1. Superluminal jets

Radio observations of galaxies show in many cases twin jets emerging from the nucleus of the galaxy. The jets are typically narrow and long, often penetrating beyond the optical extent of the galaxy. The jets are frequently one-sided, and in some cases that are favourable to observation the jets are found to be superluminal. A celebrated example is the giant elliptical galaxy M87 at the centre of the Local Supercluster, whose jet is observed over a broad range of wavelengths, including optical wavelengths. Hubble Space Telescope observations, Figure 1, show blobs in the M87 jet moving across the sky at approximately 6c.

![Figure 1: The left panel shows an image of the galaxy M87 taken with the Advanced Camera for Surveys on the Hubble Space Telescope. The jet appears about 5,000 lightyears long. A jet, bluish compared to the starry background of the galaxy, emerges from the galaxy’s central nucleus. Radio observations, not shown here, reveal that there is a second jet in the opposite direction. Credit: STScI/AURA. The right panel is a sequence of Hubble images showing blobs in the jet moving superluminally, at approximately 6c. The slanting lines track the moving features, with speeds given in units of c. The upper strip shows where in the jet the blobs were located. Credit: John Biretta, STScI.](image)

(a) Spacetime diagram

Draw a spacetime diagram of the situation, in Earth’s frame of reference. Assume that the velocity of the galaxy M87 relative to Earth is negligible. Let the $x$-axis be the direction to M87. Choose the $y$-axis so the jet lies in the $x$–$y$-plane. Let the jet be moving at velocity $v$ at angle $\theta$ away from the direction towards us on Earth.
(b) Distance moved
Argue that in Earth coordinates \(\{t, x, y\}\), the jet moves in time \(t\) a distance

\[
\begin{align*}
  l_x &= -vt \cos \theta, \\
  l_y &= vt \sin \theta.
\end{align*}
\]

(c) Apparent transverse velocity
Argue that during an Earth time \(t\), the jet has moved a distance \(l_x\) nearer to the Earth (the distances \(l_x\) and \(l_y\) are both tiny compared to the distance to M87), so the apparent time as seen through a telescope is not \(t\), but rather \(t\) diminished by the light travel time \(l_x\) (units \(c = 1\)). Hence conclude that the apparent transverse velocity on the sky is

\[
v_{\text{app}} = \frac{v \sin \theta}{1 - v \cos \theta}.
\]

(d) Maximum transverse velocity
Sketch the apparent velocity \(v_{\text{app}}\) as a function of \(\theta\) for some given velocity \(v\). In terms of \(v\) and the Lorentz factor \(\gamma\), what are the values of \(\theta\) and of \(v_{\text{app}}\) at the point where \(v_{\text{app}}\) reaches its maximum? What is the smallest possible velocity \(v\) of the jet in M87?

(e) Redshift of the jet
What is the expected redshift \(1 + z\), or equivalently blueshift \(1/(1+z)\), of the jet as a function of \(v\) and \(\theta\)? By expressing \(v\) in terms of \(v_{\text{app}}\) and \(\theta\) using equation (3), show that the blueshift factor is

\[
\frac{1}{1+z} = \sqrt{1 + 2v_{\text{app}} \cot \theta - v_{\text{app}}^2}.
\]

(f) Opening angle
In terms of \(v_{\text{app}}\), at what value of \(\theta\) is the blueshift (i) infinite, or (ii) zero? What are these angles in the case of M87? If the redshift of the jet were measurable, could you deduce the velocity \(v\) and opening angle \(\theta\)? Unfortunately the redshift of a superluminal jet is not usually observable, because the emission is a continuum of synchrotron emission over a broad range of wavelengths, with no sharp atomic or ionic lines to provide a redshift.

(g) Invisible jet?
Why is the opposing jet not visible?