ASTR 2030 Black Holes Spring 2006. Homework 3. Due in class Fri Nov 10.

Your name and ID:

Sagittario

The animation of infrared observations by Eckart, Genzel et al. (2002) at `http://www.mpe.mpg.de/www_ir/GC/` (see Figure 1) shows stars buzzing like bees around the position of SgrA*, the unresolved point radio source thought to mark the black hole Sagittario at the center of our Galaxy. In a 'virialized' gravitating system like this (meaning one that has reached an equilibrium where it is neither collapsing nor expanding), the average velocity v of stars at radius r from the center is

$$v = \sqrt{\frac{GM}{r}} \tag{1}$$

where M is the enclosed mass interior to r, including both black hole and stars. [Incidentally, this is Kepler's third law of planetary motion, in disguise. And no, in case you noticed, there's not a missing factor of $\sqrt{2}$ here; this is an average velocity, not an escape velocity.]

Astronomers deduce the distances r of stars from SgrA* from their angular separation on the sky multiplied by the known distance 8,000 pc, eight thousand parsecs, to the Galactic center. A parsec is a unit of distance, equal to about 3 lightyears.

1. Figure 2 shows measurements of the enclosed mass M, deduced from equation (1), plotted versus the radial distance r in parsecs from the Galactic center. [Enclosed mass means the mass contained inside a sphere of radius r.] What aspect of the curve of enclosed mass M versus radius r suggests that there is a large mass of small radius – a black hole, Sagittario – at the Galactic center? [If there were a large point mass, would the enclosed mass increase, decrease, or remain constant as a function of radius r from the point?] What is the mass $M_\bullet$ of Sagittario, in $M_\odot$ (solar masses), according to the curve?

   The mass $M_\bullet$ of Sagittario is ——————————————————— $M_\odot$.

2. Near Sagittario, the mass density ρ interior to radius r is the mass $M_\bullet$ of Sagittario, divided by the volume $\frac{4}{3}\pi r^3$:

$$\rho = \frac{M_\bullet}{\frac{4}{3}\pi r^3} . \tag{2}$$

   Plot this interior density ρ as a function of radius r as a **straight line** on the attached graph, Figure 3. [Hint: Try some representative values, such as $r = 10^{-2}$ pc, $r = 10^{-4}$ pc, and $r = 10^{-6}$ pc. To convert to the density units of gm cm^{-3}, you need to know that $1\,M_\odot = 2 \times 10^{33}$gm and $1\,\text{pc} = 3 \times 10^{18}$cm.]

3. A star of density ρ_* will be tidally torn apart if the density ρ interior to its distance r from the black hole exceeds the stellar density (this is different from the condition that you will be torn apart: a star is held together by gravity, whereas you are held together by electric forces):

$$\rho > \rho_* . \tag{3}$$

On the graph, Figure 3, mark and label the points where a star would be torn apart if it were: (A) a star like the Sun, whose density is $\rho_* \approx 1\,\mathrm{gm\,cm^{-3}}$, and (B) a red giant star like Betelgeuse, whose density is $\rho_* \approx 10^{-8}\,\mathrm{gm\,cm^{-3}}$,

The Sun would be torn apart if it came within ——————— parsecs of Sagittario.

Betelgeuse would be torn apart if it came within ————— parsecs of Sagittario.

4. From Figure 2, read off the distance, in parsecs, of closest approach of the star S2 to Sagittario. [Hint: It's the innermost point plotted on the graph. Incidentally, the unit AU mentioned on the graph is an Astronomical Unit, which is the Earth–Sun distance, about 5×10^{-6}pc.]

The distance of closest approach of S2 to Sagittario is ——————— parsecs.

From your graph, read off the minimum density that the star S2 must have in order not to be torn apart by Sagittario. Mark and label this point on your graph, Figure 3.

The density of S2 must exceed ————————————— $\mathrm{gm\,cm^{-3}}$.

Could the star S2 be as dense as the Sun? Yes/No (circle one).

Could the star S2 be as dense as Betelgeuse? Yes/No (circle one).

5. On Figure 2, the curve of enclosed mass M versus radius r at large radii fits approximately the power-law relation (the dot-dash line on the graph)

$$M_{\mathrm{stars}} = 1.5 \times 10^6\,\mathrm{M_\odot} \left(\frac{r}{1\,\mathrm{pc}}\right)^{1.2} \tag{4}$$

which can be interpreted as the mass M_{stars} of stars closer to the black hole than r. Continue this dot-dash line as a **straight line** on the graph which I've added as an extension below the original graph. **Label** the horizontal and vertical axes of the extended graph.

The animation in Figure 1 shows a region of radius about 10 lightdays, equivalent to about $r = 0.01\,\mathrm{pc}$ (a hundredth of a parsec). From your extension to Figure 2, read off the enclosed mass M_{stars} of stars, in $\mathrm{M_\odot}$, within this radius $r = 0.01\,\mathrm{pc}$. Mark and label this point on your graph.

The enclosed mass of stars within $r = 0.01\,\mathrm{pc}$ is ————————— $\mathrm{M_\odot}$.

Roughly how many stars might you expect there to be in the region shown by the animation? [Hint: It is reasonable to suppose that the average mass of a star is about $1\,\mathrm{M_\odot}$, one solar mass.]

There should be about ——————— stars in the region covered by the animation.

6. (No credit) What might be the reason that the actual number of stars in the animation is much less than this?
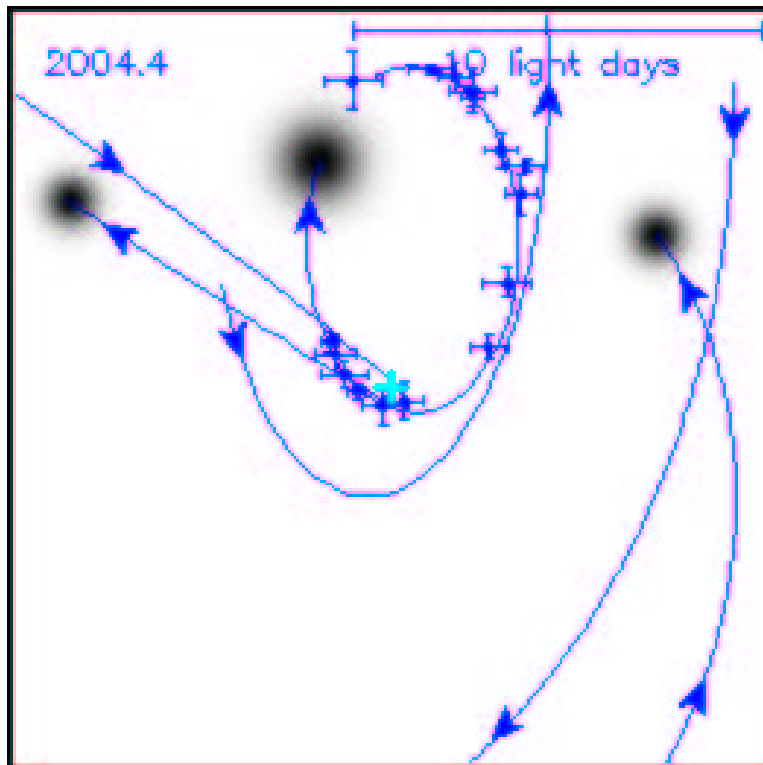
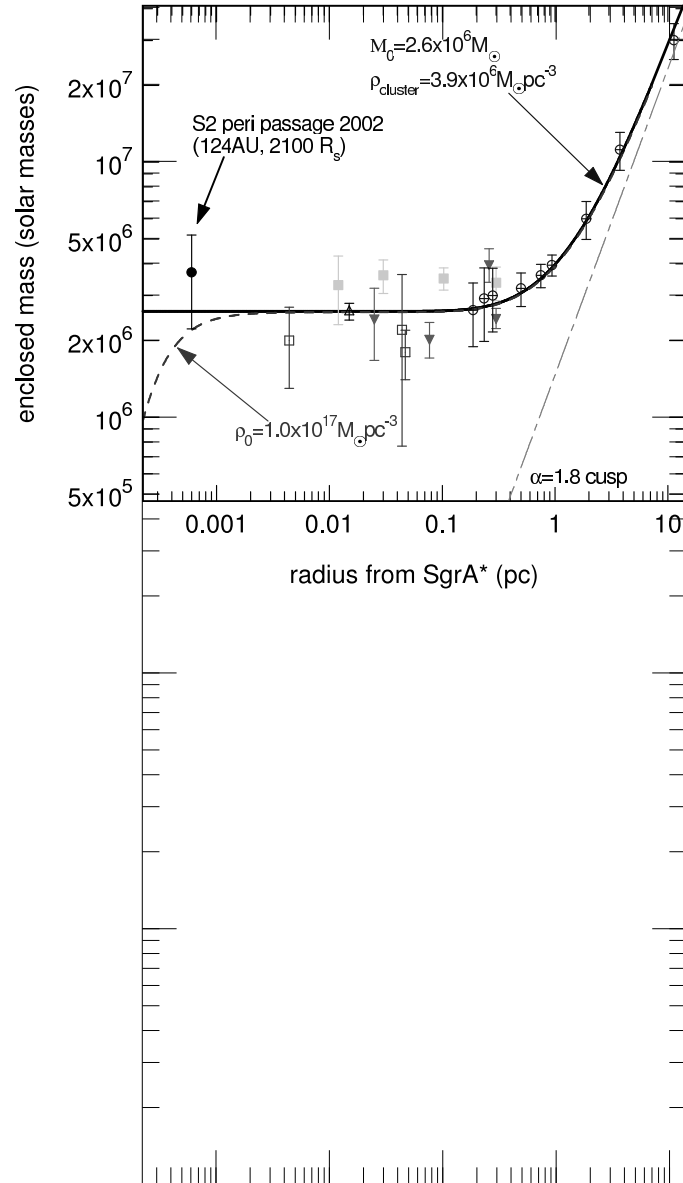Figure 1: `http://www.mpe.mpg.de/www_ir/GC/`
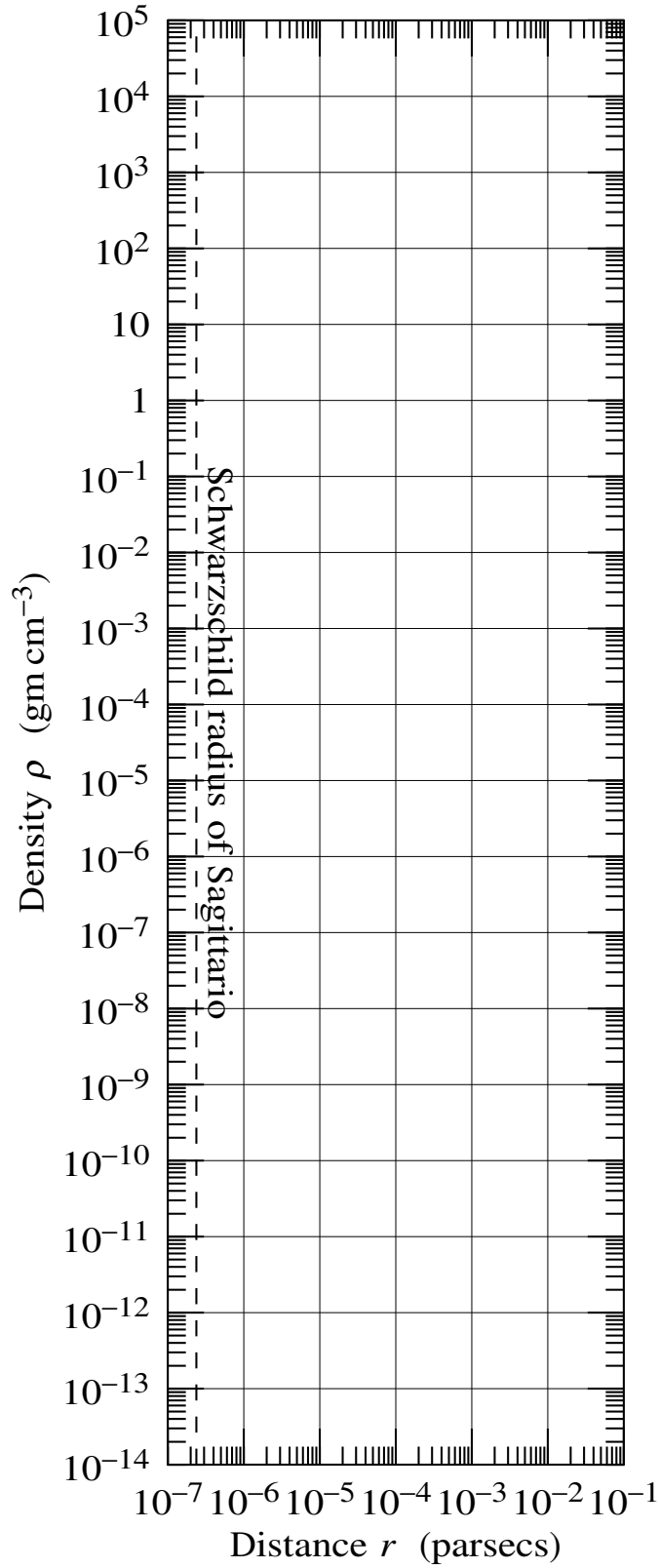
Figure 2: Schödel et al. 2002, Nature 419, 694.

Figure 3: Density ρ, equation (2), as a function of radial distance r from Sagittario.