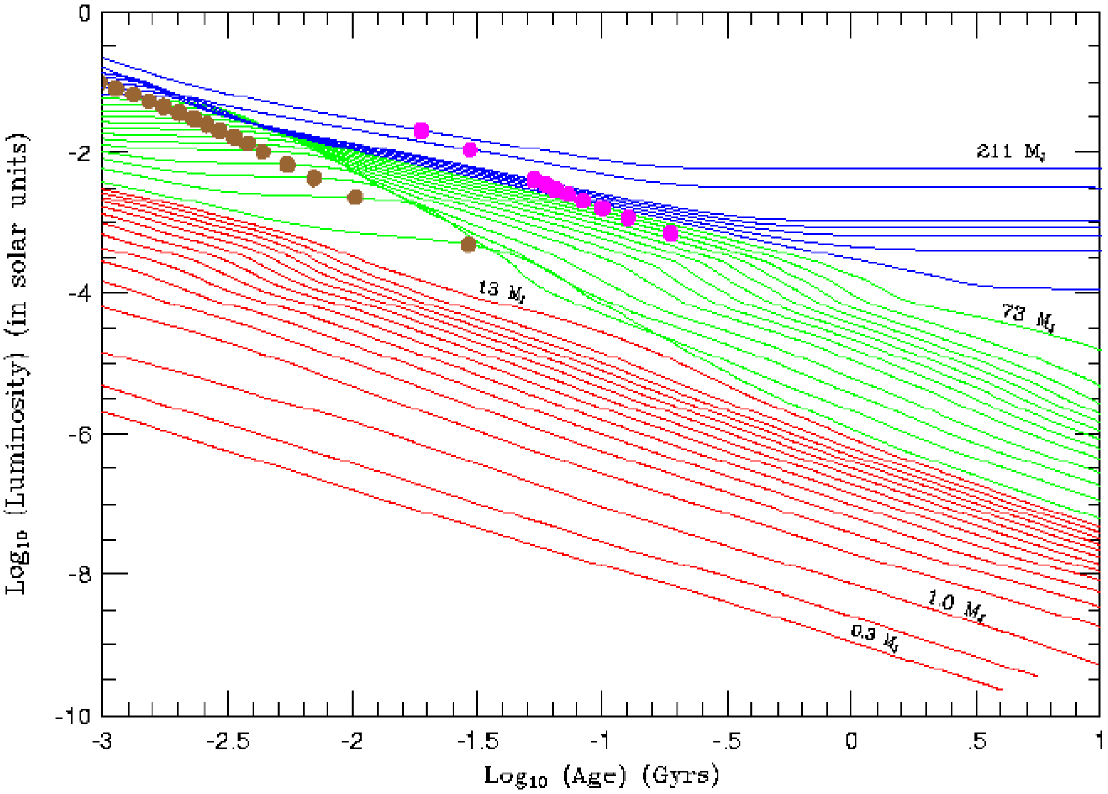
Brown dwarfs

Observationally, pre-main-sequence stars are found above the zero age main sequence, ie at higher luminosities and larger radii. Evolution involves quasi-static contraction, which ceases once the rising core temperature ignites nuclear reactions.

Brown dwarfs are objects in which contraction and rising core temperature is prevented by the onset of degeneracy pressure. Hence, L never stabilises and there is no significant hydrogen burning.

Detailed models (Burrows, astro-ph/0103383) suggest this definition makes sense:



In brown dwarfs,

- Ions never become degenerate.
- Electrons remain non-relativistic.

$\rightarrow$ pressure has contributions from nonrelativistic, degenerate electrons, and ideal gas of ions. Use an *approximate* equation of state to look at the physics.

For fully ionized hydrogen,

$$P = nKT = 2\frac{k}{m_p}\rho T.$$

Protons and electrons each contribute,

$$P_{\text{ideal}} = \frac{k}{m_p}\rho T = 8.3 \times 10^7 \rho T$$

in cgs units. Electron degeneracy pressure is,

$$P_{\text{deg}} = \left(3\pi^2\right)^{2/3} \left(\frac{\hbar^2}{m_e}\right) \left(\frac{\rho}{m_p \mu_e}\right)^{5/3} \simeq 10^{13} \rho^{5/3}$$

where,

$$\mu_e^{-1} = \sum_j \left(\frac{Z_j}{A_j}\right) X_j.$$

...assuming full degeneracy.

Define a parameter,

$$\xi = \frac{P_{\rm ideal}}{P_{\rm deg}} = 8 \times 10^{-6} T \rho^{-2/3},$$

measuring the relative importance of the two pressure sources. Note, if the brown dwarf is fully convective and non-degenerate then the entropy,

$$S = \text{const} + \frac{N_A k}{\mu} \ln \frac{T^{3/2}}{\rho}$$

is constant with radius. Thus $T\rho^{-2/3}$ is constant, and ξ has one value for the whole star.

This is also approximately true if the plasma is degenerate (but not proven in this course).

In terms of ξ, the ion pressure,

$$P_{\rm ion} = \xi P_{\rm deg}$$

and the electron pressure,

$$\begin{aligned} P_e &= P_{\rm deg} \quad \xi \ll 1 \\ P_e &= \xi P_{\rm deg} \quad \xi \gg 1. \end{aligned}$$

Hence write the total pressure in the form,

$$P \simeq 10^{13} \text{ dyn cm}^{-2} \rho^{5/3} f(\xi) \equiv K \rho^{5/3}$$

where the function $f(\xi)$ has the limits,

$$f(\xi) \to 1 + \xi$$

for $\xi \ll 1$ and,

$$f(\xi) \to 2\xi$$

for $\xi \gg 1$.

Convective stars are described by $n = 3/2$ polytropes. Numerically, relation between mean density and central density is,

$$\rho_c = 5.99 \bar{\rho}$$

where,

$$\bar{\rho} = \frac{3M}{4\pi R^3}.$$

Radius is related to the central density via,

$$R = 3.65 \left(\frac{(n+1)K}{4\pi G \rho_c^{1/3}} \right)^{1/2}$$

These relations give $R \propto K M^{-1/3}$, or numerically,

$$R \simeq 2.8 \times 10^9 \text{ cm} \left(\frac{M_\odot}{M}\right)^{1/3} f(\xi).$$

Since ξ is (approximately) constant, can evaluate it at the center,

$$\xi = \xi_c = 8 \times 10^{-6} T_c \rho_c^{-2/3} \propto T_c M^{-2/3} R^2.$$

Gives:

$$\xi = 3 \times 10^{-9} T_c \left(\frac{M_\odot}{M}\right)^{4/3} f^2(\xi)$$

ie,

$$T_c = 3 \times 10^8 \text{ K} \left(\frac{\xi}{f^2(\xi)}\right) \left(\frac{M}{M_\odot}\right)^{4/3}.$$

If we knew $f(\xi)$, this equation would allow us to evaluate the central temperature as a function of stellar radius R as a fixed mass M of gas contracts.

All we know for certain are the limits of $f(\xi)$. Expect,

- $\frac{\xi}{f^2(\xi)}$ increases as ξ for small ξ.
- Decreases as ξ^{-1} for large ξ.

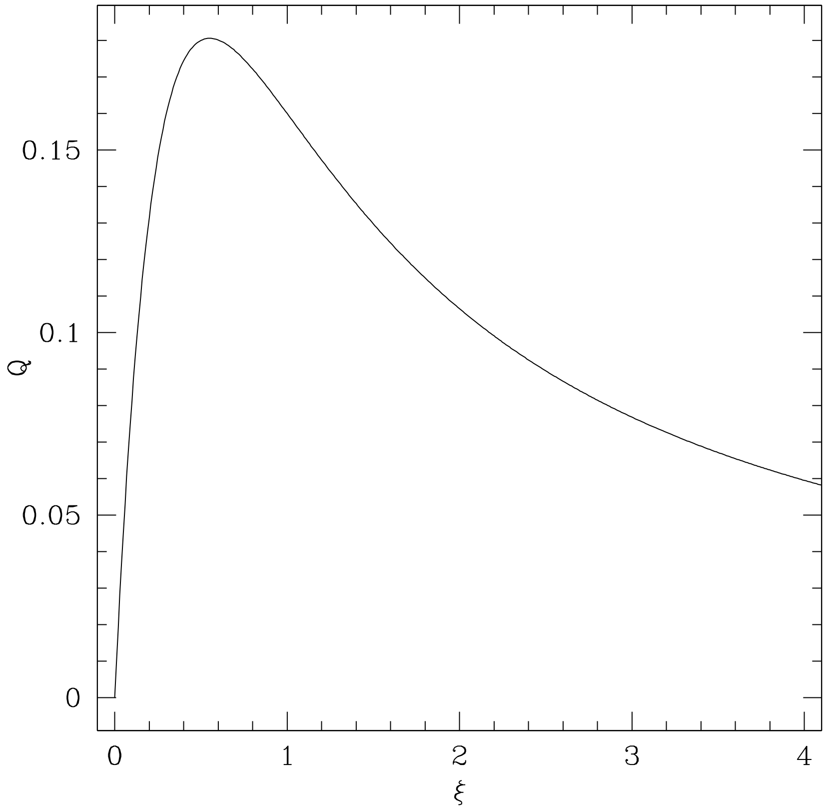
For a fudge that captures the right behavior, patch the limits together with some smooth function. eg,

$$f(\xi) = 1 + \xi + \frac{\xi^2}{1+\xi}.$$

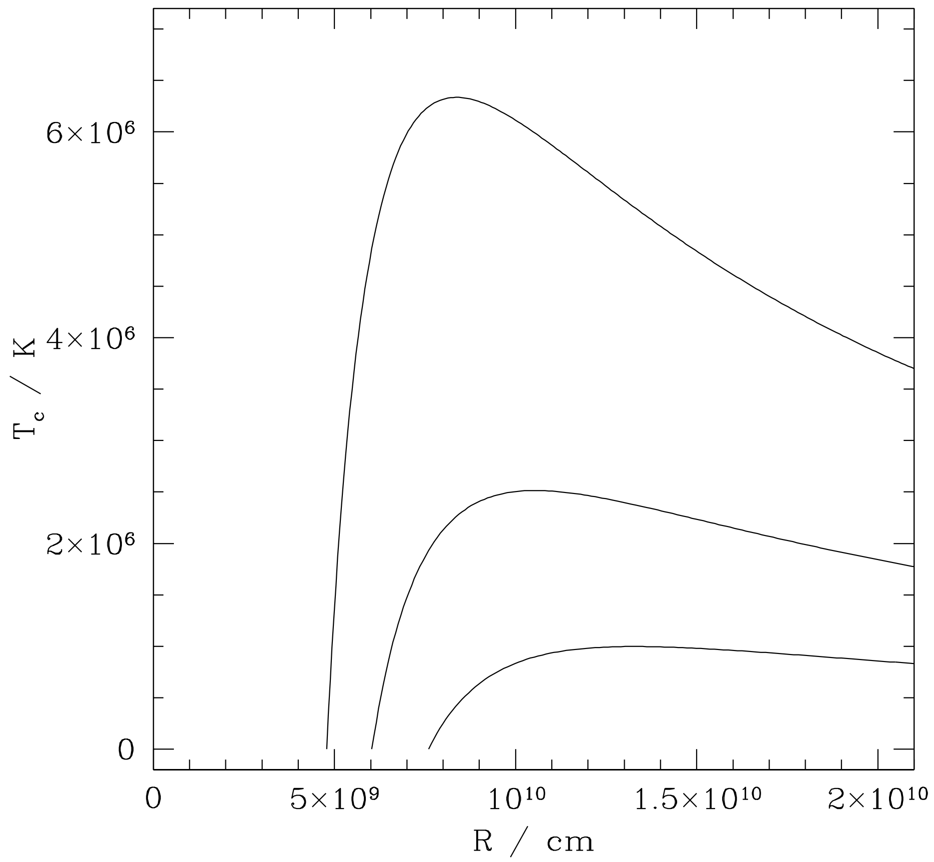
With this choice,

$$T_c = 3 \times 10^8 \text{ K} \left(\frac{M}{M_\odot}\right)^{4/3} Q(\xi)$$

where the function $Q(\xi)$ looks like:



Translated to radius, T_c has the form,



where the curves are for 0.2 $M_\odot$, 0.1 $M_\odot$ and 0.05 $M_\odot$. There is a maximum T_c, which drops as the stellar mass decreases.

The exact value of the maximum value of the central temperature, $T_{\max}$, depends upon the composition. For $\mu = 1.15$,

$$T_{\max} = 8 \times 10^7 \text{ K} \left(\frac{M}{M_\odot}\right)^{4/3}.$$

To estimate the brown dwarf boundary, compare the PP luminosity with the energy released from gravitational contraction.

Approximately, the PP luminosity at the maximum temperature is,

$$L_{PP} \approx \epsilon_{PP}(T_c)\rho_c R_c^3$$

and R_c is some measure of the core radius (say where $\rho = \rho_c/2$, which is at 0.38 R for an $n = 3/2$ polytrope). This gives,

$$L_{PP} = 0.08 M \epsilon_{PP}(T_c).$$

Using the expression for ϵ for the PP chain,

$$\epsilon_{PP} = 2.4 \times 10^6 \rho X^2 T_6^{-2/3} e^{-33.8 T_6^{-1/3}} \text{ erg g}^{-1}\text{s}^{-1}$$

find,

$$L_{PP} = 4 \times 10^{36} \left(\frac{M}{M_\odot}\right)^3 F(T_{\rm max}) \text{ erg s}^{-1}$$

where,

$$F(T_{\rm max}) = 2.4 \times 10^6 T_6^{-2/3} e^{-33.8 T_6^{-1/3}}$$

and T_6 is temperature in units of 10^6 K.

Compare this with the gravitational energy release. Total energy radiated when the contracting star has reached a radius $R(T_{\rm max})$ is the gravitational energy of an $n = 3/2$ polytrope,

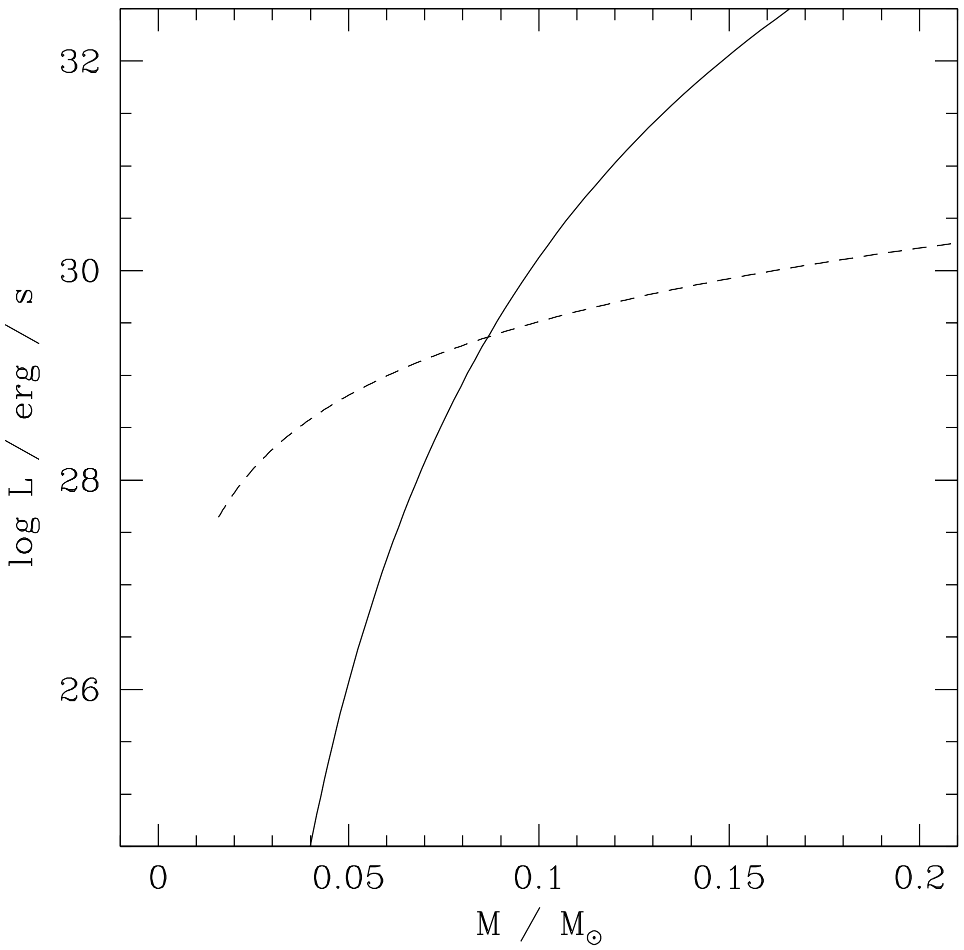
$$E = \frac{3}{7}\frac{GM^2}{R(T_{\rm max})} \approx 2 \times 10^{49} \left(\frac{M}{M_\odot}\right)^{7/3} \text{ erg.}$$

Assuming this is radiated over $\tau = 10^{10}$yr, then a lower limit on the gravitational contraction luminosity is,

$$L_{\rm grav} = \frac{E}{\tau} \approx 7 \times 10^{31} \left(\frac{M}{M_\odot}\right)^{7/3} \text{ erg s}^{-1}.$$

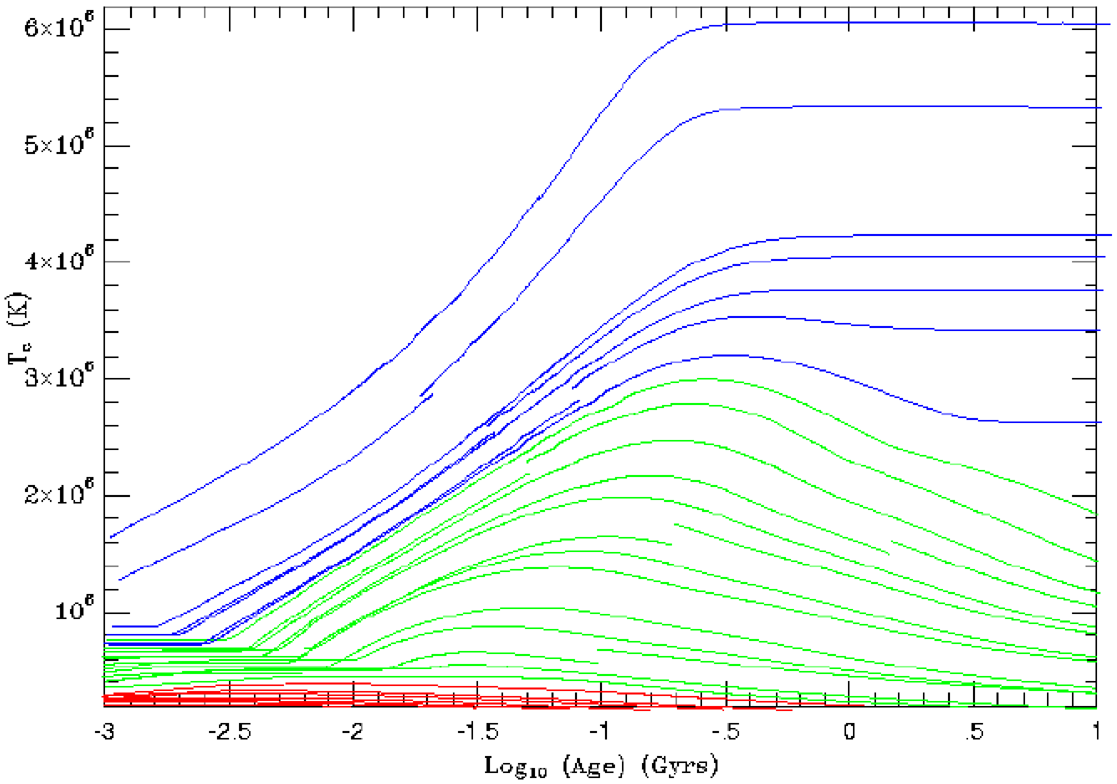
Particular choices of numerical factors are arbitrary and unimportant. Main point is that relative importance of L_{PP} *decreases* compared to $L_{\rm grav}$ at smaller masses.

With our choices of numerical factors,



$\rightarrow$ substellar boundary is at $\approx 0.08\ M_\odot$.

Numerical models of brown dwarf / low mass stellar evolution broadly consistent with this toy model,



This figure from Burrows et al. (2001), Reviews of Modern Physics, 73, 719.

See also Chabrier & Baraffe (2000), ARAA, 38, 337 for another recent review, or Stevenson (1991), ARAA, 29, 163 for more introductory aspects.

Basri (2000), ARAA, 38, 485 discusses observational aspects of brown dwarfs.