ASTR 2030 Black Holes Fall 2003. Homework 4. Due in class T Dec 9.

Please write your answers on these sheets, and **show your working**. You should be able to read off many of the answers from the attached graphs, and you should **mark the graphs** to show how you got your answer.

1. Hawking Radiation

The Hawking radiation from a black hole has a characteristic wavelength comparable to its Schwarzschild radius. The top graph of the attached set of graphs shows the Schwarzschild radius as a function of the black hole's mass. On the graph, **label** the regimes where a black hole would emit Hawking radiation in: (a) radio waves; (b) visible light; (c) gamma rays. [Hint: Thorne page 25 shows the wavelengths of the kinds of radiation.]

What is the approximate mass, in kilograms, of a black hole that radiates **visible** light in Hawking radiation?

The mass of a visible black hole is approximately ________________________ kg.

Compare this mass to that of a solar system object. [Hint: Try searching `www.google.com` for "Physical Solar System Data".]

What is the approximate Hawking luminosity, in Watts, of a black hole of that mass?

The Hawking luminosity of the visible black hole is approximately ________________ W.

How does this luminosity compare to that of a 100 Watt light bulb? [More? Less? Much more? Much less?]

2. To Eat or not to Eat

In Homework 3, you worked out that a black hole of mass M inside the Earth would eat the Earth on a timescale

$$t = 50 \text{ yr} \left(\frac{M}{M_\oplus} \right)^{-1}$$

where $M_\oplus = 6 \times 10^{24}$ kg is an Earth mass. **Draw** this relation as a **straight line** on the bottom graph.

Approximately what is the mass, in kilograms, of the black hole whose Hawking evaporation time equals the time to eat the Earth?

The mass of the black hole is approximately ___________________________ kg.

What do you think will happen to a black hole in the Earth if its mass is less than this?

3. Black Hole Evaporates inside Earth

The interior of the Earth today is heated by radioactivity, as a result of which the Earth radiates to space a luminosity of about 3×10^{13} Watts. What is the approximate mass, in kilograms, of a black hole whose Hawking luminosity has that value?

The black hole mass is approximately ______________________________ kg.

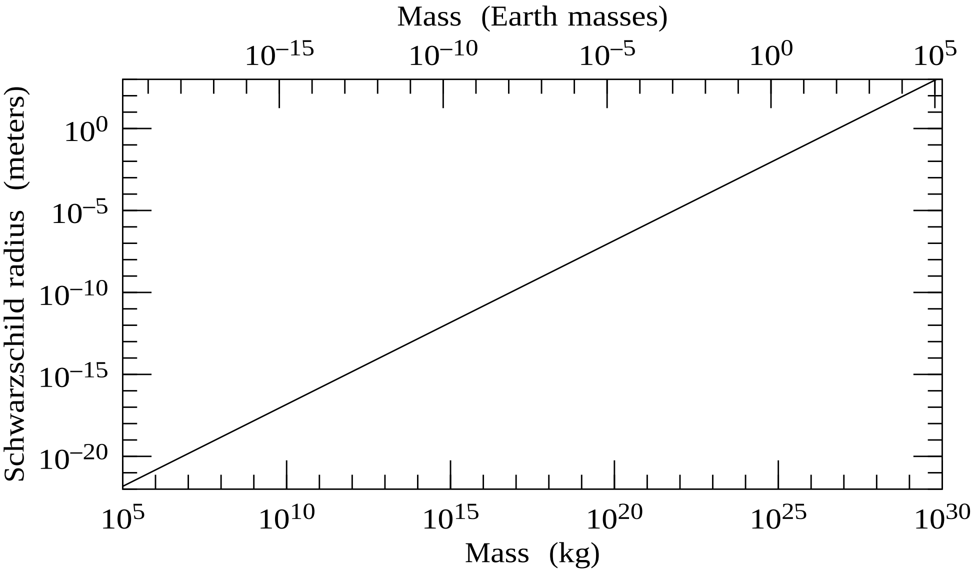
Approximately what is the evaporation time, in years, of a black hole of that mass?

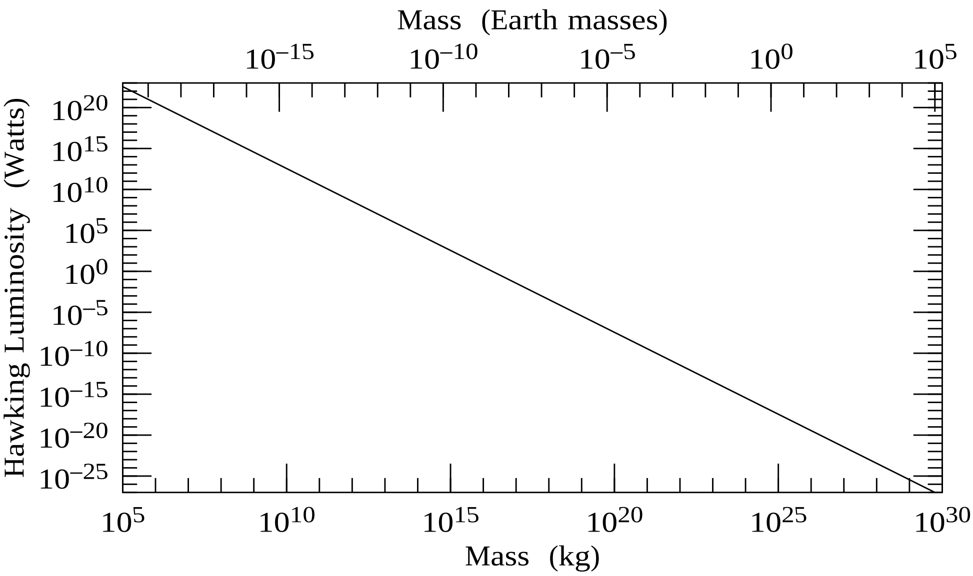
The evaporation time is approximately ______________________________ yr.

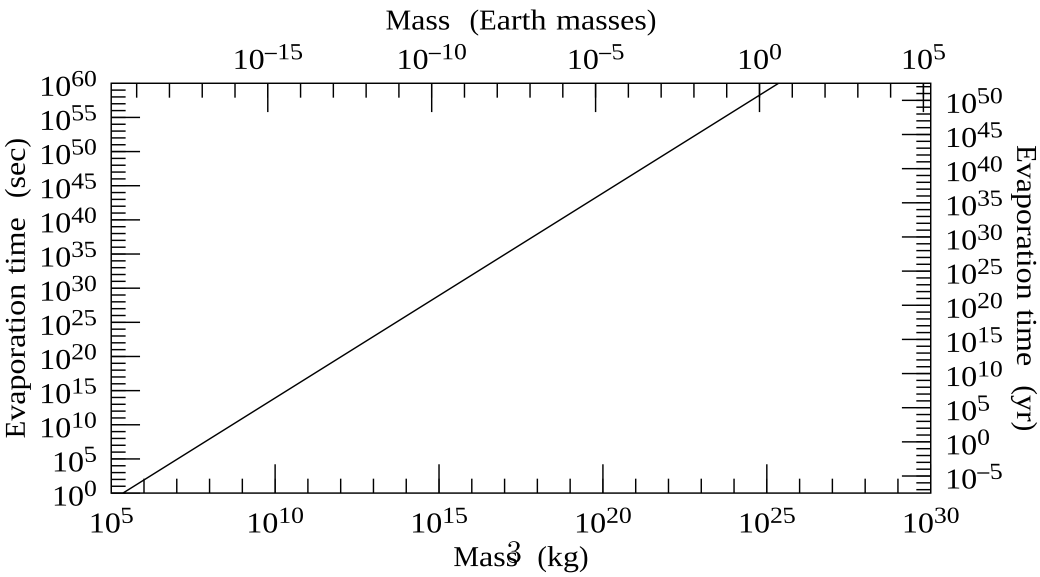
Suppose that the Earth contains an evaporating black hole. Let's assume that the Earth has no problem radiating away the black hole's energy as long as the Hawking luminosity is less than the Earth's radioactive luminosity, but that once the black hole's Hawking luminosity exceeds the Earth's radioactive luminosity, then the black hole's energy will be trapped inside the Earth, unable to leak out rapidly. In other words, assume that when a black hole evaporates inside the Earth, the amount of mass-energy that it dumps inside the Earth equals the mass you just determined above.

If a black hole evaporated inside the Earth, by roughly what temperature, in centigrade, would it raise the Earth, averaged over the entire mass of the Earth? [Hint: According to Einstein's $E = Mc^2$, mass is equivalent to energy to the tune of 2×10^{16} calories per kilogram. A calorie, the kind you eat, is defined so that it takes 1 calorie per kilogram per °C to raise the temperature of water.]

The average temperature of the Earth would increase by approximately __________ °C.


Mass (Earth masses)
10^−15 10^−10 10^−5 10^0 10^5
Schwarzschild radius (meters)
10^0 10^−5 10^−10 10^−15 10^−20
10^5 10^10 10^15 10^20 10^25 10^30
Mass (kg)


Mass (Earth masses)
10^−15 10^−10 10^−5 10^0 10^5
Hawking Luminosity (Watts)
10^20 10^15 10^10 10^5 10^0 10^−5 10^−10 10^−15 10^−20 10^−25
10^5 10^10 10^15 10^20 10^25 10^30
Mass (kg)


Mass (Earth masses)
10^−15 10^−10 10^−5 10^0 10^5
Evaporation time (sec)
10^60 10^55 10^50 10^45 10^40 10^35 10^30 10^25 10^20 10^15 10^10 10^5 10^0
Evaporation time (yr)
10^50 10^45 10^40 10^35 10^30 10^25 10^20 10^15 10^10 10^5 10^0 10^−5
10^5 10^10 10^15 10^20 10^25 10^30
Mass (kg)