## Post-main-sequence evolution

How does a red giant with a mass of several Solar masses turn into a white dwarf with $M \approx 0.6\ M_\odot$?

Main considerations:

- Envelope of a star with $R \sim 10^2 R_\odot$ is extremely weakly bound $\rightarrow$ possibility of shedding mass at a high rate.
- Hydrogen and helium burning occurs in shells, which are *less stable* to runaway nulcear reactions than core burning. Instabilities lead to large periodic changes in the luminosity of the star.

This final stage of evolution is thought to be similar for most stars with masses below about 8 $M_\odot$.

## Homologous contraction

Consider a star which is contracting (or expanding) such that the sequence of resulting models are homologous to each other. If the density is $\rho$ and pressure $P$ at some mass shell $m$, then as before,

$$\rho \propto \frac{m}{r^3}$$

and,

$$P \propto \frac{m^2}{r^4}$$

where $r = r(t)$ and we have assumed that the change in radius is sufficiently slow that hydrostatic equilibrium remains valid.

Measured at a fixed mass shell $m$, a homologous change in structure implies,

$$\begin{aligned} \frac{\dot{\rho}}{\rho} &= -3\frac{\dot{r}}{r} \\ \frac{\dot{P}}{P} &= -4\frac{\dot{r}}{r}. \end{aligned}$$

For an equation of state, $\rho \propto P^{\alpha} T^{-\delta}$,

$$\frac{\dot{\rho}}{\rho} = \alpha\frac{\dot{P}}{P} - \delta\frac{\dot{T}}{T}$$

and,

$$\frac{\dot{T}}{T} = -\frac{4\alpha - 3}{\delta}\left(\frac{\dot{r}}{r}\right).$$

## Stability of central nuclear burning

Consider a small sphere of radius $r_s$ and mass $m_s$ around the center of a star in hydrostatic equilibrium. Pressure $P$ at $r_s$ is $\approx P_c$, density is $\approx \rho_c$, where the subscript denotes central values.

Now perturb the structure such that the core undergoes a homologous contraction or expansion. We have,

$$\frac{dP_c}{P_c} = \frac{4\delta}{4\alpha - 3}\frac{dT_c}{T_c}.$$

Using the first law of thermodynamics, the heat $dQ$ per unit mass added to the central sphere is,

$$dQ = dU + PdV$$

which can be written,

$$dQ = c_P T_c \left( \frac{dT_c}{T_c} - \nabla_{ad} \frac{dP_c}{P_c} \right) \equiv c^* dT_c,$$

defining the *gravothermal specific heat* $c^*$. In terms of the specific heat at constant pressure,

$$c^* = c_P \left( 1 - \nabla_{ad} \frac{4\delta}{4\alpha - 3} \right).$$

In the center of the Sun, the gas is approximately ideal. Thus,

$$\alpha = \delta = 1$$

and,

$$\nabla_{ad} = \frac{2}{5}.$$

We find,

$$c_* = c_P \left(1 - \frac{2}{5} \times \frac{4}{4-3}\right) < 0.$$

Same result as before – if we add heat to the core of a star under ideal gas conditions, the temperature *decreases* due to the resulting expansion. Nuclear reactions are stable under these conditions.

Conversely, under degenerate conditions,

$$P \propto \rho^{5/3},$$

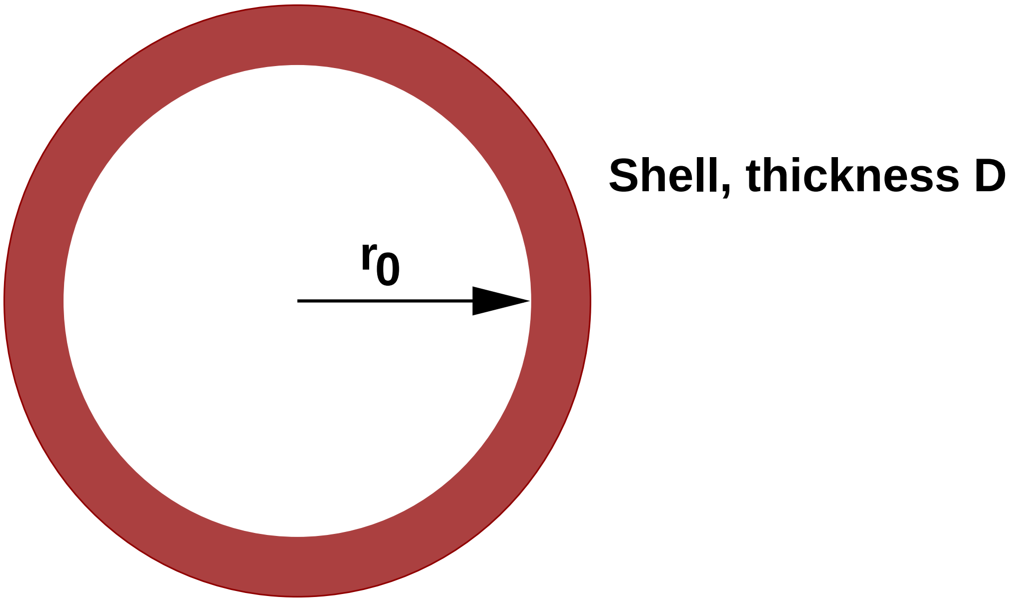
ie $\delta = 0$ and $\alpha = 3/5$. Obtain,

$$c^* > 0$$

justifying the claim that under degenerate (or partially degenerate) conditions nuclear reactions are vulnerable to runaways.

For core burning, the helium flash is the manifestation of this instability.

Consider a shell burning geometry,




Assume that the inner radius of the shell $r_0$ is fixed. The mass in the shell is,

$$m \propto \rho r_0^2 D.$$

If the shell is perturbed such that $D \to D + dD$, then using $dm = 0$,

$$\frac{\dot{\rho}}{\rho} = -\frac{\dot{D}}{D} = -\frac{r}{D}\frac{\dot{r}}{r}.$$

If we assume that the mass exterior to the shell behaves homologously, then the previous analysis carries over exactly with replacement of factor 3 by factor $r/D$ in equation for change in $\rho$.

Result becomes,

$$c^* = c_P \left( 1 - \nabla_{ad} \frac{4\delta}{4\alpha - r/D} \right).$$

Implications,

- As before, for a degenerate gas with $\delta \to 0$, obtain $c^* > 0$ and recover the flash instability. Relevant since there are circumstances where the helium flash may occur in a shell geometry.
- Even for an ideal gas, $c^* > 0$ for large values of $r/D$ – ie for thin shells. This is called a *pulse instability* of the nuclear burning.

The pulse instability has a simple physical interpretation. Consider a very thin shell. Suppose we make a *large* perturbation by doubling its thickness. Then,

- The density drops by a factor of two within the shell.
- But, the pressure acting on the shell hardly changes, because $r_0 + 2D \simeq r_0 + D$. The weight of layers above we have to hold up is almost unchanged.
- If the pressure does not change, while the density drops, the temperature must increase. Implies instability.

On the AGB, when there may be both hydrogen and helium shell burning, this instability can be periodic. eg Iben & Renzini (1993):

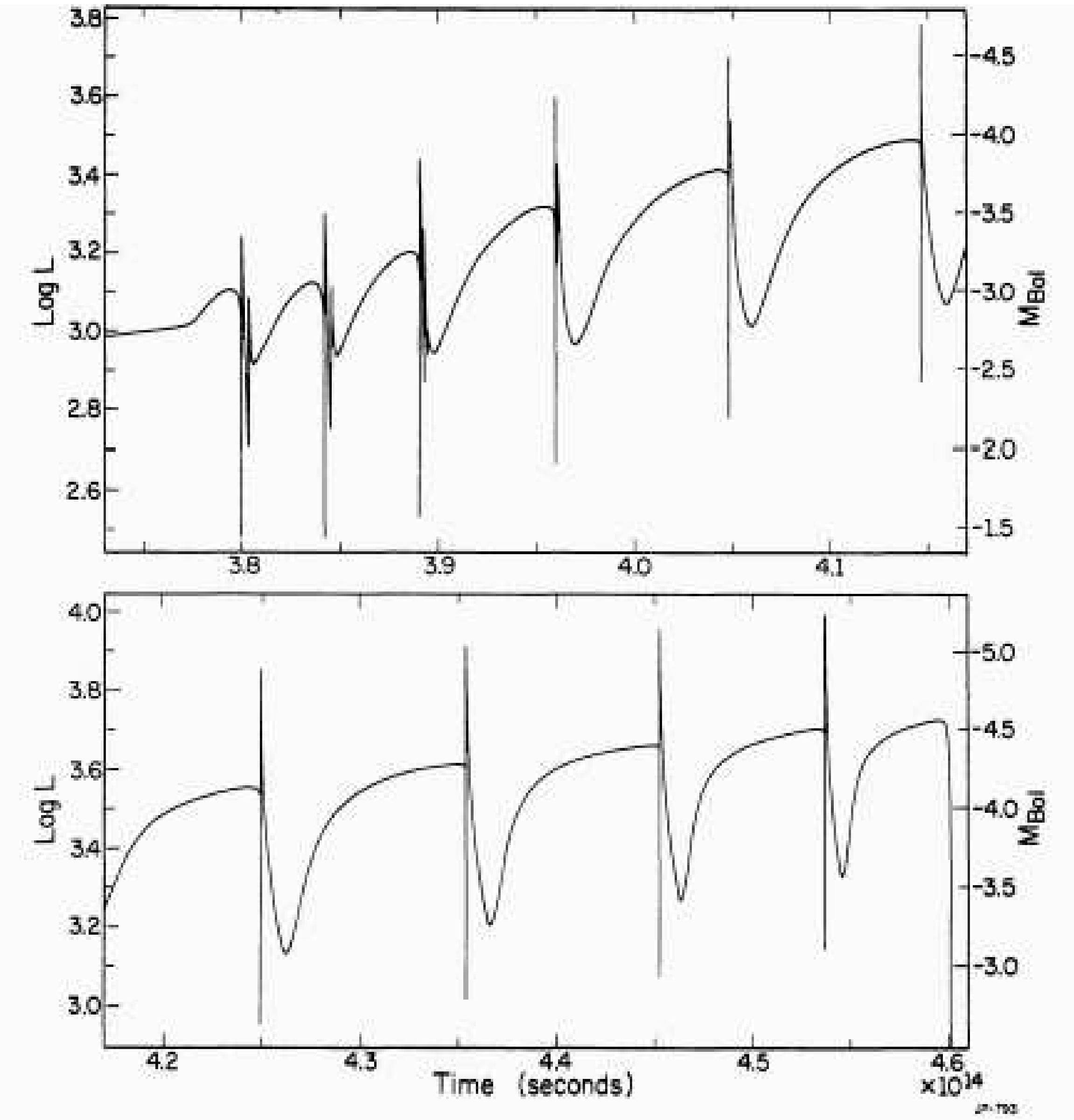


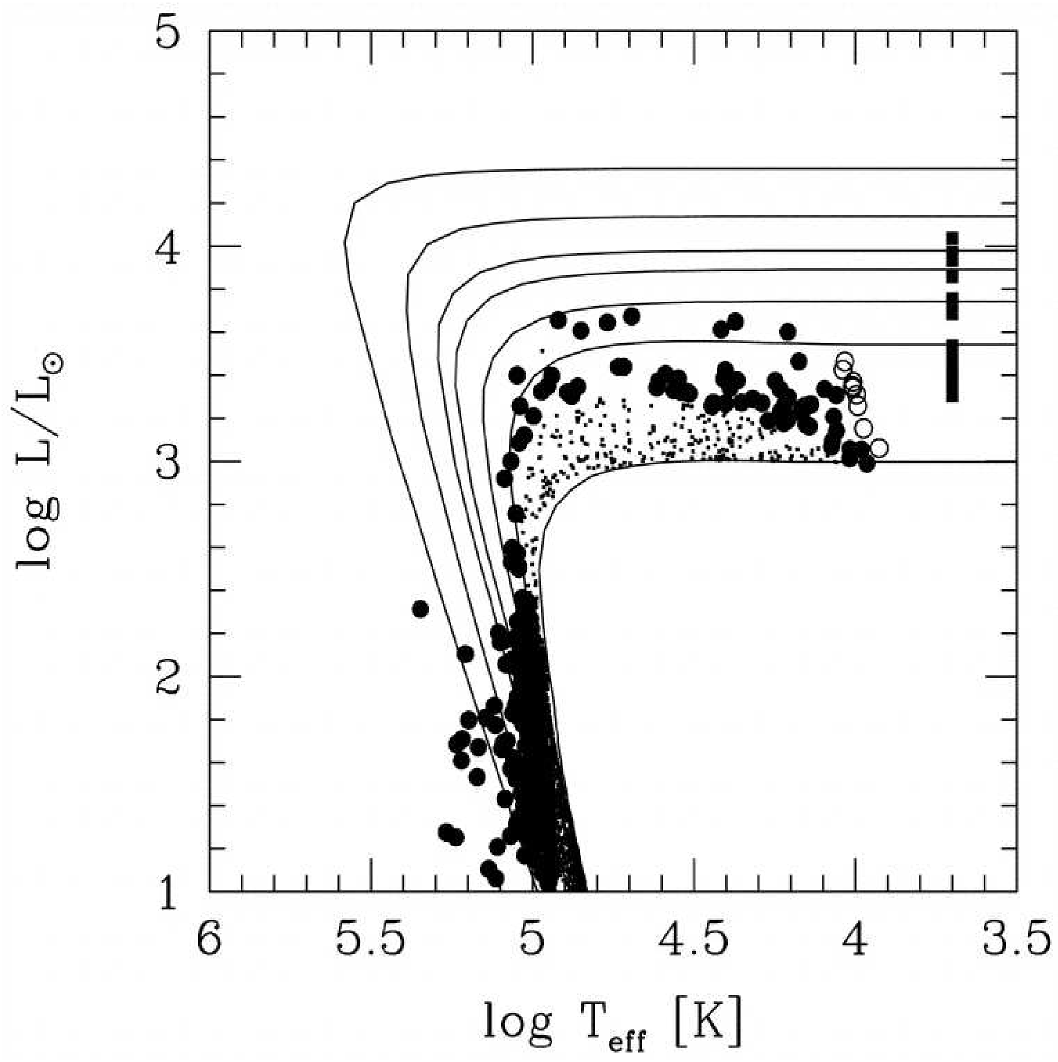

*Figure 4* Surface luminosity (in solar units) as a function of time in a model of mass $0.6\ M_\odot$, and initial composition $(Y, Z) = (0.25, 0.001)$.

Number of pulses increases rapidly for larger masses.

Final evolutionary stages of stars with $M < 8M_\odot$ are dominated by loss of the weakly bound envelope. Several stages,

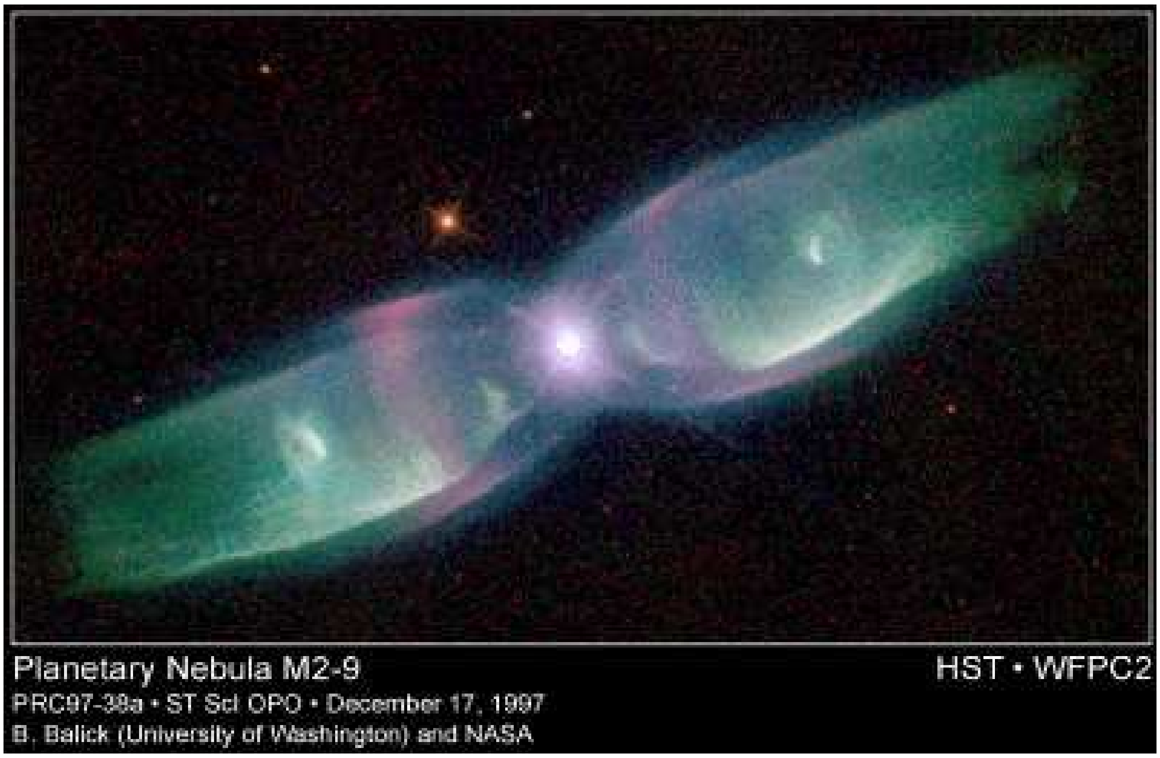
- A phase of extremely rapid mass loss $\dot{M} \sim 10^{-5}\ M_\odot \mathrm{yr}^{-1}$ which removes most of the hydrogen rich envelope (a *superwind*).
- Star shrinks in radius at constant $L$ and increasing $T_e$. Initially this phase is obscured by the slowly expanding ejecta (maybe 10 – 20 km/s). A *proto-planetary nebula.*
- Once the star reaches $\sim 10^4$ K, it starts photoionizing the surrounding material, producing a visible planetary nebula.
- Hot star produces a fast radiatively driven wind.
- Eventually, post-AGB star cools to evolve towards final white dwarf state.

Sample evolutionary tracks from Stanghellini & Renzini (2000):



## Shapes of planetary nebulae

Great deal of diversity in the morphology of planetary nebulae.




Discussion of how observed morphologies can be explained in eg Soker & Livio, ApJ, 339, 268 (1989).