Common envelopes

If mass transfer is too rapid, the accreting star is unable to accept mass at the rate provided by the donor $\rightarrow$ formation of a hot envelope around the accretor.

If this massive envelope becomes larger than the size of the Roche lobe, no longer sensible to study system within the Roche approximation.

Instead, imagine the mass losing star and the core of the accretor as orbiting within a *common envelope*.

Common envelope phase likely,

- When mass transfer occurs on a thermal timescale,

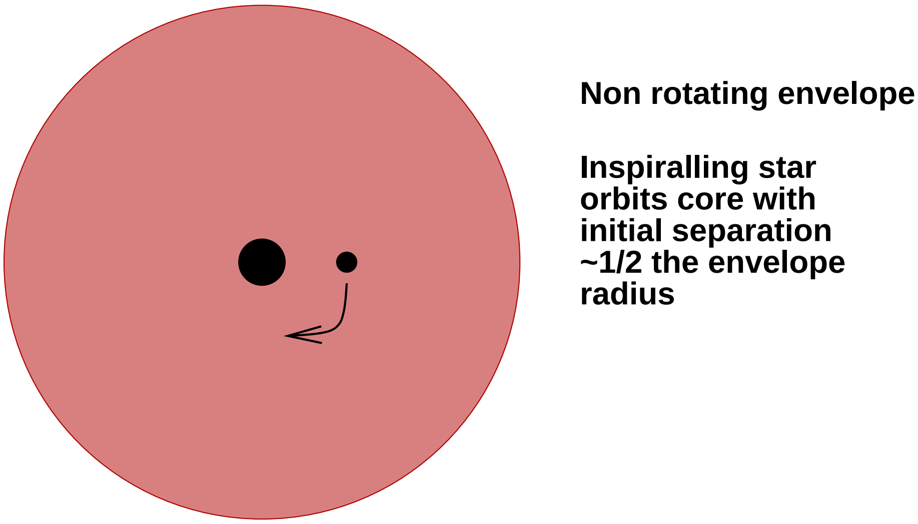
$$\tau_{KH} = \frac{GM^2}{RL}$$

  e.g. when the more massive star in a binary fills its Roche lobe as it becomes a red giant.

- In cases where there is a dynamical mass transfer instability.

Common envelope evolution

Geometry at the onset of common envelope evolution:



Relative motion of the inspiralling star and the envelope creates a 'drag' $\rightarrow$ conversion of orbital energy into thermal energy at a rate given dimensionally by:

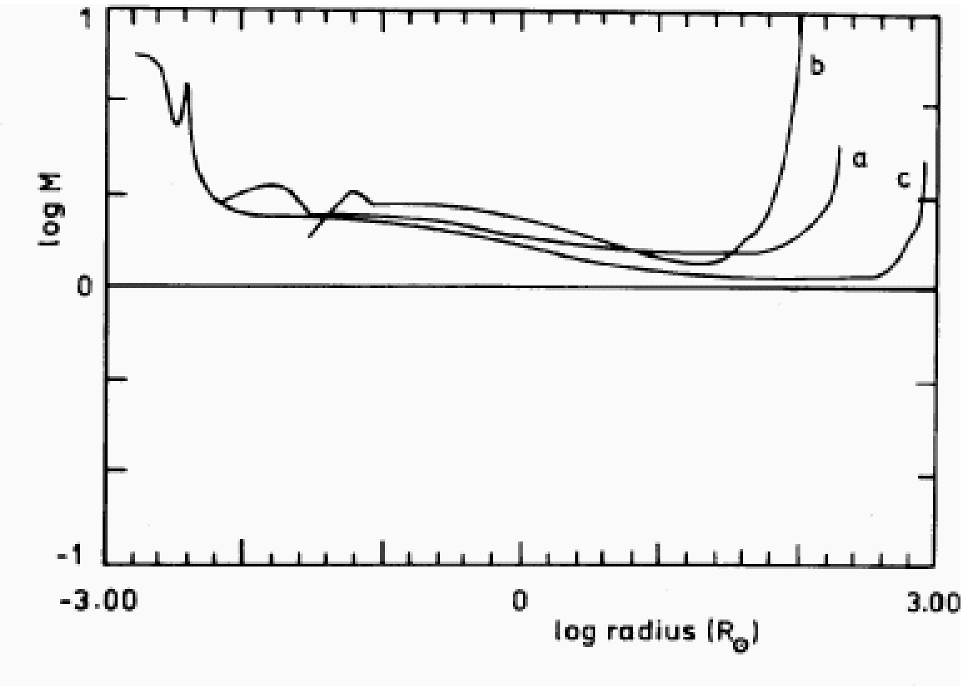
$$L_{\rm drag} = \xi \pi R_{acc}^2 \times \rho \Delta v \times (\Delta v)^2$$

ie a characteristic area times a mass flux / unit area times an energy per unit mass. Here Δv is the relative velocity between the orbiting star and the envelope and ξ is a fudge factor (see E. Ostriker 1999 for a proper calculation of drag in a collisional medium).

Generalizing the Bondi-Hoyle result to allow for transonic flow in gas with sound speed c_s,

$$R_{acc} = \frac{2GM_*}{(\Delta v)^2 + c_s^2}.$$

Expect that the velocity of the inspiralling star will be comparable to the sound speed. For a variety of giant models, de Kool (1987) obtained Mach numbers $\mathcal{M}$ for circular orbits of:



Dissipated energy must come from the orbit. If M_2 is the inspiralling secondary, and $m_1(a)$ is the mass of the primary *interior* to the orbit at radius a, then

$$L_{\rm drag} \simeq -\frac{GM_2 m_1(a)}{2a^2}\frac{da}{dt}.$$

The orbit will then decay on a timescale,

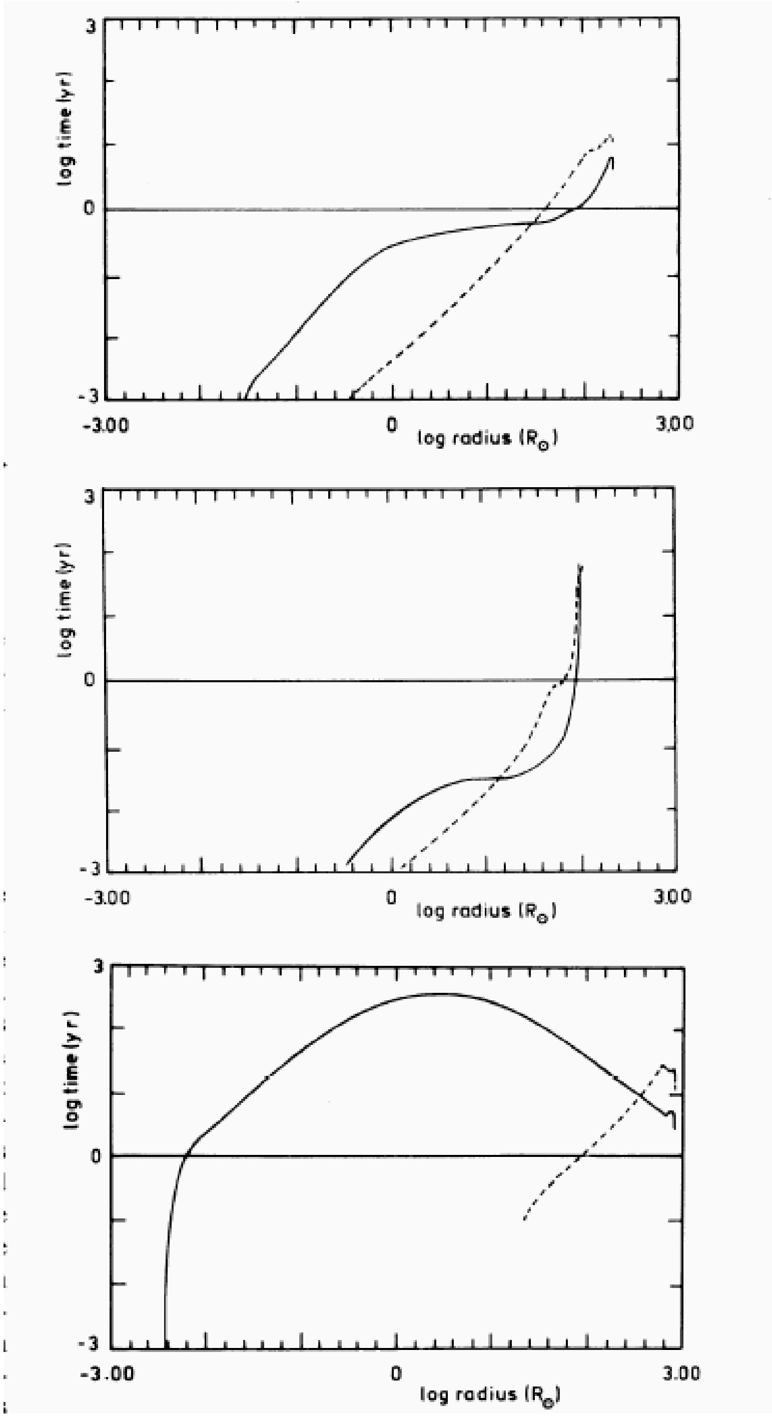
$$\tau_{\rm decay} \equiv \frac{a}{|da/dt|}.$$

How fast is the inspiral? Consider a $1.4M_\odot$ neutron star embedded within a $10M_\odot$ giant at an initial radius of $50R_\odot$. Estimate:

- Orbital velocity $\Delta v \simeq 200$ km s^{-1}.
- Accretion radius $R_{acc} \simeq 10^{12}$ cm.
- Taking $\rho = 3 \times 10^{-7}$ g cm^{-3} (from Fryer et al. 1996), $L_{\rm drag} \sim 7.5 \times 10^{39}$ erg s^{-1} (fudge factor assumed to be unity).
- $\rightarrow da/dt \sim 100$ km s^{-1} – ie inspiral occurs on dynamical timescale.

Better estimates reach same conclusion – inspiral is an extremely rapid process.

e.g. Iben & Livio (1993):



Result of common envelope evolution may be,

- Merger of the inspiralling star with the other core.
- Ejection of the envelope before merger occurs, leaving a much closer binary system.

Define a parameter α_{CE} measuring the efficiency with which orbital energy goes into unbinding the envelope:

$$\alpha_{CE} \equiv \frac{\Delta E_{bind}}{\Delta E_{orbital}}.$$

If star M_1 has a core mass M_c and an envelope mass M_{env} (assumed exterior to the initial orbit of the inspiralling star) then,

$$\Delta E_{orbital} = G M_c M_2 \left(\frac{1}{2a_f} - \frac{1}{2a_i} \right)$$

where a_i, a_f are the initial and final separations.

The binding energy of the envelope is *very approximately*,

$$\Delta E_{bind} = \frac{G M_{env} (M_1 + M_2)}{2a_i}$$

assuming that the typical size of the envelope when common envelope evolution begins is twice the initial orbital separation.

For a particular system (known masses), these equations give a_f in terms of a_i for specified α_{CE}.

Numerous factors might influence the magnitude of α_{CE},

- If the envelope is convective, energy might be transported to the surface and radiated on a timescale shorter than $\tau_{decay} \rightarrow$ decrease in α_{CE}.
- Pulsations, winds driven by induced rotation, or enhanced nuclear burning might all increase α_{CE}.

Numerical calculations (eg Zhang & Fryer 2001) and models of the population of observed compact binaries suggest $\alpha_{CE} \approx 0.5$,