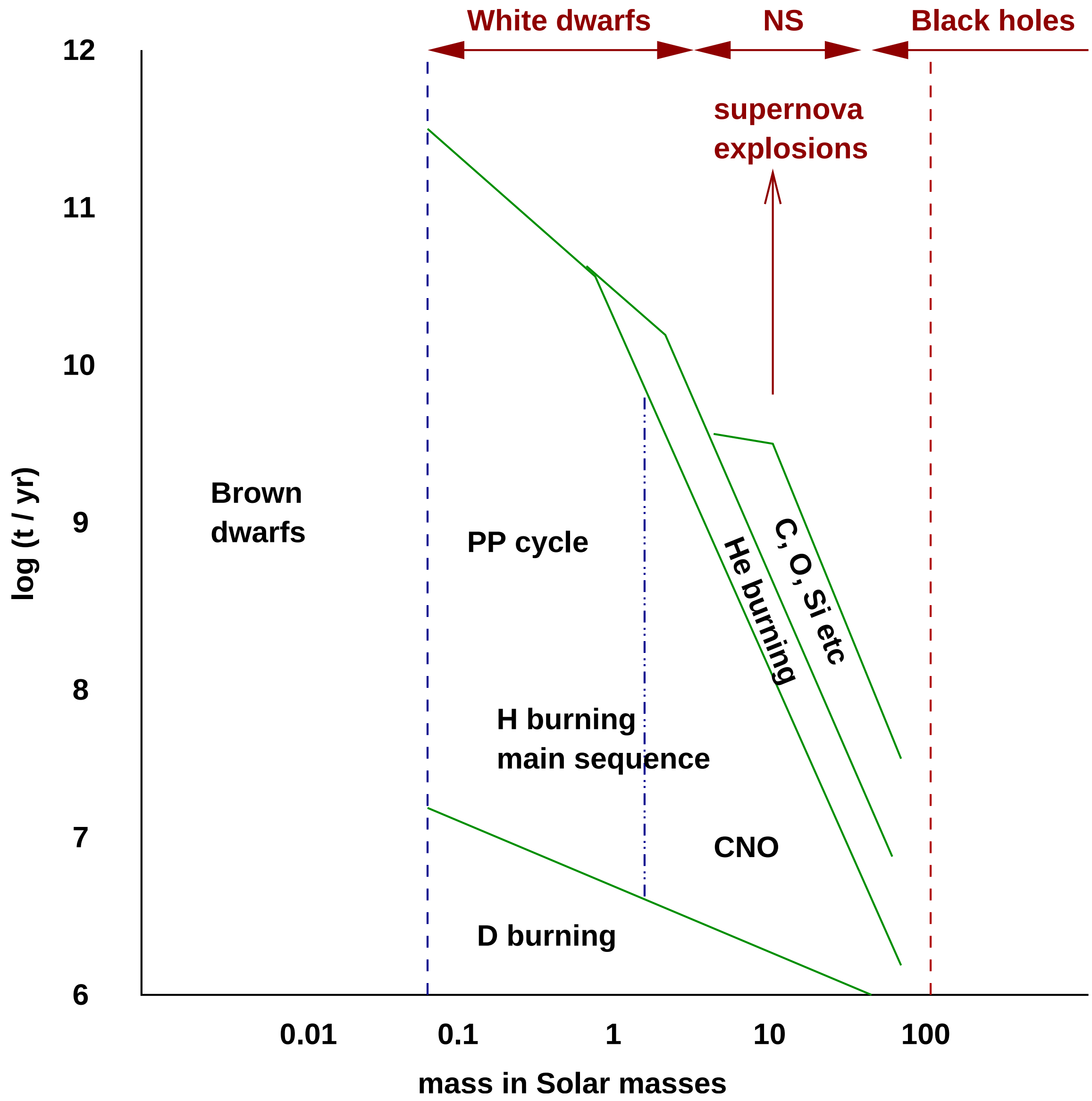
Post-main-sequence evolution



Need to consider:

- Track in HR diagram (very luminous).
- Mass loss (source of uncertainty).
- Final endpoint, WD, NS or black hole?

Summary of probable endpoints

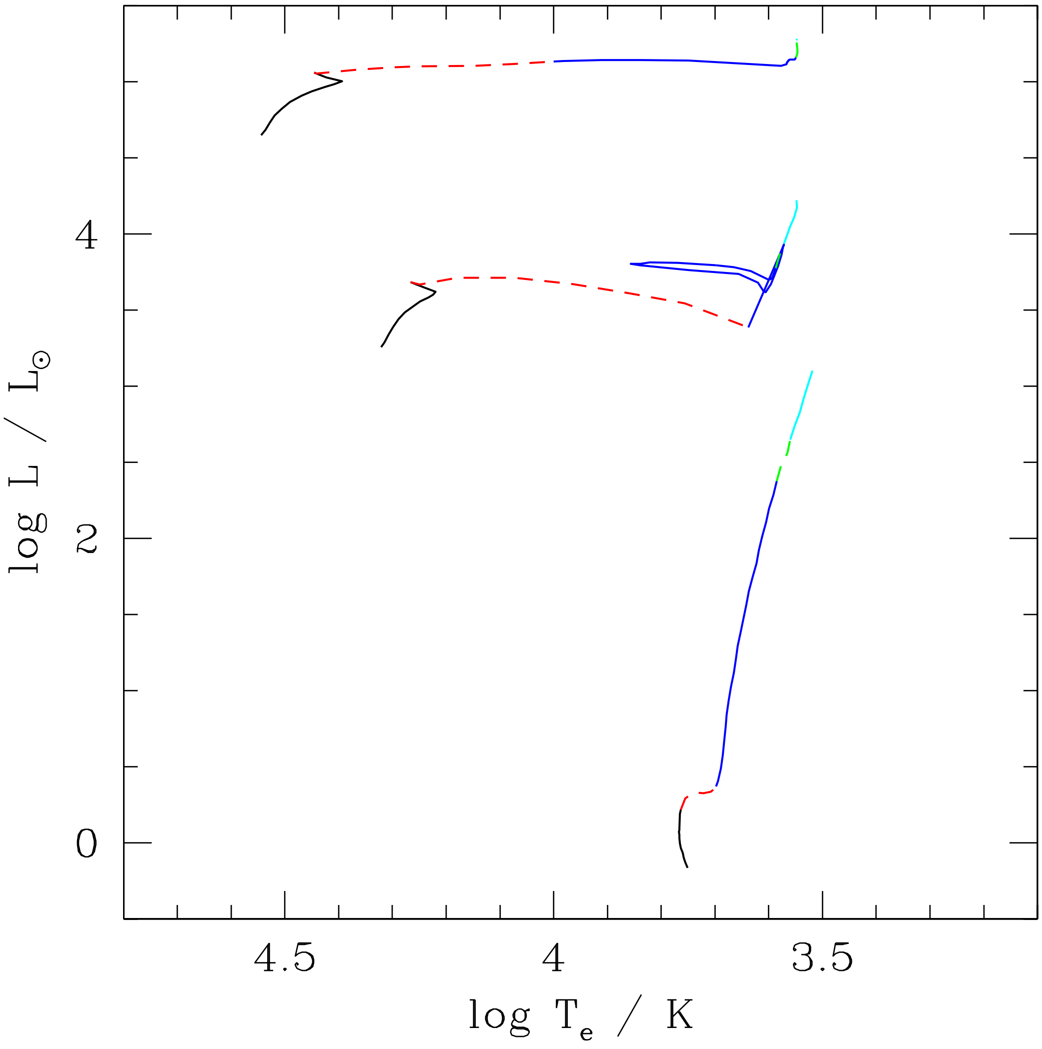
Uncertainties in the boundaries due to eg,

- Treatment of convection.
- At high masses, extent of stellar winds.

$M < 0.08 M_\odot$	Brown dwarf
$0.08 M_\odot < M < 0.5 M_\odot$	Central hydrogen burning
	Formation of a degenerate core
	No helium ignition
	Helium white dwarf
$0.5 M_\odot < M < 2 M_\odot$	Central hydrogen burning
	Helium flash
	CO white dwarf
$2 M_\odot < M < 8 M_\odot$	Central hydrogen burning
	Helium ignites in nondegenerate core
	CO white dwarf
$8 M_\odot < M < 20 M_\odot$	Numerous burning phases
	Bulk of heavy element enrichment
	from $M > 10 M_\odot$
	Type II supernova
	Neutron star
$M > 20 M_\odot$	**Black hole**

Calculated evolutionary tracks

Data from Schaller et al., A&A Sup. Ser., 96, 269 (1992). Tracks for $1M_{\odot}$, $7M_{\odot}$, $20M_{\odot}$.



Solid lines show track during, core hydrogen burning, helium burning and carbon burning respectively.

Note different evolution of low mass and higher mass stars once they leave the main sequence. Several reasons,

- Low mass stars have no convective core $\rightarrow$ continuous gradient in composition in central regions.
- Low mass stars have cores that are closer to the degeneracy boundary than high mass stars $\rightarrow$ less evolution needed to create a degenerate core.
- Low mass stars are closer to the Hayashi line.

Dividing line at around $2.3 M_{\odot}$.

Basic evolution involves,

- Contraction of the core, accompanied by expansion of the envelope.
- $\rightarrow$ reduced T_e, increased L.
- Sometimes repeated crossings of the HR diagram during He burning.

Evolution past critical core mass

For a core composed of an ideal gas, used the virial theorem to derive an expression for the core pressure P_s as a function of core radius R_c.

Found that there was a maximum in this function $\rightarrow$ maximum core mass that can match smoothly onto an envelope.

For small R_c, onset of degeneracy provides additional pressure support. In non-relativistic regime,

$$P \propto \rho^{5/3},$$

and taking core density $\rho \propto M_c/R_c^3$ we obtain,

$$P = \frac{C_4 M_c^{5/3}}{R_c^5}.$$

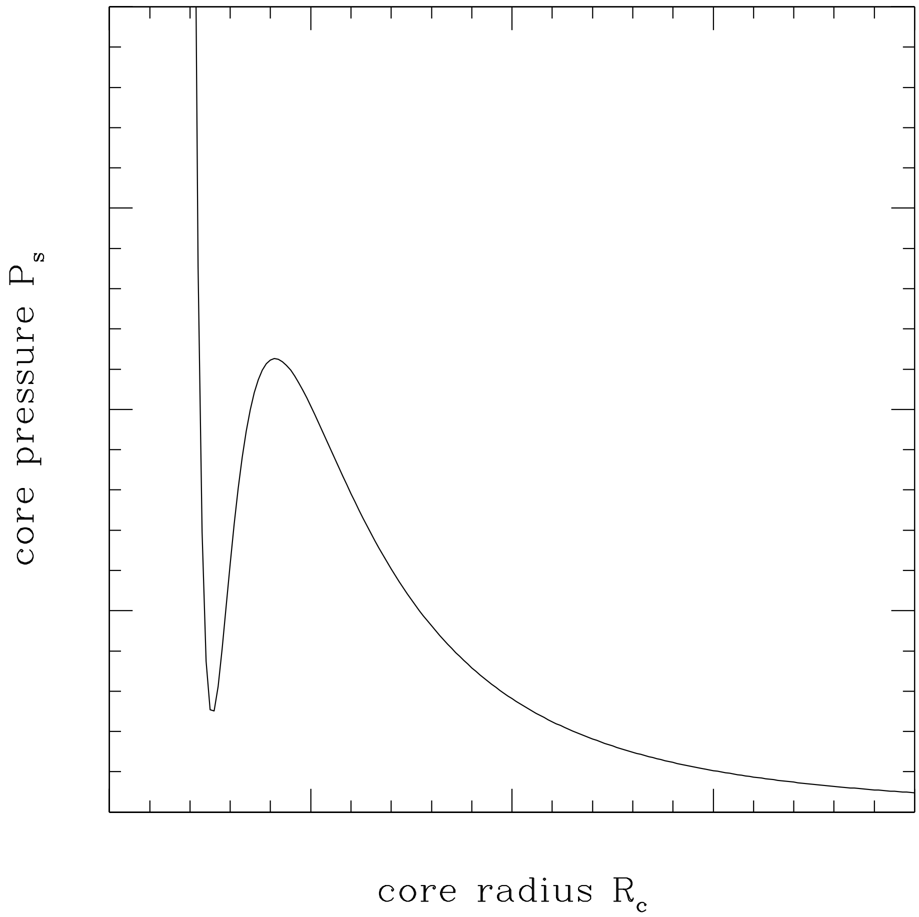
Adding this in to our previous expression, obtain,

$$P_s = \frac{C_1 M_c T_c}{R_c^3} - \frac{C_2 M_c^2}{R_c^4} + \frac{C_4 M_c^{5/3}}{R_c^5}.$$

Third term rises for small R_c $\rightarrow$ allows star to find solution with degenerate core which does not exist if the core is assumed to remain an ideal gas.

If M_c is large enough, this function has two turning points.

Note: straightforward but not much gained by adding in the various constants.



As before, need to match P_s onto the envelope pressure, which is a straight line in this diagram.

Evolution for a star with $M > 3M_\odot$ has three phases,

- Initially, $q < q_{SC}$. One, stable solution corresponding to a large, non-degenerate core.
- As q increases, reach a point where there are 3 possible solutions. Middle solution is thermally unstable, others represent a degenerate or non-degenerate core holding up the envelope. Continuity suggests we remain on the 'large R_c' branch.
- When q exceeds q_{SC}, only solution corresponds to the degenerate core. Star must make a rapid transition to this structure.

Timescale for core contraction will be the Kelvin-Helmholtz time,

$$t_{KH} = \frac{GM^2}{RL}.$$

This timescale is short $\sim 10^6$ yr for a star with $M = 5M_\odot$. Therefore few stars are observed populating the transition from high to low $T_e \rightarrow$ this region in the HR diagram is the **Hertzprung gap**.

Easy to see why the core must contract. Numerical calculations show that this is accompanied by an expansion of the envelope. Numerous explanations, mostly of questionable validity, for why this should be so.

Example from Padmanabhan:

Consider timescales shorter than the Kelvin-Helmholtz time. Both energy conservation,

$$\Omega + U = \text{constant}$$

and the virial theorem,

$$\Omega + 2U = \text{constant}$$

must hold over such timescales. Thus, Ω and U must be conserved separately.

For $M_c \gg M_{env}$,

$$|\Omega| \approx \frac{GM_c^2}{R_c} + \frac{GM_cM_{env}}{R}$$

where R is the radius of the star and we have assumed that the binding energy of the envelope is dominated by the gravity of the core.

If we take M_c to be constant (hence M_{env} also) then,

$$-\frac{GM_c^2}{R_c^2}\frac{dR_c}{dt} - \frac{GM_cM_{env}}{R^2}\frac{dR}{dt} = 0$$

which implies,

$$\frac{dR}{dR_c} = -\left(\frac{M_c}{M_{env}}\right)\left(\frac{R}{R_c}\right)^2$$

ie the envelope expands as the core contracts.

Lengthy discussions of 'why stars become red giants' can be found in the literature,

- Iben, ApJ, 415, 767 (1993), *'the transition from main sequence to giant branch involves a complicated interplay between a core, an envelope, and a nuclear-burning shell'*.
- Renzini & Ritossa, ApJ, 433, 293 (1994), expansion is driven by increased opacity in the envelope.
- Laughlin, Bodenheimer & Adams, ApJ, 482, 420 (1997), expansion is due to (1) increased core luminosity, (2) μ gradients, (3) atmospheric opacity.

Difficulty is in finding an intuitive explanation – numerical results are clear and in general agreement with observations.

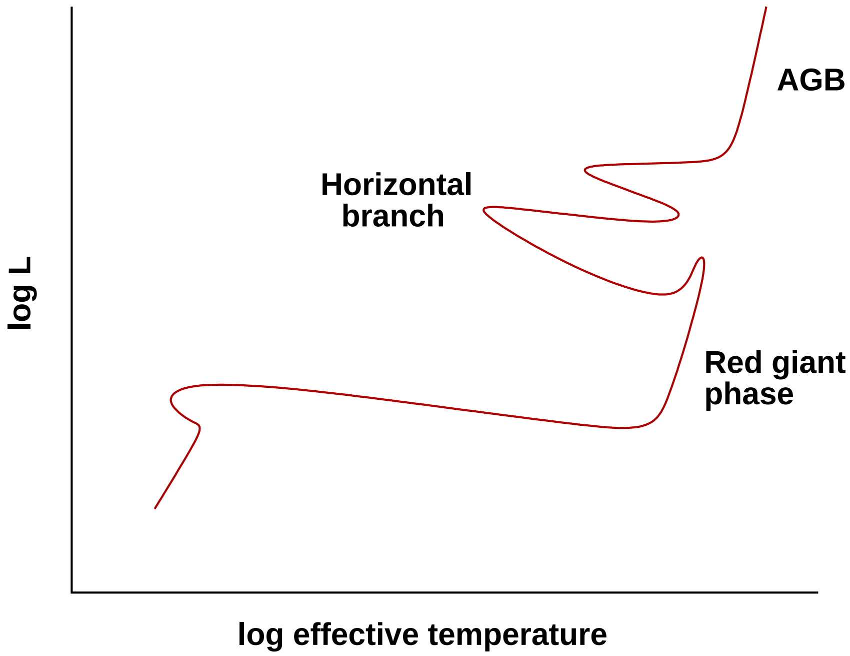
Once the envelope has expanded, opacity in the photosphere increases due to H^- in the outer regions $\rightarrow$ establishment of a surface convection zone.

Near vertical evolution along a Hayashi track.

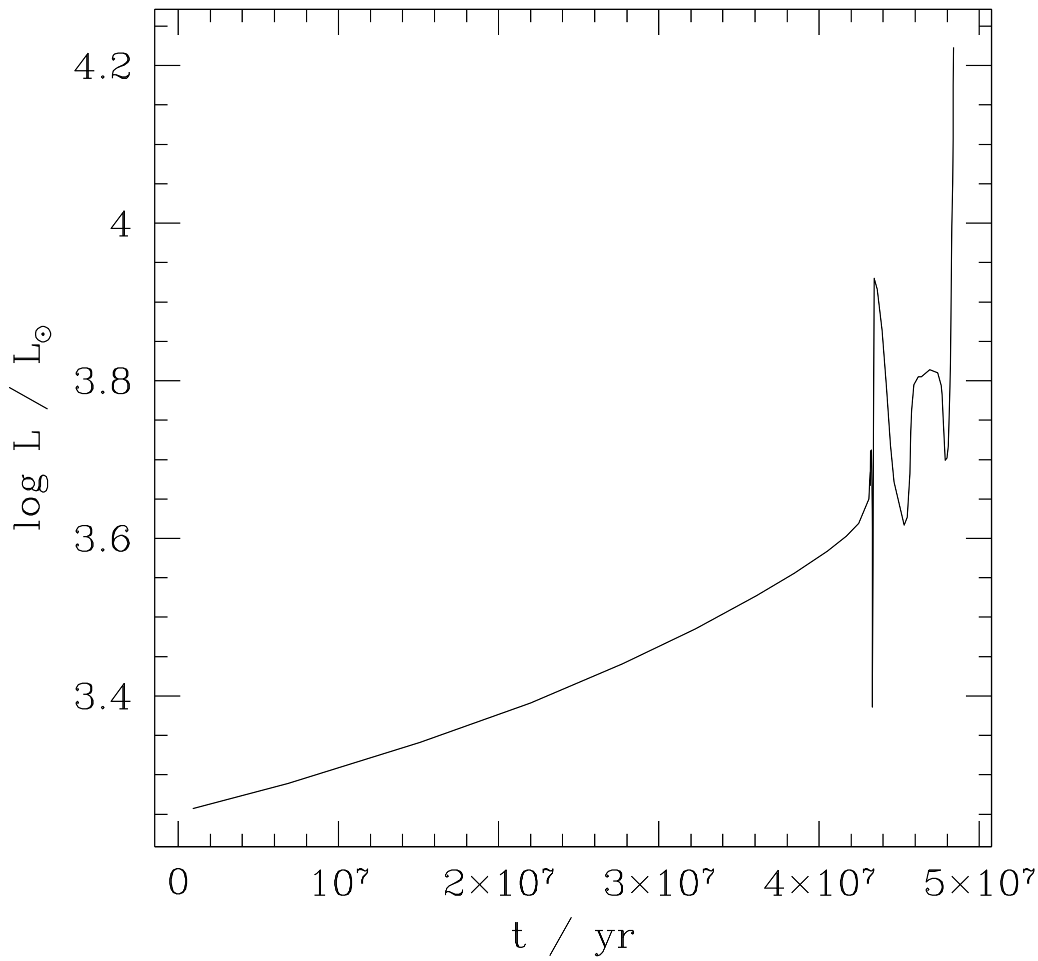
Convection zone can be deep enough to retrieve products of nuclear burning '**dredge-up**'.

Onset of helium burning in the core (plus hydrogen burning in a shell) leads to an excursion to higher T_e. The **horizonal branch**.

Next phase of Hayashi track evolution when core helium burning ceases is the **asymptotic giant branch**.



Calculated luminosity evolution for 7 $M_\odot$:



Calculated radius evolution for 7 $M_\odot$: