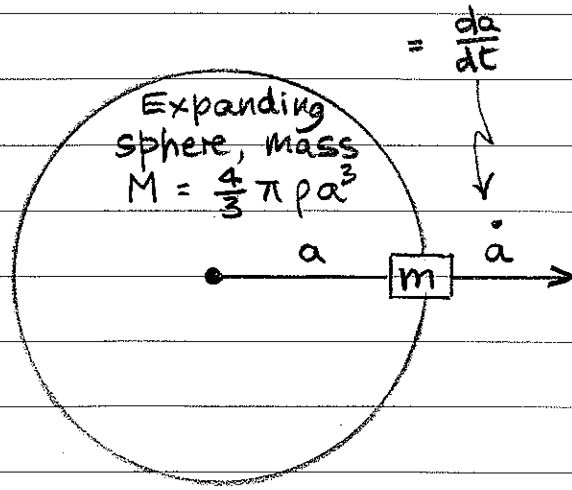
Feq

Friedmann equations

Einstein equations applied to FRW metric
$\Rightarrow$ 2 Friedmann equations, which describe
(i) curvature of universe
(ii) evolution of $a(t)$ with time

Newtonian "derivation" of Friedmann equations

1. Energy equation



Newtonian energy equation for small mass m at edge of sphere M is

$$\underbrace{\frac{1}{2} m \dot{a}^2}_{\text{Kinetic Energy}} - \underbrace{\frac{GmM}{a}}_{\text{Potential Energy}} = \underbrace{-\frac{\kappa}{2} mc^2}_{\text{constant}}$$

Divide by $\frac{1}{2} m a^2$:

$$\Rightarrow \boxed{\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{\kappa c^2}{a^2}}$$

$\frac{\dot{a}}{a} = H =$ Hubble "constant"
constant in space, not time

Einstein's equations are required to reveal that κ is the curvature constant in the FRW metric

Feq.

2. 1st law of thermodynamics (energy conservation)

$$\underset{\substack{\text{Energy}\\\text{in Volume } dV}}{dE} + \underset{\text{pressure}}{p}\ \underset{\substack{\text{Volume}\\\text{element}}}{dV} = 0$$

$$\Rightarrow d(\rho a^3) + p\, d(a^3) = 0$$

ρ = mass-energy density $\quad$ p = pressure (units $c=1$) = momentum density

$$\Rightarrow a^3\, d\rho + (\rho + p)\, d(a^3) = 0$$

$$\Rightarrow \frac{d\rho}{dt} + \frac{3(\rho+p)}{a}\frac{da}{dt} = 0$$

First Friedmann energy equation is:

$$\dot{a}^2 = \frac{8}{3}\pi G \rho a^2 - \kappa c^2$$

Differentiate:

$$2\dot{a}\ddot{a} = \frac{8}{3}\pi G\left(\dot{\rho} a^2 + 2\rho a \dot{a}\right)$$

$$\dot{\rho} = -3(\rho+p)\frac{\dot{a}}{a}$$

$$= \frac{8}{3}\pi G\, a\dot{a}\,(-\rho - 3p)$$

$$\Rightarrow \boxed{\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p)}$$

Comment:

Derivation is only heuristic.

If started from $m\ddot{a} = -\dfrac{GmM}{a^2}$

would get $\dfrac{\ddot{a}}{a} = -\dfrac{4}{3}\pi G\rho$, which is wrong.

Feq Omega

Formal definition of Ω

Define critical density ρ_c to be the density at which the universe is flat, $k = 0$.
In this case, Friedmann energy equation is

$$\boxed{H^2 = \frac{8}{3}\pi G \rho_c} \quad \text{defines } \rho_c$$

Then define

$$\boxed{\Omega \equiv \frac{\rho}{\rho_c}} \, .$$

Two important inferences from Friedmann equations

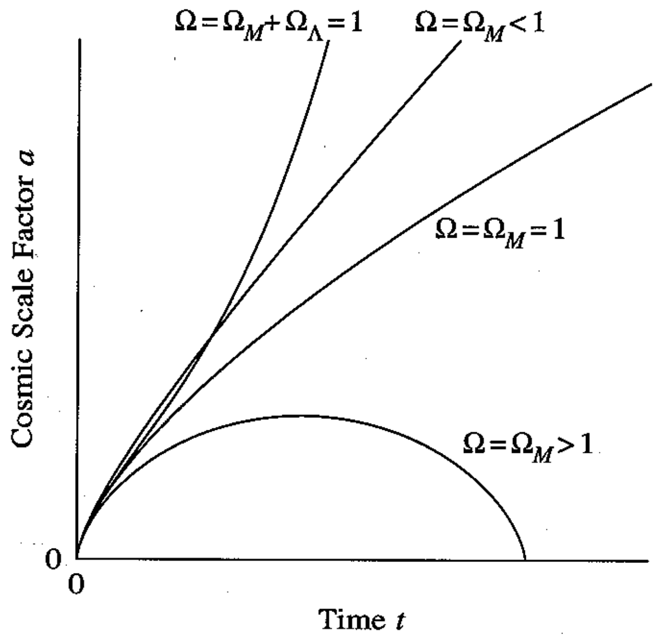
1. The universe is

closed	$(k > 0)$		$\Omega > 1$
flat	$(k = 0)$	as	$\Omega = 1$
open	$(k < 0)$		$\Omega < 1$

2. The acceleration (or deceleration) of the Universe depends on both mass-energy ρ AND pressure p.

Species		p/ρ	$(\rho+3p)/\rho$	
"Radiation"	relativistic particles: photons, neutrinos	$\frac{1}{3}$	2	attractive
"Matter"	non-relativistic massive particles	0	1	attractive
"Curvature"		$-\frac{1}{3}$	0	neutral
"Vacuum"	same as Einstein's cosmological constant	-1	-2	repulsive

Examples of evolution of Cosmic Scale Factor



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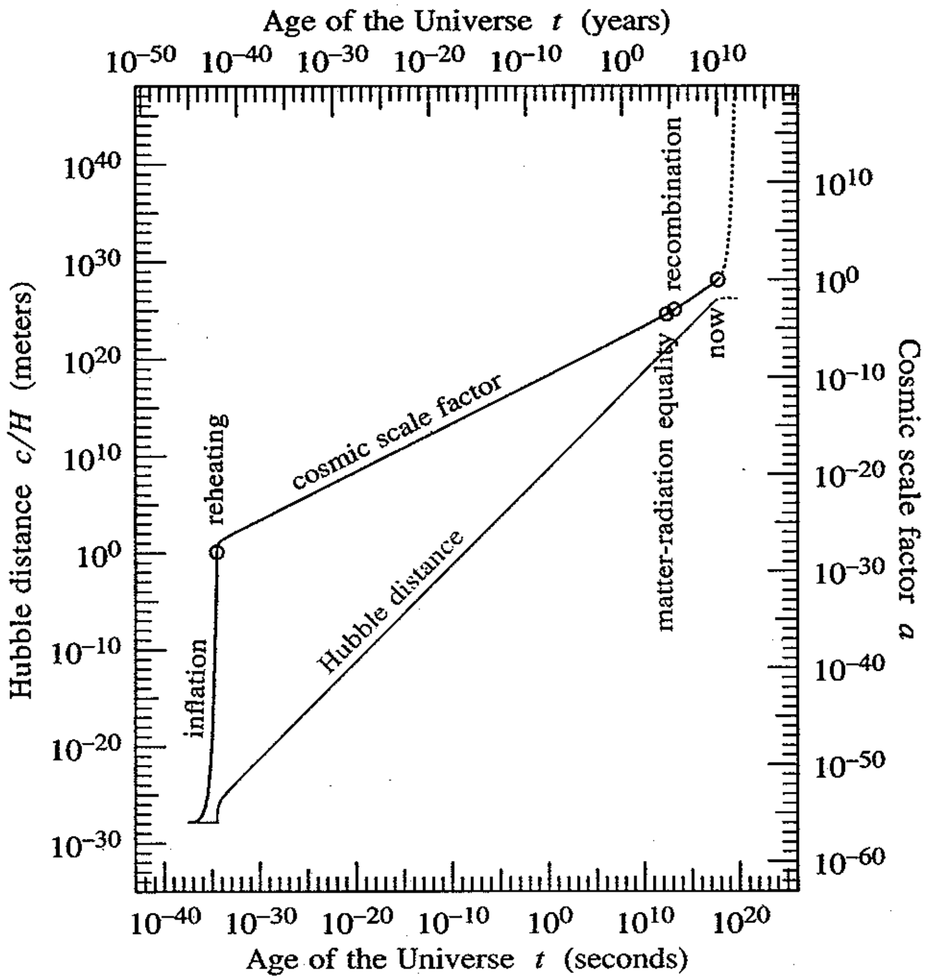


Figure 10.11 Cosmic scale factor a and Hubble distance c/H as a function of cosmic time t, for a flat ΛCDM model with the same parameters as in Figure 10.9. In this model, the Universe began with an inflationary epoch where the density was dominated by constant vacuum energy, the Hubble parameter H was constant, and the cosmic scale factor increased exponentially, $a \propto e^{Ht}$. The initial inflationary phase came to an end when the vacuum energy decayed into radiation energy, an event called reheating. The Universe then became radiation-dominated, evolving as $a \propto t^{1/2}$. At a redshift of $z_{\rm eq} \approx 3200$ the Universe passed through the epoch of matter-radiation equality, where the density of radiation equalled that of (non-baryonic plus baryonic) matter. Matter-radiation equality occurred just prior to recombination, at $z_{\rm rec} \approx 1090$. The Universe remained matter-dominated, evolving as $a \propto t^{2/3}$, until relatively recently (from a cosmological perspective). The Universe transitioned through matter-dark energy equality at $z_\Lambda \approx 0.4$. The dotted line shows how the cosmic scale factor and Hubble distance will evolve in the future, if the dark energy is a cosmological constant, and if it does not decay into some other form of energy.

Figure 10.12 shows the mass-energy density ρ as a function of time t for the same flat ΛCDM model as shown in Figure 10.11. Since the Universe here is taken to be flat, the density equals the critical density at all times, and is proportional to the inverse square of the Hubble distance c/H plotted in Figure 10.11.

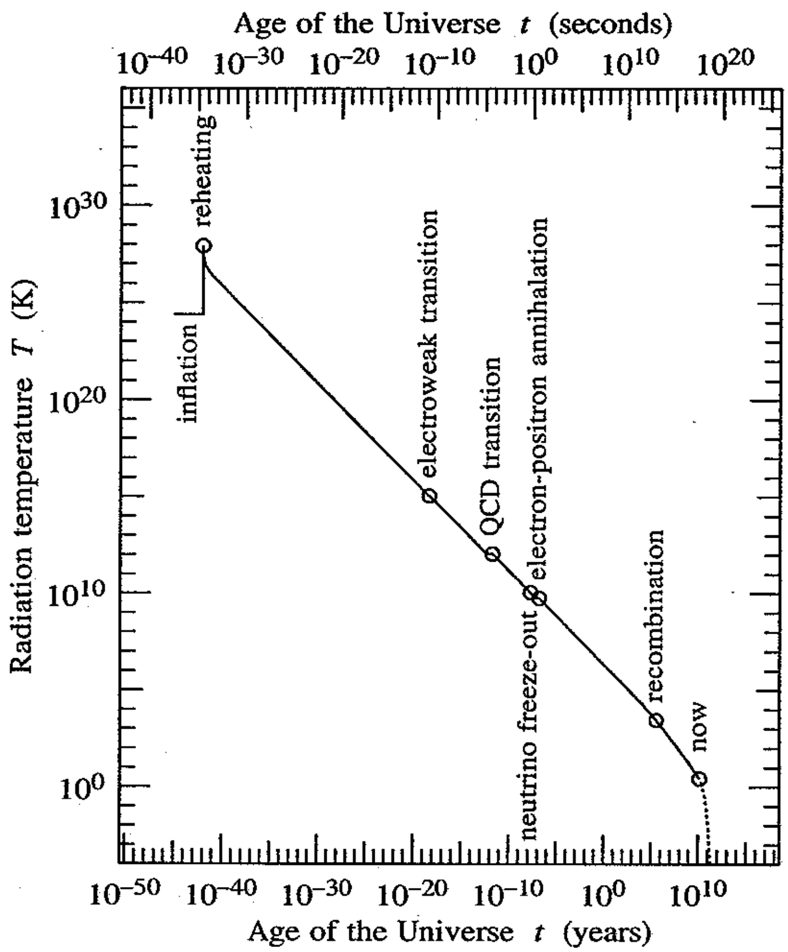


Figure 10.13 Radiation temperature T of the Universe as a function of cosmic time t corresponding to the evolution of the cosmic scale factor shown in Figure 10.11. The temperature during inflation was the Hawking temperature, equal to $H/(2\pi)$ in Planck units. After inflation and reheating, the temperature decreases as $T \propto a^{-1}$, modified by a factor depending on the effective entropy-weighted number g_s of particle species, equation (10.98). In this plot, the effective number g_s of relativistic particle species has been approximated as changing at three discrete points, electron-positron annihilation, the QCD phase transition, and the electroweak phase transition, Table 10.4.

Figure 10.13 shows the radiation (photon) temperature T as a function of time t corresponding to the evolution of the scale factor a and temperature T shown in Figures 10.11 and 10.12.

10.25 Neutrino mass

Neutrinos are created naturally by nucleosynthesis in the Sun, and by interaction of cosmic rays with the atmosphere. When a neutrino is created (or annihilated) by a weak interaction, it is created in a weak eigenstate. Observations of solar and atmospheric neutrinos indicate that neutrino species oscillate into each

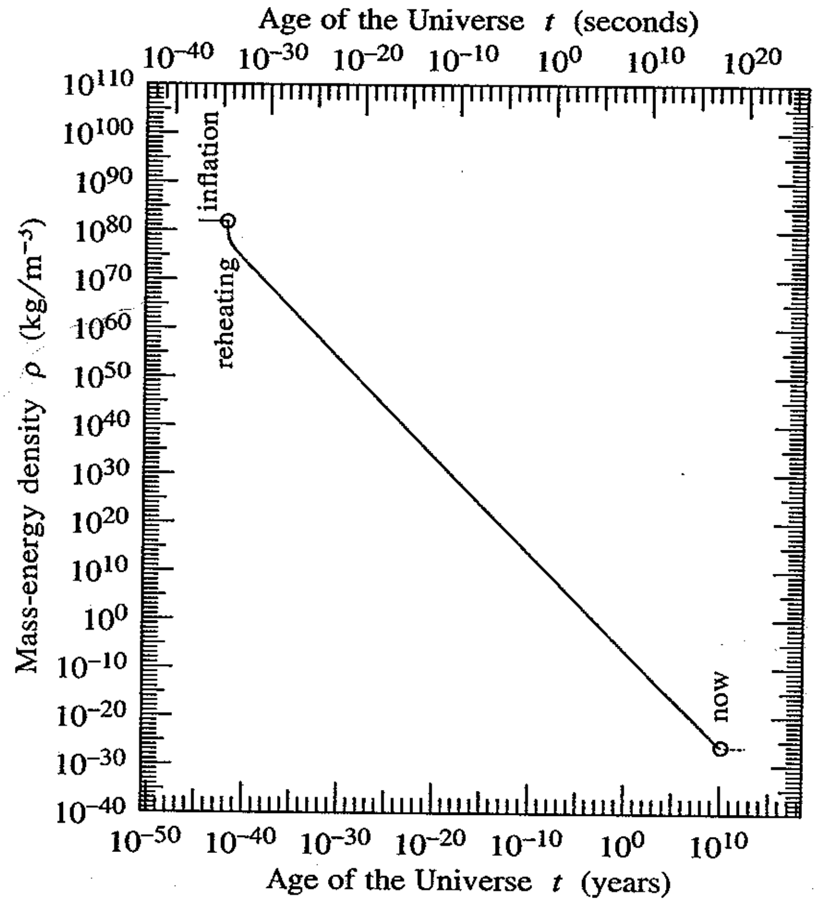


Figure 10.12 Mass-energy density ρ of the Universe as a function of cosmic time t corresponding to the evolution of the cosmic scale factor shown in Figure 10.11.

The energy density is constant during epochs dominated by vacuum energy, but decreases approximately as $\rho \propto t^{-2}$ at other times.

10.24 Evolution of the temperature of the Universe

A system of photons in thermodynamic equilibrium has a blackbody distribution of energies. The CMB has a precise blackbody spectrum, not because it is in thermodynamic equilibrium today, but rather because the CMB was in thermodynamic equilibrium with electrons and nuclei at the time of recombination, and the CMB has streamed more or less freely through the Universe since recombination. A thermal distribution of relativistic particles retains its thermal distribution in an expanding FLRW universe (albeit with a changing temperature), Exercise 10.12.

The evolution of the temperature of photons in the Universe can be deduced from conservation of entropy. The Friedmann equations imply the first law of thermodynamics, §10.9.2, and thus enforce conservation

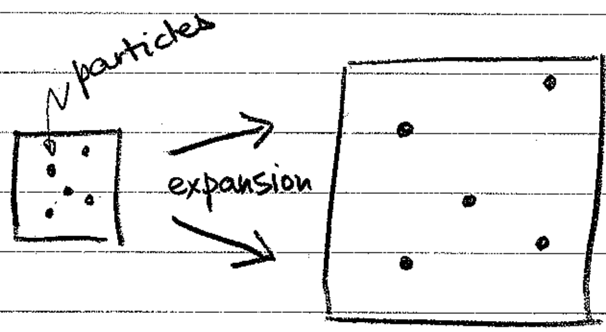
The 3 important types of mass-energy

1. Matter
2. Radiation
3. Vaccum

1. "Matter" includes baryons, non-baryonic Cold Dark Matter

Nonrelativistic stuff (velocities $\ll c$)
$\Rightarrow$ zero pressure

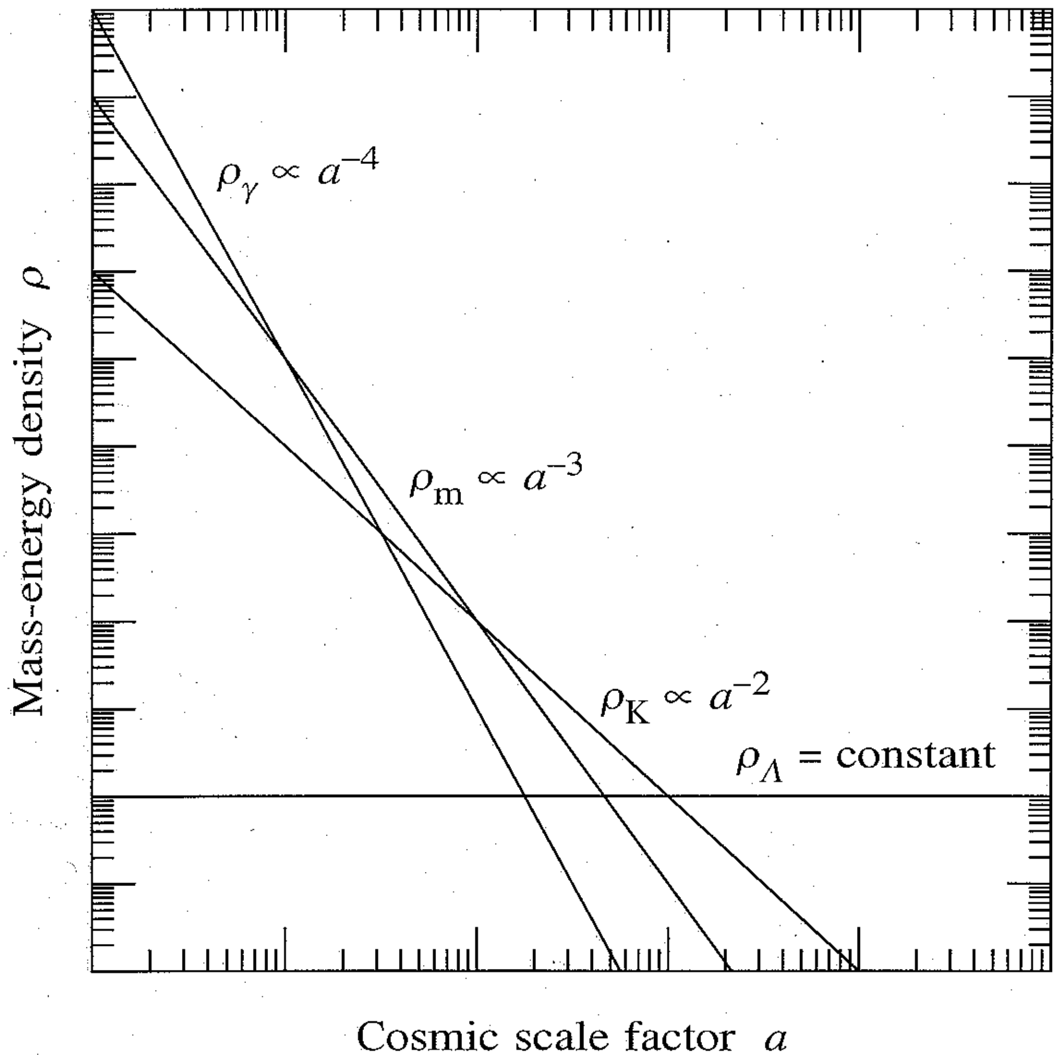
Conservation of mass in comoving volume



$$\Rightarrow \quad \boxed{\rho \propto a^{-3}} \qquad \text{"matter"}$$

Can show (PS # 6) that in a
flat matter-dominated Universe

$$\boxed{a \propto t^{2/3} \qquad (\Omega = \Omega_M = 1)}$$



$\rho_\gamma \propto a^{-4}$
$\rho_m \propto a^{-3}$
$\rho_K \propto a^{-2}$
ρ_Λ = constant
Mass-energy density ρ
Cosmic scale factor a

2. "Radiation" includes photons, light neutrinos

Relativistic stuff (velocities $\approx c$)
$\Rightarrow \; p = \frac{1}{3} \rho$

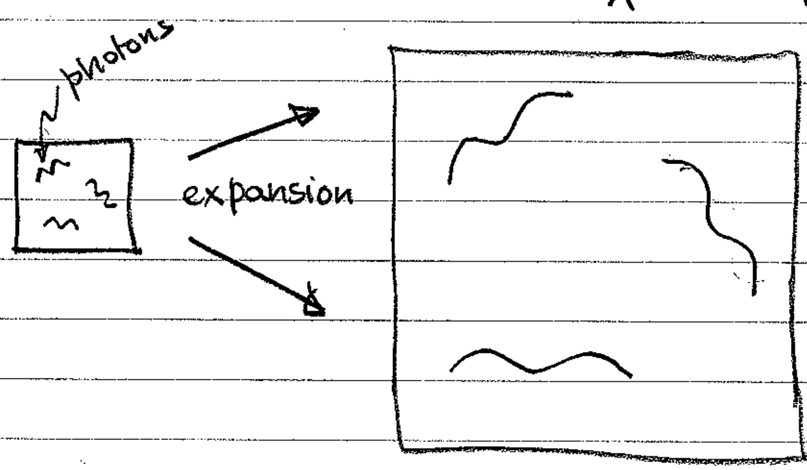
Note $\rho + 3p = 2\rho$,
so radiation gravitates twice as much as pressureless matter.

Number of particles in comoving volume is conserved, so $n \propto a^{-3}$

number density of photons/neutrinos

But in addition wavelength of particles expands with the Universe $\lambda \propto a$.
(Exact result in GR; not entirely trivial to derive). So energy per particle decreases as $E = h\nu = \dfrac{hc}{\lambda} \propto \dfrac{1}{a}$.

redshifts!



$$\Rightarrow \quad \boxed{\rho \propto a^{-4}} \qquad \text{"radiation"}$$

Can show (PS #7) that in a flat radiation-dominated Universe

$$\boxed{a \propto t^{1/2} \qquad (\Omega = \Omega_{rad} = 1)}$$

<u>When was Universe radiation-dominated?</u>

CMB dominates energy density in photons today.

$$\boxed{\Omega_{CMB}} = \frac{\rho_{CMB}}{\rho_c}$$

Stefan-Boltzmann radiation density constant

$$= \frac{a T_{CMB}^4 / c^2}{\frac{8\pi G}{3H_0^2}}$$

Blackbody formula

ρ_c

$$\boxed{= 2.48 \times 10^{-5}\ h^{-2}}$$

Ω in CMB today.

where $h = \dfrac{H_0}{100\ \mathrm{km/s/Mpc}}$

and $T_{CMB} = 2.725$ K observed.

But $\dfrac{\rho_{CMB}}{\rho_M} \propto \dfrac{a^{-4}}{a^{-3}} \propto a^{-1}$ (redshift factor)

If $\Omega_M \approx 0.35$ now then

$$\frac{\rho_{CMB}}{\rho_M} = \frac{\Omega_{CMB}}{\Omega_M} \approx 10^{-4} \text{ now}$$

cosmic scale factor today

so $$\frac{\rho_{CMB}}{\rho_M} = 10^{-4}\left(\frac{a_0}{a}\right)$$

$$= 1 \quad \text{when} \quad \frac{a}{a_0} \approx 10^{-4}$$

Conclusion: Universe was radiation-dominated when its size (a) was less than $\approx 10^{-4}$ times its current size

Notion of "redshift" of an epoch

$$\underbrace{1+z}_{\text{"redshift"}} \overset{\text{usual}}{\underset{\text{definition}}{\equiv}} \frac{\lambda_{obs}}{\lambda_{emit}} \overset{\text{cosmological}}{\underset{\text{case}}{=}} \frac{a_0}{a}$$

CSF now

CSF at time of emission.

so there is correspondence between redshift z and cosmic scale factor a.

Example:

Universe was radiation-dominated at redshifts $z \gtrsim 10^4 \underbrace{-1}_{\text{ignore this compared to } 10^4}$

3. "Vacuum" same as Cosmological Constant

$\rho = \rho_\Lambda =$ constant; $\quad p = -\rho$.

Einstein's cosmological constant Λ is related to ρ_Λ by

$$\Lambda = 8\pi G \rho_\Lambda$$

Can show (PS #7) that in a flat vacuum-dominated Universe

$$\boxed{a \propto e^{H_\Lambda t} \qquad (\Omega = \Omega_\Lambda = 1)}$$

Hubble constant, here constant in time as well as space

Pressure vs. equation of state

Index n of $\rho \propto a^{-n}$ and pressure p are related.
Can show from $d(\rho a^3) + p\, d(a^3) = 0$

$$[\equiv dE + p\,dV = 0]$$

that $\boxed{n = 3\left(1 + \frac{p}{\rho}\right)}$

4. "Curvature density" not really a form of energy

Define curvature density ρ_K by

$$\frac{8}{3}\pi G \rho_K \equiv -\frac{k c^2}{a^2}$$

Then $\boxed{\rho_K \propto a^{-2}}$

Then

$$\underset{\substack{\text{critical}\\\text{density}}}{\rho_c} = \underset{\text{radiation}}{\rho_\gamma} + \underset{\text{matter}}{\rho_M} + \underset{\text{curvature}}{\rho_K} + \underset{\text{vacuum}}{\rho_\Lambda}$$

Cute way to treat curvature as if it were a form of energy, which it is not.

Equivalently

$$1 = \Omega_\gamma + \Omega_M + \Omega_K + \Omega_\Lambda$$

which is really just a cute way to re-write the first Friedmann equation.

Cosmic Microwave Background Radiation (CMB)

Important concepts to grasp:

1. Universe was once in Thermodynamic Equilibrium.
2. $\lambda \propto a \implies T \propto \frac{1}{a}$

   Wavelength $\propto$ Cosmic Scale Factor Temperature $\propto$ 1/Cosmic Scale Factor.
3. Horizon problem.
4. Isotropy problem.
5. The power spectrum of fluctuations in the CMB has yielded precise measurements of cosmological parameters.
6. The next huge step:
   Future measurements of the power spectrum of <u>polarization</u> of the CMB will (!)* measure primordial gravitational waves generated during inflation.

* Discovered by BICEP team March 2014.

CMB

Predicted by Gamow (1948)
Dicke & Peebles (1964)
Discovered by Penzias & Wilson (1964) Nobel prize

Observational facts

- Planck, ie. blackbody, spectrum to high accuracy (COBE 1990)

$$\boxed{T_{CMB} = 2.725 \pm .001 \text{ K}}$$ (Fixsen et al 2002)

- Almost uniform. Main departure from uniformity is dipole pattern, consistent with notion that the Local Group of galaxies is moving through the CMB at

$$\boxed{627 \pm 22 \text{ km/s}}$$

towards galactic longitude (ℓ), latitude (b)

$$\boxed{\ell = 276° \pm 3°, \quad b = 30° \pm 3°}$$

in constellation of Hydra.
(These numbers are after correction for rotation of solar system about MW center, and small correction for motion of MW in the Local Group.
See Kogut el al 1993 ApJ 419, 1 for details).

- After subtraction of dipole, residual fluctuations are at level

$$\boxed{\frac{\Delta T}{T} \sim \text{few} \times 10^{-5}} \qquad \text{(COBE 1990)}$$

ie. CMB is amazingly isotropic.

Detailed character of fluctuations described by its power spectrum C_ℓ

$$C_\ell \equiv \left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle_\ell$$

which is the mean square temperature fluctuation at harmonic number ℓ.

CMB — Theoretical Interpretation

- Remnant of Big Bang fireball, cooled (redshifted) by expansion of Universe

$$T \propto \nu \propto \frac{1}{\lambda} \propto \frac{1}{a}$$

T: measure of energy per photon; ν: frequency; $\frac{1}{\text{wavelength}}$; $\frac{1}{\text{cosmic scale factor}}$.

- Comes from epoch of <u>Recombination</u>

$$t \sim 4 \times 10^5 \text{ years old}, \quad T \approx 3000\,\text{K}$$

when H (and He) (re)combined from ionized to neutral.

Q: Why is the temperature $T \approx 3000\,\text{K}$ at Recombination approximately the same as the surface temperature $T_\odot \approx 6000\,\text{K}$ of the Sun?

- Most distant thing we can see in electromagnetic radiation;
Before Rec, Universe was <u>ionized</u>, <u>opaque</u>;
after Rec, " " <u>neutral</u>, <u>transparent</u>.

- Before Recombination, "baryonic" matter was tightly coupled to radiation; no tendency for baryons to cluster gravitationally. Sound speed = fraction of c.

  Jean's criterion (rigorous formulation exists):

sound speed > gravitational escape velocity	:	fluctuations oscillate as sound waves.
sound speed < gravitational escape velocity	:	fluctuations collapse.

  After Recombination, sound speed in baryonic matter drops precipitously, and it can start to cluster, forming stars, galaxies ...

- Details of gravitational growth of fluctuations before (and after) Recombination depends on the relative amounts of:
  - radiation
  - baryonic matter
  - neutrinos (=Hot Dark Matter)
  - non-baryonic non-relativistic particles (= Cold Dark Matter).

  These affect the power spectrum C_ℓ of temperature fluctuations of the CMB in various ways.

Wilkinson Microwave Anisotropy Probe, launched Fall 2000

Cosmological parameters from CMB power spectrum measured by ~~WMAP (2003, updated 2006)~~
Planck satellite (2013)

Amazingly, a 6-parameter "vanilla" model fits all the data precisely.

1. Hubble constant H_0
Actually, this is the one parameter that is NOT well measured by WMAP alone.
Can be measured from
- distances to "standard candles" (Cepheids)
- galaxy clustering.

Density Ω of various species

2. $\Omega_{tot} = 1.00 \pm .01$ nearly flat
3. $\Omega_{CDM} = 0.26$ non-baryonic Cold Dark Matter
4. $\Omega_b = 0.05$ ordinary "baryonic" matter

The flatness of the Universe was first established from CMB observations by the Boomerang balloon experiment (2000), which measured $\Omega_{tot} = 1.0 \pm .1$.

The CMB provides the most direct evidence for non-baryonic Cold Dark Matter. Consistent with ubiquitous evidence for unseen Dark Matter in galaxies and galaxy clusters.

$\Omega_{tot} = 1$ and $\Omega_M = \Omega_{CDM} + \Omega_b = 0.3$ are in nice agreement with galaxy clustering, which indicates $\Omega_M = 0.2 - 0.3$ in matter and with the Hubble diagram of high redshift supernovae, which indicate $\Omega_\Lambda = 0.7$ in Dark Energy.

$\Omega_b = 0.04$ in ordinary matter is in excellent agreement with independent measurements from comparing the predictions of Big Bang Nucleosynthesis to the observed abundances of light elements (H, He, D, ^3He, Li).

<u>Parameters describing primordial fluctuations</u>

5. A = amplitude
6. n = slope = 0.96 (slightly less than 1).

Quantum fluctuations in the vacuum generated during inflation predict fluctuations that are:

- "Gaussian" = most random kind of fluctuation
- "Adiabatic" = all kinds of radiation, matter fluctuate together, in proportion to each other
- "Scale-invariant" = definite slope n which should be slightly less than 1.

Remarkably, the observed fluctuations agree precisely with all these predictions!

The Hubble distance

$$\boxed{\text{Hubble distance} \equiv \frac{c}{H}}$$

occasionally called "the cosmological horizon", but it is not actually the horizon.

Sets approximate <u>proper</u> distance within which objects are in causal contact at any epoch.

$$\boxed{\text{Comoving Hubble distance} \quad x_H \equiv \frac{c}{aH}}$$

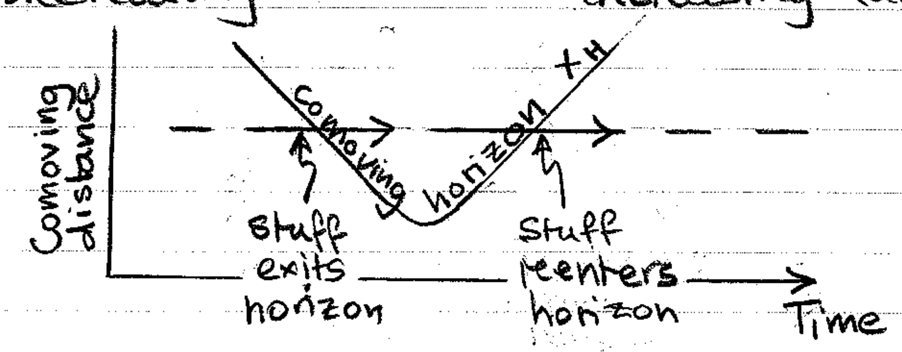
is corresponding <u>comoving</u> distance within which objects are in causal contact.

Are objects "coming in over the horizon" or "going out over the horizon"?

Have $x_H = \dfrac{c}{a\underbrace{H}_{=\frac{\dot a}{a}}} = \dfrac{c}{\dot a}$

So

x_H is increasing if $\dot a$ is decreasing (decelerating)
" " decreasing " " " increasing (accelerating)



Example: Hubble distance in flat, matter-dominated ($\Omega = \Omega_M = 1$) Universe

PS # 6 $\Rightarrow$ $a \propto t^{2/3}$

ie $t \propto a^{3/2}$ in this case.

Have

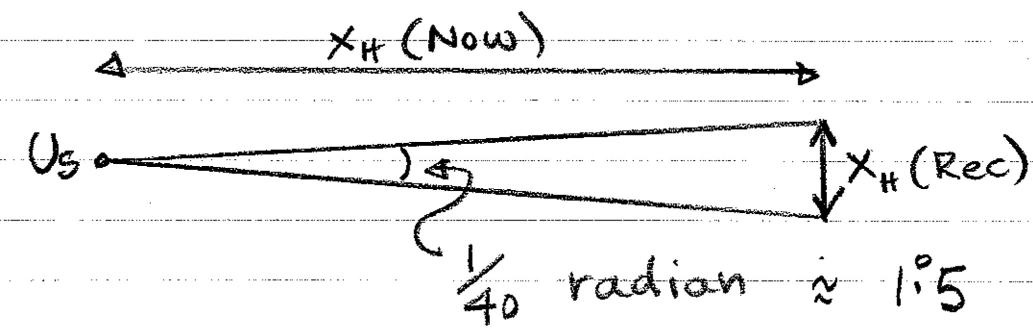
$$x_H = c\frac{dt}{da}$$
$$\propto a^{1/2}$$

Example:

redshift at Recombination

$$\frac{x_H\,(\text{Recombination})}{x_H\,(\text{Now})} = \left(\frac{a_R}{a_0}\right)^{\frac{1}{2}} = \left(1 + z_R\right)^{-1/2}$$
$$\approx 1300^{-1/2}$$
$$\approx 1/40$$

Hence estimate of angular scale over which objects in CMB were in causal contact at Recombination:



Horizon size at Recombination

for matter-dominated Universe
corresponds to

Omega today

$$\text{Angle on CMB} = \left(\frac{\Omega_0}{1+z_R} \right)^{1/2}$$

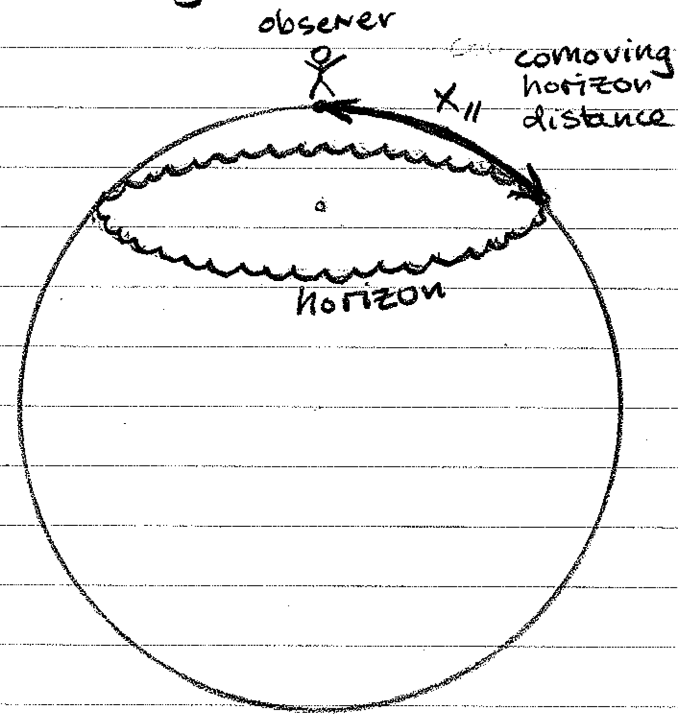
redshift $z_R \approx 1300$
at Recombination

$$\boxed{\text{Angular size of horizon at Rec, as observed on CMB} = 1^\circ 59 \; \Omega_0^{1/2}}$$

or "Particle horizon"

Horizon (mathematics)

Light rays follow null-geodesics.
In FRW geometry, spatial geodesics are great circles, which photons follow.



Null geodesic satisfies

$$ds^2 = dt^2 - a^2 dx_{\parallel}^2$$
$$= 0$$

whence

$$\frac{a\,dx_{\parallel}}{dt} = 1$$

Horizon is how far light can travel from Big Bang $(t=0)$ to now $(t = t_0)$.
So comoving horizon distance is

$$x_{\parallel} = \int_0^{t_0} \frac{dt}{a} .$$

Convert this to comoving distance evaluated in present day units by multiplying by cosmic scale factor today a_0.
So comoving horizon distance in today's units is

$$\boxed{a_0 x_{\parallel} = a_0 \int_{0}^{t_0} \frac{dt}{a}}$$

now

Big Bang

Example

Matter-dominated flat Universe, $\Omega = \Omega_M = 1$. ("matter")

Here $a \propto t^{2/3}$.

So horizon distance is

$$a_0 x_{//} = a_0 \int_0^{t_0} \frac{dt}{a}$$
$$= t_0^{2/3} \int_0^{t_0} \frac{dt}{t^{2/3}}$$
$$= t_0^{2/3} \left[3t^{1/3} \right]_0^{t_0}$$
$$= 3t_0 \qquad \text{(note units } c = 1\text{)}.$$

Restoring c,
comoving horizon distance is

$$\boxed{a_0 x_{//} = 3ct_0 \qquad (\text{case } \Omega = \Omega_M = 1)}$$

Or in terms of Hubble constant:

$$H = \frac{da}{a\,dt} \underset{a \propto t^{2/3}}{\overset{\text{for}}{=}} \frac{2}{3t}$$

whence

$$\boxed{a_0 x_{//} = \frac{2c}{H_0} \qquad (\text{case } \Omega = \Omega_M = 1)}$$

e.g. $\sim 10^{-43}$ s Planck inflation
or $\sim 10^{-34}$ s GUT inflation

INFLATION

A hypothesized epoch in the very early Universe when the mass-energy density of the Universe was dominated by vacuum energy.

Vacuum energy has $p = -\rho$, so is gravitationally repulsive ($\rho + 3p < 0$).

Inflation solves:

1. The Horizon Problem
2. The Flatness Problem
3. Why the Universe is expanding
4. Where matter / radiation came from (vacuum energy decayed into it)
5. Origin of fluctuations in the Universe (quantum mechanical vacuum fluctuations generated during inflation).

Inflation predicts (in the simplest models)

- $\Omega \overset{\text{very}}{\underset{\text{nearly}}{=}} 1$ today, with high probability
- near "scale-invariant" spectrum of fluctuations today

$\langle \phi^2 \rangle$ = same on all scales.

ϕ = gravitational potential.

$\sin\theta_w$ 8

where $\alpha_i = g_i^2/4\pi$, are so different from the values observed at low energies that grand unification appears to be ruled out immediately. However, it must be remembered that the couplings are scale-dependent and so we must use the renormalization group equations of Section 2.7 to continue these relations from M_X down to the energies at which the α_i have been measured.

In the one-loop (or leading log) approximation, the solution (2.88) to the renormalization group equation gives

$$\frac{1}{\alpha_{GUT}} \equiv \frac{1}{\alpha_i(M_X^2)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{4\pi}\log\left(\frac{M_X^2}{\mu^2}\right) \tag{7.14}$$

with

$$\begin{aligned} b_3 &= 11 - \tfrac{4}{3}N_g, \\ b_2 &= \tfrac{22}{3} - \tfrac{4}{3}N_g - \tfrac{1}{6}N_H, \\ b_1 &= \quad\;\; - \tfrac{4}{3}N_g - \tfrac{1}{10}N_H \end{aligned} \tag{7.15}$$

(here $b_i = (b_0)_i$ of (2.89) with $i = 3, 2, 1$ for $SU(i)$), where N_g is the number of families (or generations) of fermions and N_H is the number of Higgs doublets in the electroweak sector. The individual terms in (7.15) correspond, respectively, to the contributions to b_i from the gauge boson, fermion and Higgs loops, like Fig. 2.6. The behavior (7.14) of the $1/\alpha_i$ with $N_g = 3$ and $N_H = 1$ is shown in Fig. 7.1 and reflects the property that

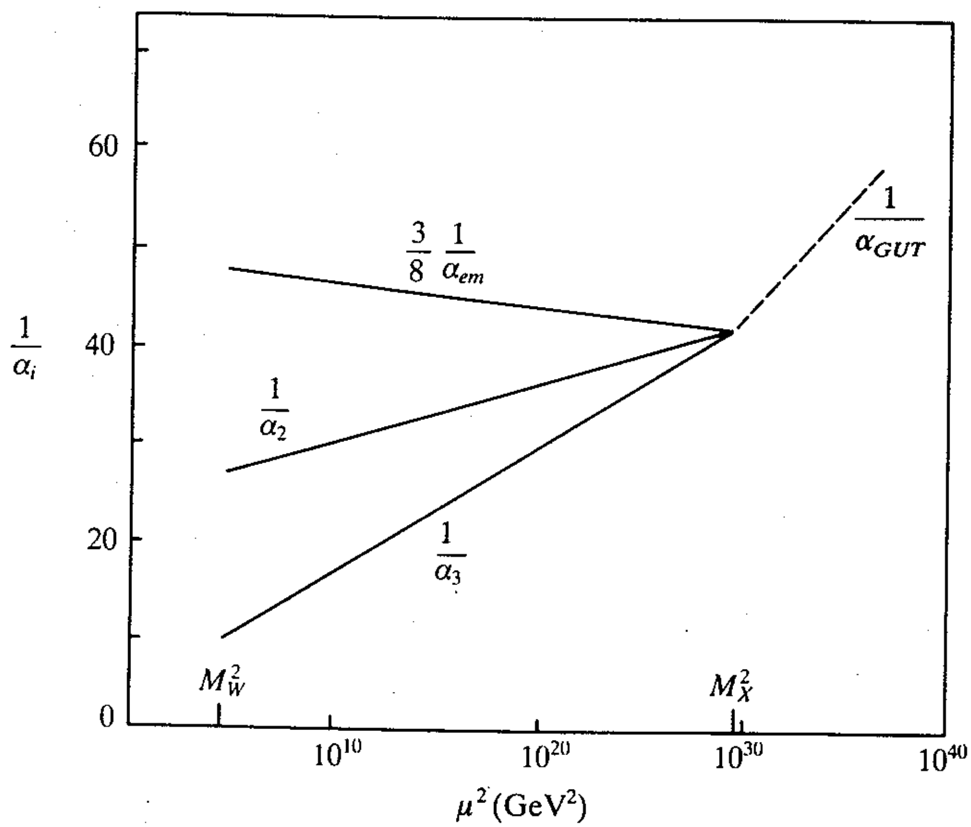
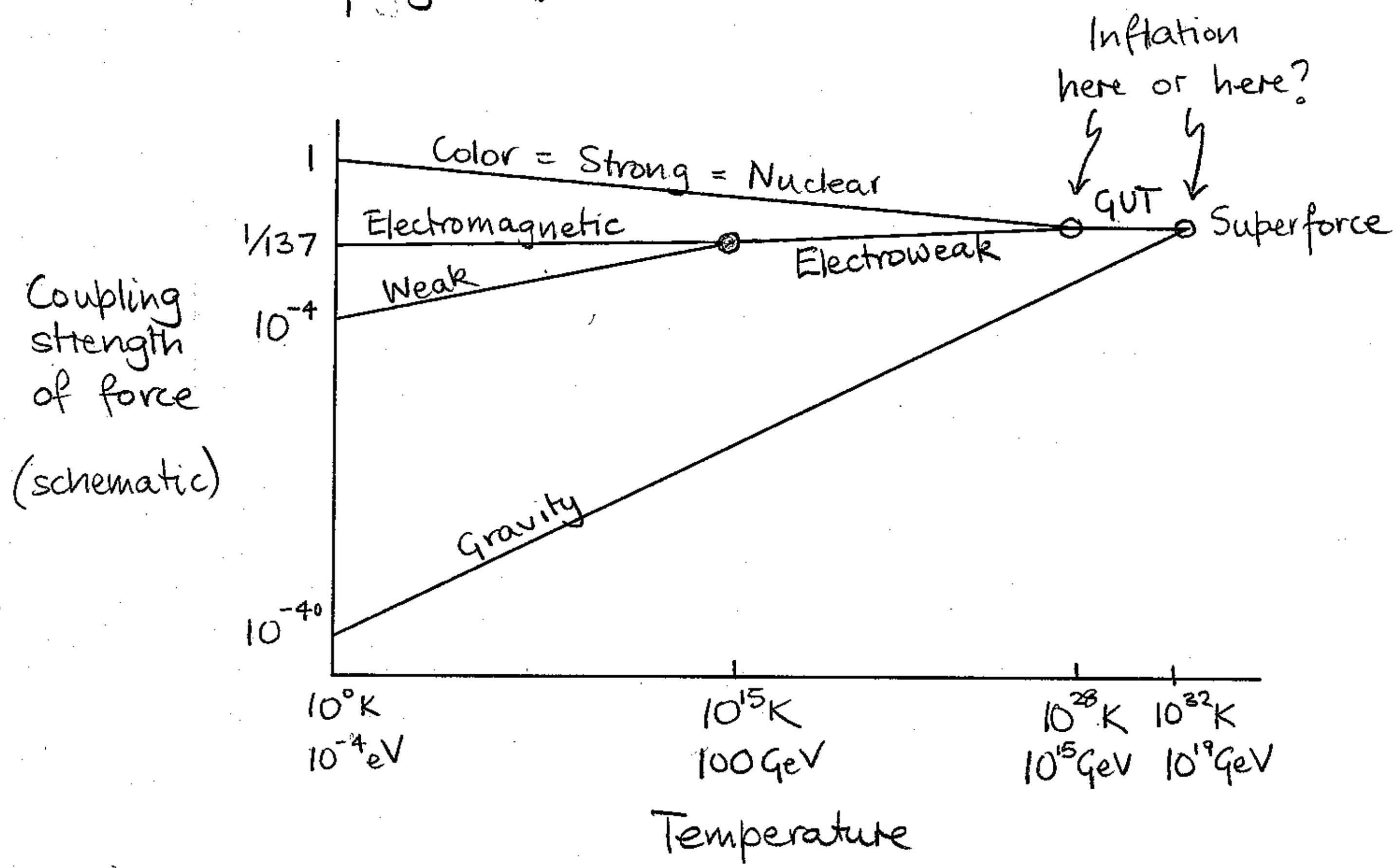


Fig. 7.1 The variation of the effective coupling constants, $\alpha_i(\mu)$ with the energy scale μ; see (7.14).

Motivation for Inflation

Particle physics!



Steven Weinberg, Abduus Salam won Nobel Prize (1974) for their "SU(2) × U(1)" Electroweak theory. This remains only unification well tested in particle accelerators.

like water ⇔ ice

(de) Unification is like phase transition: Vacuum energy is "latent heat" of transition.

4. Where matter/radiation came from

Inflation must have ended
- vacuum energy today is tiny
- galaxies have clustered (attractive gravity).

Presumably vacuum energy decayed into matter/radiation at end of inflation.

Interesting problem:

To solve the horizon/flatness problems requires $\gtrsim 70$ e-folds of inflation.

If vacuum energy can decay, why didn't it decay immediately? like water freezes into ice as soon as the temperature drops below 0°C.

Possible answer 1:

The decay occurs unnaturally slowly.

Possible answer 2:

Inflation occurs near the Planck scale.

Argument: use Planck units $c = \hbar = G = 1$ for simplicity.

S'pose transition mass/energy/temperature is

$$m = E = T.$$

Heisenberg uncertainty principle $\Delta t . \Delta E \approx \hbar$

$$\Rightarrow \quad t_{decay} \approx \frac{1}{E} = \frac{1}{m}$$

By comparison, expansion time is

$$t_{exp} \approx \frac{1}{H} \approx \frac{1}{\rho^{1/2}} \approx \frac{1}{T^2} = \frac{1}{m^2}$$

Then $t_{decay} > t_{exp} \quad \Rightarrow \quad \boxed{m > 1 \text{ Planck mass}}$

5. Generation of fluctuations during inflation

Picture:

quantum mechanical

- During inflation, random "vacuum" fluctuations in the gravitational potential ϕ freeze in when they expand over the horizon.

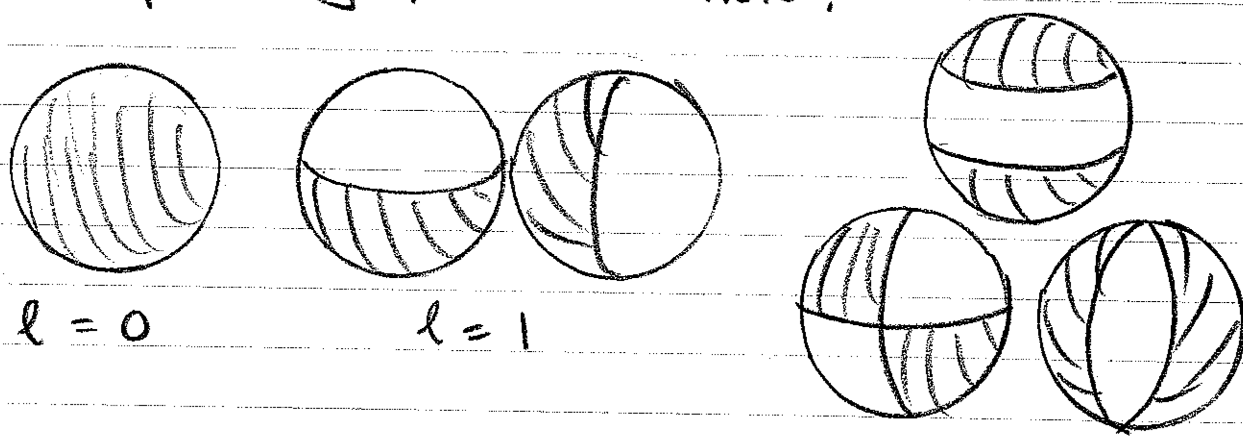
- These same fluctuations come back in over the horizon today, producing
  - CMB fluctuations
  - seeds which grow by gravity into galaxy clusters, galaxies, stars.

Power Spectrum

Step 1

Resolve temperature fluctuations in the CMB into $\boxed{\text{harmonics}}$, labeled by harmonic number ℓ.

modes of vibration of a sphere. with definite frequency
Each harmonic produces a pure "note".
For each harmonic ℓ, there are $2\ell+1$ separate modes producing the same "note".



$\ell = 0$ $\ell = 1$ $\ell = 2$

Step 2

Define power spectrum C_ℓ to be the mean square fluctuation in the ℓ'th harmonic modes

$$\boxed{C_\ell = \left\langle \left(\frac{\delta T}{T} \right)^2_\ell \right\rangle}$$

average over $2\ell+1$ modes of ℓ'th harmonic

mean $= 2.728\,K$

Evolution of fluctuations

0. Origin of fluctuations

Scale-invariant spectrum (C_ℓ = constant)
laid down during inflation.

1. Comoving wavelengths > horizon

Amplitudes of fluctuations are frozen
in while outside the horizon

2. Comoving wavelengths < horizon, Before Recombination

(a) Baryons tightly coupled to photons
Baryon-photon fluid oscillates
as sound waves. comoving wavelength remains constant, like photons
The sound waves redshift with expansion,
so their amplitude decreases with time.
Friction further decreases amplitude
at smaller wavelengths.

(b) Non-baryonic Dark Matter,
Not coupled to photon-baryon fluid.
(i) Cold Dark Matter (non relativistic).
CDM density fluctuations
grow by gravity.
Growth is faster after $1+z_{eq} \approx 10^4$,
when Universe transitions from
radiation- to CDM matter-dominated.

(ii) Hot Dark Matter (relativistic)
i.e. neutrinos.
Relativistic free-streaming tends to erase fluctuations inside horizon.

3. <u>Comoving wavelengths < horizon, After Recombination</u>

Baryons decouple from photons.

(a) Photons stream freely through the Universe, producing the CMB today.

(b) Baryons fall into the gravitational potential wells of CDM fluctuations.

(c) Sufficiently massive neutrinos become non-relativistic, allowing HDM fluctuations to start growing.

(d) CDM + baryons (+ HDM ?) begin process of collapsing into galaxies.

3.49

Lineweaver & Barbosa (1998) ApJ 496, 624.

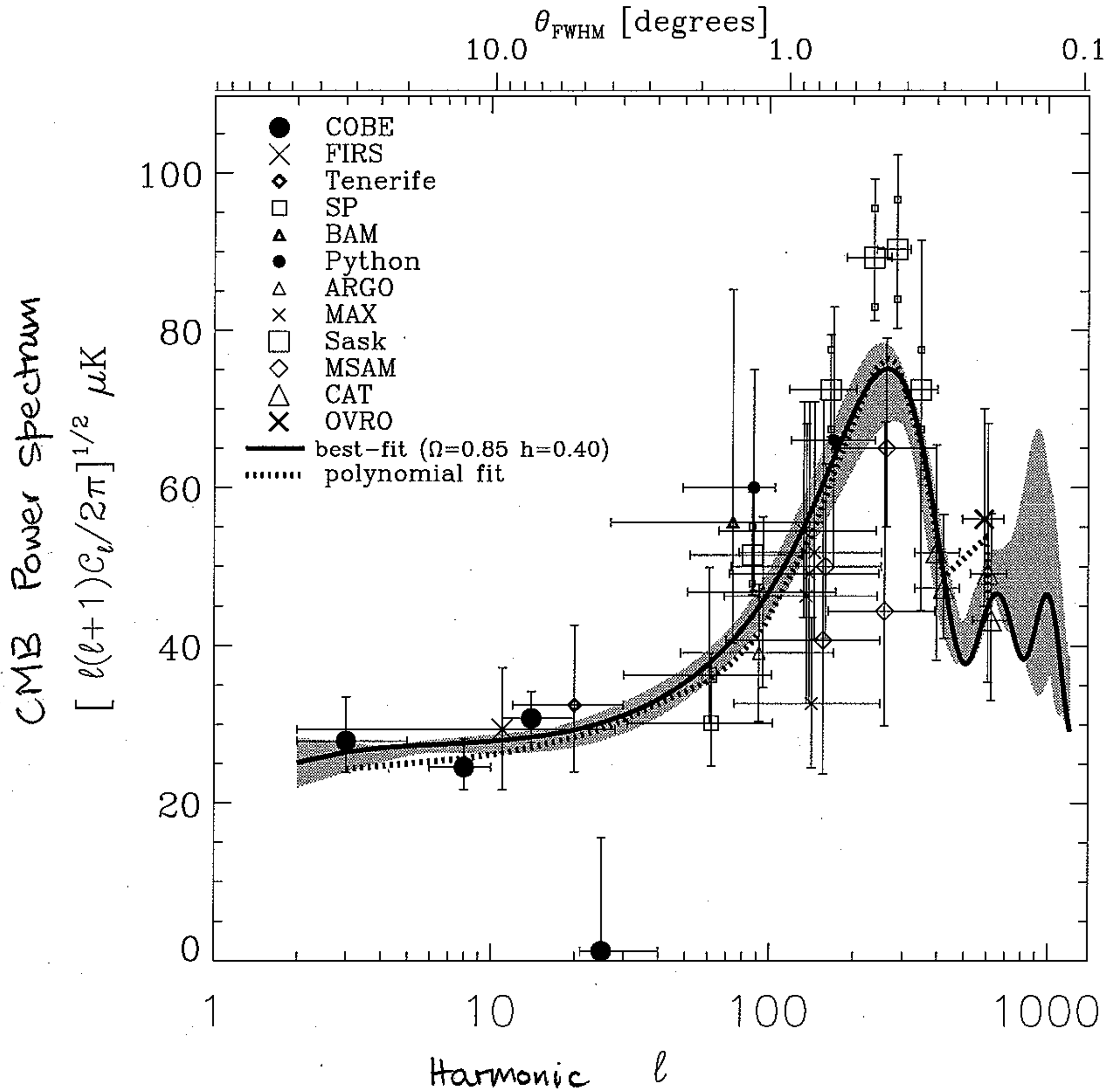


Fig. 1.— Recent CMB observations compared with the best-fit model from Figure 4 The dotted line is a sixth order polynomial fit to the data which has a peak amplitude and position: $A_{peak} \approx 77\,\mu$K and $\ell_{peak} \approx 260$. The grey region represents the $\sim 1\sigma$ contour in Figure 4; that is, the power spectra from models within $\sim 1\sigma$ of the best-fit model are contained within the grey region. The small squares above and below the 5 Saskatoon points represent the 7% correlated calibration uncertainty (Leitch 1998). The best-fit model has $n = 0.91$, $Q_{10} = 18.0\,\mu$K and $\Omega_b h^2 = 0.026$.

an axis yielding 1, 2, 3 and 4σ confidence intervals is described in Press *et al.* (1992 p 690) (see also Avni 1976). These conditions are: i) the errors are normally distributed and ii) the model is linear in the parameters *or* that a linear approximation reasonably represents the models within

The End of Cosmology?

The power spectrum of temperature fluctuations in the CMB that will be measured by the MAP, then Planck, satellites WILL* determine:

* Done.

- the curvature of the Universe, from the position of the first acoustic peak in the power spectrum, which should occur at the horizon size

$$\boxed{\Omega \approx \left(\frac{200}{\ell_{peak}}\right)^2}$$

Ω: total Omega in everything

eg. $\ell_{peak} = 200 \Rightarrow \Omega = 1$

$\ell_{peak} = 400 \Rightarrow \Omega = .25$

- other cosmological parameters

H_0, Ω_b (bargyons), Ω_M (matter (CDM)), Ω_Λ (cosmo constant), Ω_ν (neutrinos (HDM)), tilt (departure from scale-invariance of primordial fluctuations)

from detailed shape of the power spectrum.

See: • Max Tegmark's
http://www.sns.ias.edu/~max/cmb/experiments.html
for latest data on CMB spectrum,
• Wayne Hu's
http://www.sns.ias.edu/~whu/physics/physics.html
for superb presentation of theory.

CMB Satellites

COBE (Cosmic Background Explorer)

http://www.gsfc.nasa.gov/astro/cobe/cobe_home.html
NASA. Launched Nov 1989.
Spent next 4 years mapping the microwave sky at 7° resolution.

MAP (Microwave Anisotropy Probe)

http://map.gsfc.nasa.gov/
NASA. Approved 1997 for launch in Fall 2000.
Mapped the microwave sky to $\sim 1/3^\circ$ resolution.
9 years' observation. Final results 2012

Planck

http://astro.estec.esa.nl/SA-general/Projects/Planck/
ESA (European Space Agency).
Launched 2009.
1st year observations published 2013

Evidence for DM (Dark Matter)

Astronomers conventionally characterize amount of matter in galaxies by Mass-to-Light ratio $\frac{M}{L}$ in solar units.

Mean Luminosity density of nearby Universe is measured to be

$$\mathcal{L} = 1.2 \times 10^{8} \frac{L_\odot}{Mpc^3} \qquad (H_0 = 65 \text{ km/s/Mpc})$$

To achieve $\Omega = 1$ requires

$$\boxed{\left.\frac{M}{L}\right|_{critical} = 1000 \frac{M_\odot}{L_\odot}} \qquad (H_0 = 65 \text{ km/s/Mpc})$$

1. Rotation curves of spiral galaxies

$\Rightarrow$ increasing fraction of DM outward.

eg. $\frac{M}{L} \approx 3 \frac{M_\odot}{L_\odot}$ in Milky Way at solar neighborhood.

In well-observed cases (eg. NGC 3198) $\frac{M}{L}$ rises to $\sim 15 \frac{M_\odot}{L_\odot}$ in outer parts.

NGC 3198

3.56

Binney & Tremaine ('87) Galactic Dynamics p. 602.

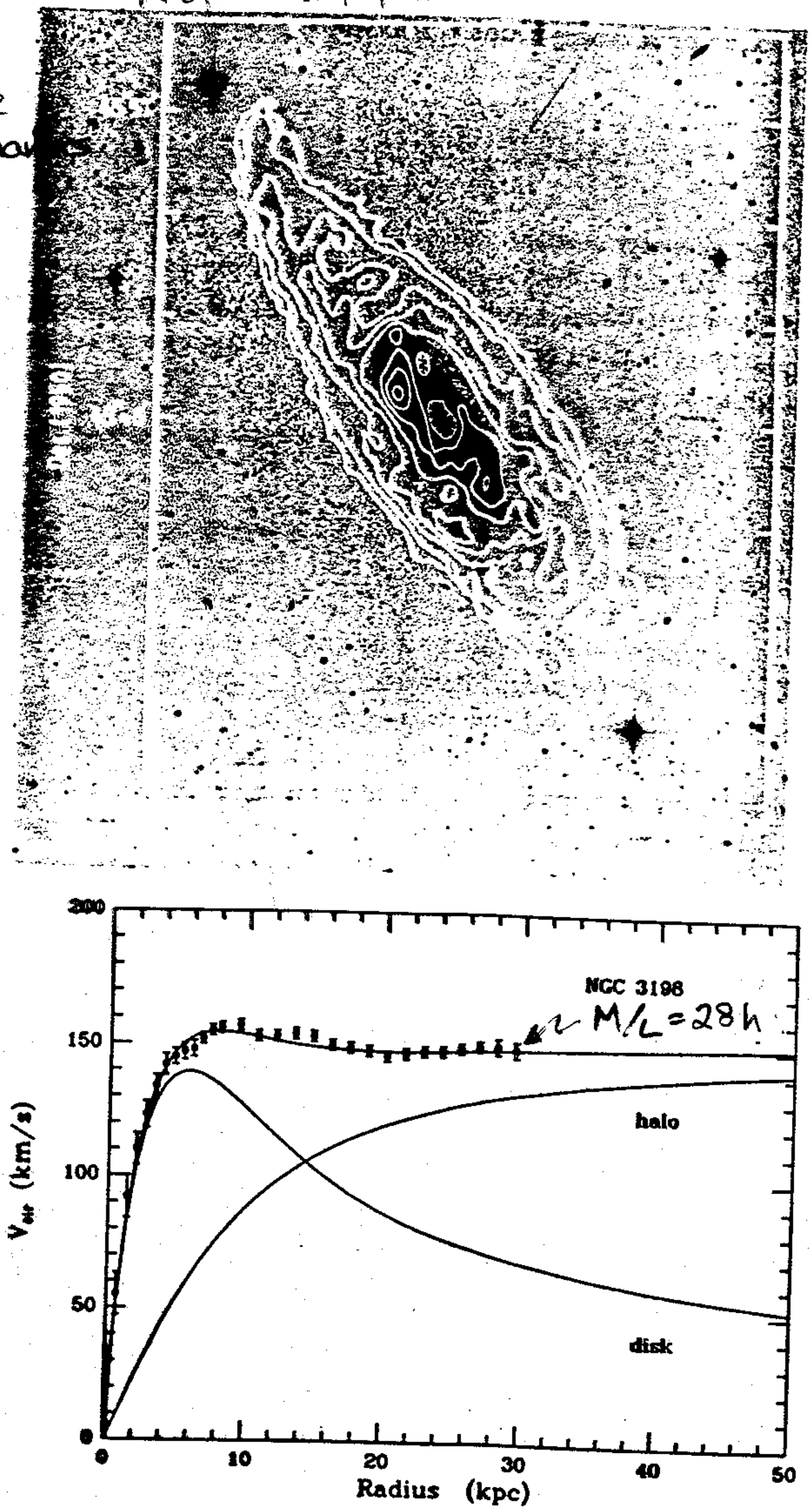


Figure 10-2. The Sc galaxy NGC 3198. Top: neutral hydrogen column density contours superimposed on an optical photograph. Bottom: circular-speed curve plus model fits using an exponential disk with constant mass-to-light ratio and the halo density profile (10-10). The model curve is for the maximum possible disk mass-to-light ratio. The horizontal scale assumes $h = 0.75$. Reprinted from van Albada et al. (1985), by permission of *The Astrophysical Journal.*

2. Dynamics of groups & clusters of galaxies

e.g. in Local Group
dynamics of MW – Andromeda
(enough mass to turnaround in age of Universe)

$$\Rightarrow \frac{M}{L} \approx 100$$

In rich clusters, such as Coma,

$$\boxed{\frac{M}{L} \approx 300}$$

To derive M, use variants of virial theorem

$$\frac{GM}{R} = \langle v^2 \rangle$$

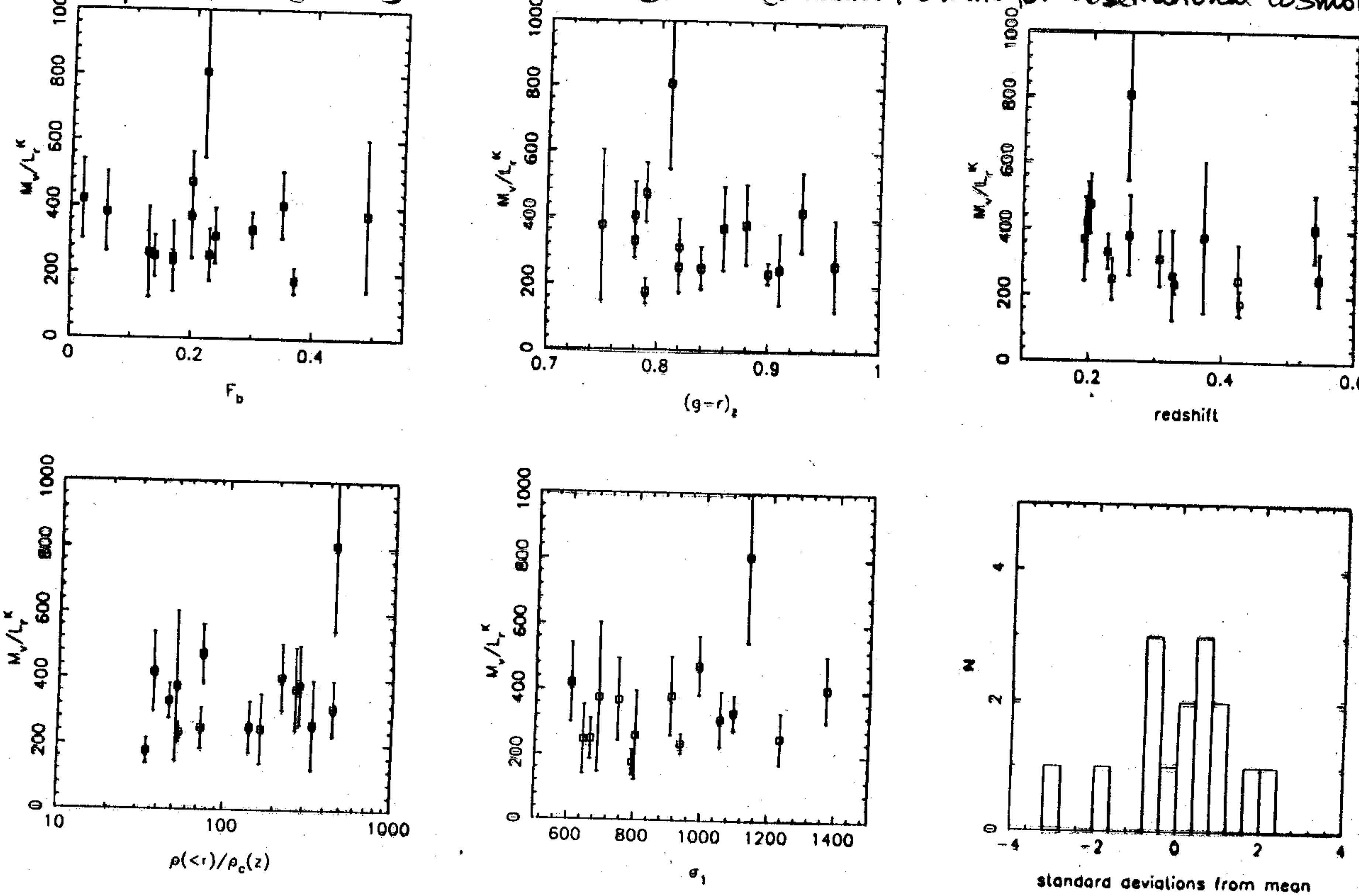
radius of cluster

mean velocity^2 of galaxies

Instead of $\langle v^2 \rangle$, can use temperature T of x-ray emitting gas in cluster

$$\langle v^2 \rangle \approx kT.$$

Carlberg et al (1996) ApJ 462, 32
M/L for galaxy clusters in CNOC (Canadian Network for Observational Cosmology)



3. 5.—M_v/L ratio as a function of (*from left to right, top to bottom*) total blue fraction, color, redshift, mean interior density, and velocity dispersions, for −18.5 mag. The distribution of differences from the mean, ratioed to the estimated error, is in the lower right.

3.58

Evidence for non-baryonic DM

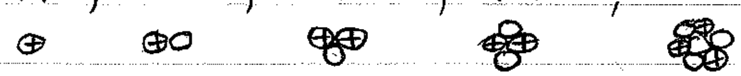
***½ 2. Solves Isotropy Problem

Non-baryonic matter can start to cluster before Recombination
(whereas baryons are tightly coupled to photons, and cannot cluster yet).

After Recombination, baryons fall into gravitational potential wells of non-baryonic DM.

*** 3. Primordial Nucleosynthesis

predicts observed abundances of light elements
H, D, ^3He, ^4He, ^7Li



only if

$$\boxed{\Omega_{baryon} = .04}$$

whereas dynamics of galaxy clusters indicates
$\Omega \approx .25$.
Seems to be a gap there...

***** 1. Cosmic Microwave Background

Ratios of amplitudes of 1st, 2nd, 3rd

especially the height of this one

$$\Rightarrow \boxed{\Omega_{CDH} = .26}$$

Schramm & Turner (1997)

3.60

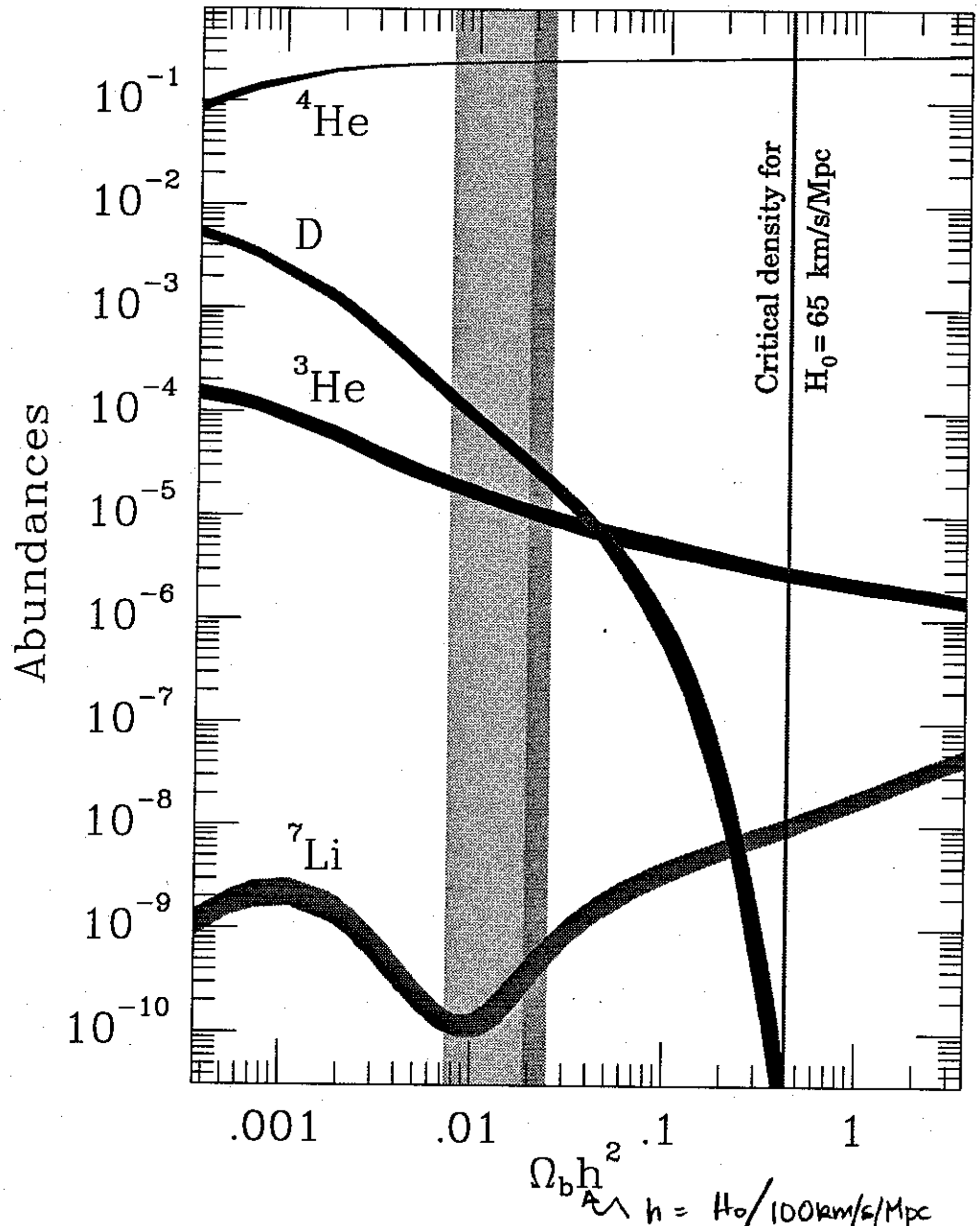


Figure 1: Summary of big-bang production of the light elements. The widths of the curves indicate the 2σ theoretical uncertainties, and the vertical band is the Copi et al [16] consistency interval where the predicted abundances of all four light elements agree with their measured primeval abundances. The darker band in the consistency interval corresponds to Tytler et al's determination of the primeval deuterium abundance (Figure courtesy of K. Nollett).

CDM (Cold DM) vs. HDM (Hot DM)

HDM

Virtue: candidate particles exist
— neutrinos!

Drawbacks;
- predicts not enough structure on "small scales" — galaxies, galaxy groups
- predicts galaxies formed "yesterday" from fragmentation of superclusters, which is not what is observed

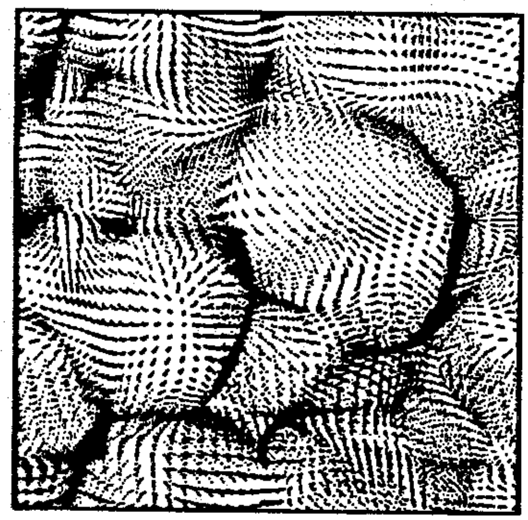
CDM

Drawback: what is it?!

Virtues:
- fits galaxy distribution quite nicely (possibly a pinch of HDM might help...)
- seems to be working for the CMB power spectrum C_ℓ too...

Ben Bromley (1993) PhD thesis

CDM



HDM

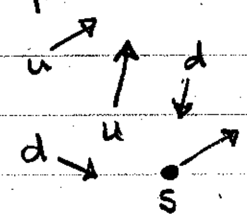
Figure 7.1: Particle plots from CDM and HDM simulations showing two dimensional projections of a $320^2 \times 32$ Mpc^3 slice through the simulation volumes. In both cases, 64^3 particles were evolved by PM force calculations on a 128^3 mesh.

PRIMORDIAL NUCLEOSYNTHESIS

Boesgaard & Steigman (1985) Ann. Rev. Astr. Astrophysics 23, 319

Steps:

1. $T \sim 3 \times 10^{15}$ K   quark soup $\longrightarrow$ p, n
   300 GeV



p: u u d   n: u d d

2. $T \sim 10^{13}$ K   p, n go nonrelativistic
   1 GeV
   ↑
   rest mass energy of p, n

3. $T \sim 10^{10}$ K   weak interactions (radioactivity)
   1 MeV

$n + \nu \longleftrightarrow p + e$ stop
$n + \bar{e} \longleftrightarrow p + \bar{\nu}$

freeze-out $\Rightarrow$ $\dfrac{n}{p} \approx \dfrac{1}{7}$ thermodynamic equilibrium ratio at $T \sim 1$ MeV

n is 1.293 MeV heavier than p,
hence lower abundance of n.

Q: If $\overset{p}{\oplus}$ and $\overset{n}{\bigcirc}$ subsequently combine entirely into $\overset{H}{\oplus}$ and $\overset{^4He}{\oplus\bigcirc\bigcirc\oplus}$,
what is resulting ratio $\dfrac{H}{He}$ by mass?

4. $T \sim 10^9 - 10^{10}$ K Nucleosynthesis begins

$\cdot 1 - 1$ MeV

$\uparrow$

n-p mass difference

Critical reaction:

$$p + n \longrightarrow D$$

Once D is formed, quickly adds another p and n to form ^4He.

5. $T \lesssim 10^9$ K Nucleosynthesis stops.

$\cdot 1$ MeV

$$H + {}^4He \rightarrow \text{mass } 5?$$

$$^4He + {}^4He \rightarrow \text{mass } 8?$$

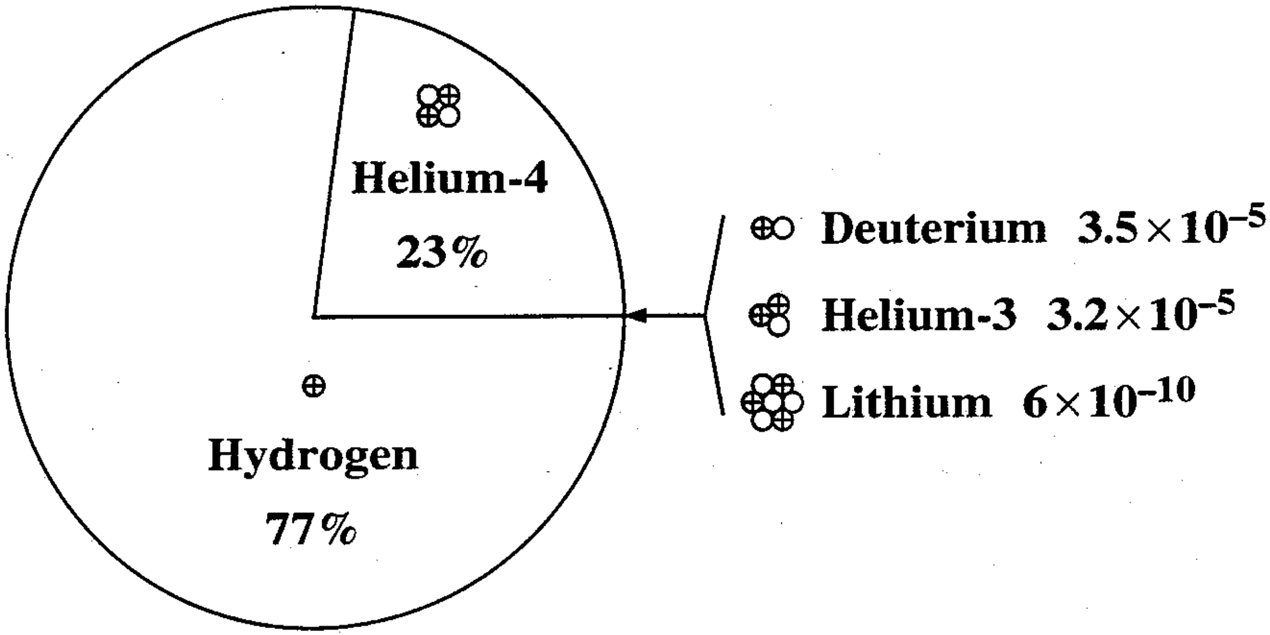
don't happen, because no stable nuclei of mass 5 or 8 exist.

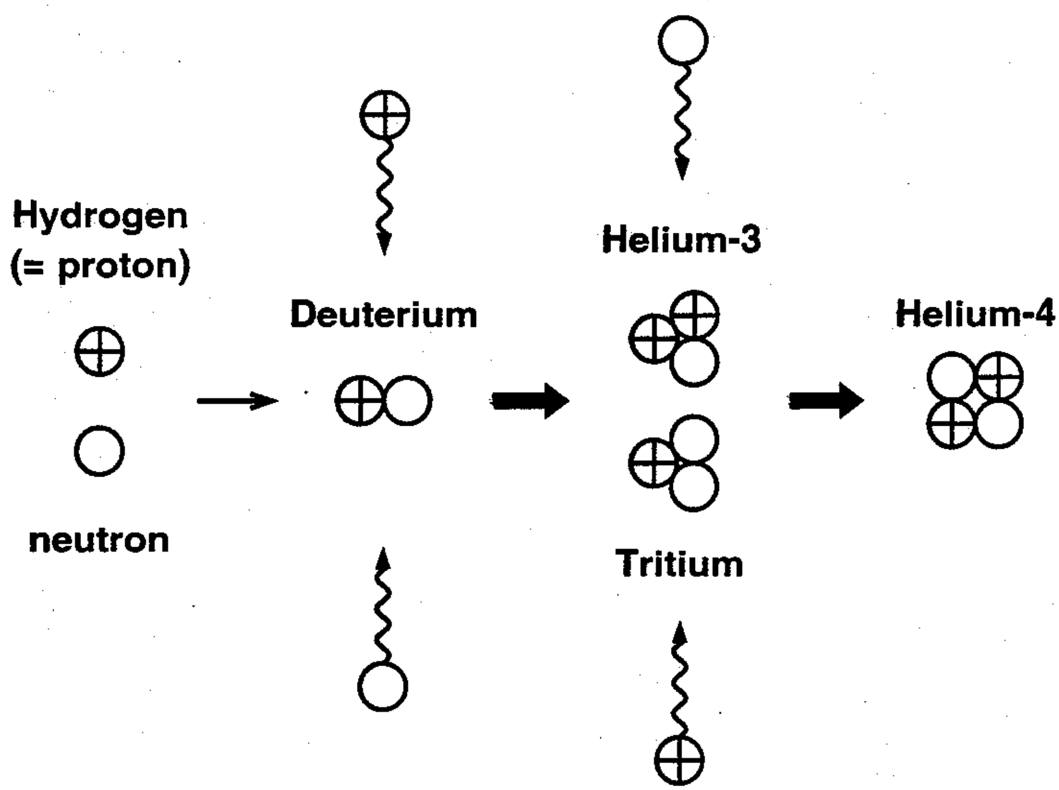
Q: How do stars make heavy elements?

A: "Triple alpha" reaction in dense cores of red giants

$$3\,{}^4He \longrightarrow {}^{12}C$$

Abundances (by mass) of Elements Synthesized in the Big Bang





Q: What is temperature?
A:

Q: When can an object be described by a temperature?
A:
Thermodynamic Equilibrium

A — a "state" of system e.g. of this room

B — another "state"

Ex/ (1) H + energy $\xrightarrow{\text{ionization}}$ p + e
of hydrogen

(2) e + e $\xrightarrow{\text{collision}}$ e + e
of electrons

(3) Wavefunction of Universe → Wavefunction of Universe

Note A can convert into B only if reaction satisfies conservation of energy, etc.

Principle of "microscopic reversibility"

rate (A → B) = rate (B → A)

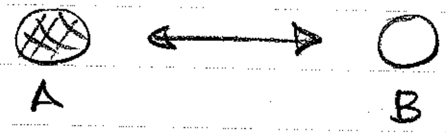
rate (A → B): to go from one state to another

rate (B → A): to go back

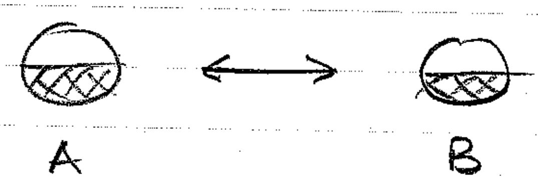
expresses time reversibility of all reactions

often called "T" symmetry.

Start with system all in A



As time passes, some of the probability is in B.
After long time, system spends
  ½ time in state A
  ½ time in state B



This is the condition of thermodynamic equilibrium: ½ in A, ½ in B.
Notice condition is irreversible!
If you start with ½ & ½, it will remain so.

Conclusion;
  Time reversibility
  $\Rightarrow$ irreversible approach to thermodynamic eq.

$$\boxed{\text{Reversibility} \Rightarrow \text{Irreversibility}}$$

Entropy

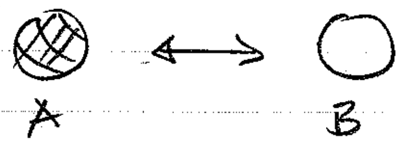
Definition:

Entropy $S = \ln(\#\text{ states accessed by system})$

natural logarithm

Ex/

(1)

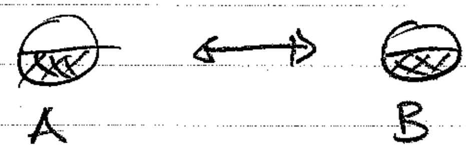


All in state A : # states = 1

$\Rightarrow S = \ln 1 = 0$

Zero entropy = condition of maximal "order".

(2)



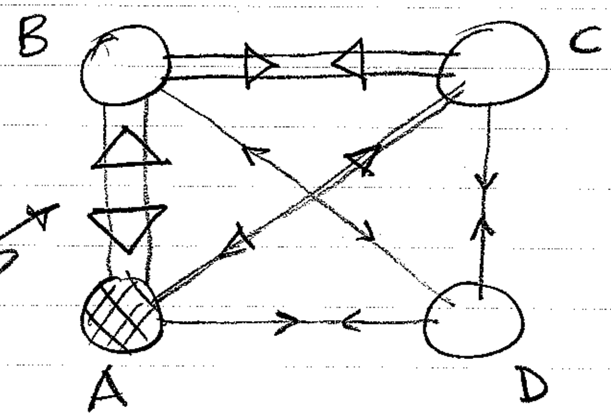
½ in state A, ½ in state B:

states = 2

$\Rightarrow S = \ln 2$ = whatever (about .6)

Point is, entropy is larger

(3)



Complicated system with many states.

thick pipe means rates $A \leftrightarrow B$ rapid.
Quickly achieve equilibrium.

Result: entropy increases from $S = \ln 1$ (all in A)
to $\ln 2$ (A & B)
to $\ln 3$ (A & B & C)
to $\ln 4$ (A, B, C & D).

General theorem:

$\boxed{\text{Entropy increases}}$

Cosmological Arrow 1

CMB has blackbody spectrum
$\Rightarrow$ Universe used to be in Thermo Eq.

Q: How did Univ get out of TE ?
A: $\boxed{\text{Expansion}}$

Cosmological Arrow 2

Gravity has negative specific heat:
remove energy from gravitating system
$\rightsquigarrow$ it heats up ("temperature" increases).

$\Rightarrow$ $\boxed{\text{Gravity is thermodynamical unstable}}$

Gravity, the ultimate perpetual motion machine

Gravitational instability causes:
- galaxies to cluster
- intergalactic/interstellar gas to collapse
- stars to ignite
- stars to evolve ever hotter at their centers
- supernovae (Type II ie. core collapse)
- accretion disks to glow
-

Ultimate condition : = Black Hole

$$S_{BH} = \frac{A}{4}$$

A = area of horizon $= 4\pi r_s^2$

in Planck units $(c = G = \hbar = 1)$

S_{BH} = entropy of BH

(Hawking 1975).

Ex/

$S_{CMB} \approx$ number of CMB photons

$\approx 10^{88}$ within our horizon.

To match this entropy requires BH of

$r_s \approx 10^{44}$ Planck lengths ($\approx 10^{-35}\,m$)

$\approx 10^6$ km

whence

$M_{BH} \approx 10^6 M_\odot$.

ie the BH at the center of our galaxy contains as much entropy as all the CMB!

Big Crunch (if there is one)

does not look like Big Bang ← smooth, low entropy

Big Crunch ← full of BHs, high entropy.

Antimatter

hence negative frequency

Antimatter is negative mass matter going backwards in time.

— Richard Feynmann.

Observed fact:

Universe is dominated by matter, not antimatter.
(Specifically, there is ≈ 1 proton for every 10^9 cMB photons.)

Why?

Excess of matter over antimatter requires 3 conditions:

1. Non TE ✓ (from expansion)
2. Baryon - lepton violation:
   somehow protons p must be able to convert to positrons $\bar{e}$ (= anti-electrons) and other light particles (neutrinos, photons).
3. T - violation.

T - violation

Is observed in one obscure case:
decay of neutral kaon K°
(and analogous decay of B meson)

Mass eigenstates: T- eigenstates:

$$K_L \approx K^\circ + \bar{K}^\circ$$

long lived

$$K_S \approx K^\circ - \bar{K}^\circ$$

short lived

would be exact
if T were exact symmetry

Example of explicit T-violation:

$$K_L \xrightarrow{\text{via } K^0} \pi^- e^+ \nu \quad \text{is faster than}$$

$$K_L \xrightarrow{\text{via } \bar{K}^0} \pi^+ e^- \bar{\nu} \quad \text{by } 1.65 \times 10^{-3}$$

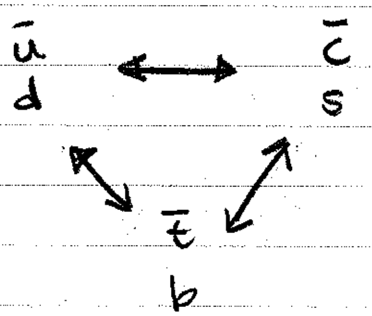
Mystery of 3 generations

	electronic	muonic	tauonic
neutrino:	ν_e	ν_μ	ν_τ
	e	μ	τ
quarks	u up	c charm	b bottom
	d down	s strange	t top

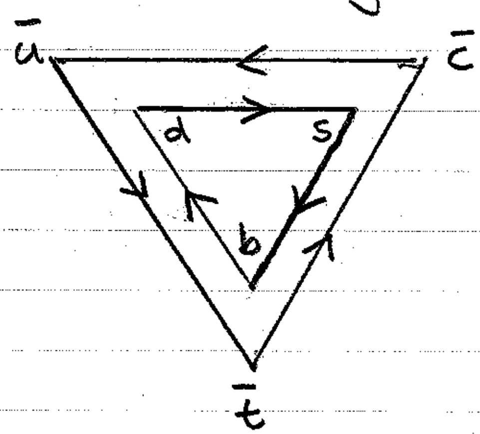
There are oscillations between the 3 generations of quarks, mediated by weak interactions between quarks and "Higgs" particles.

How obscure can you get?

T - symmetric:



T - violating:



Need 3 generations to allow T violation hence to allow matter to exist in Universe today.