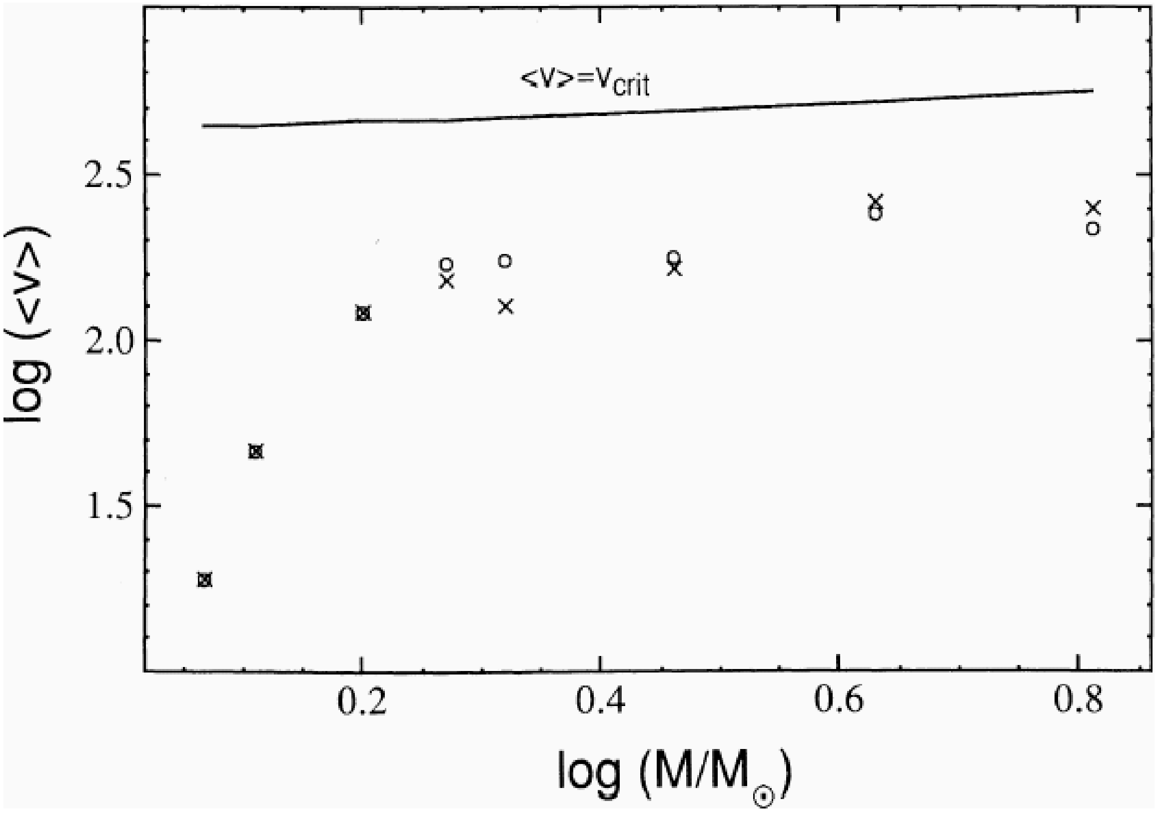
Influence of rotation

Obvious that rapid stellar rotation, with equatorial velocity $v_{\mathrm{eq}} \sim \sqrt{GM/R}$, may introduce large changes as compared to spherically symmetric models. However even slow rotation, as in the Sun, raises interesting questions:

- *Why* is the rotation slow, given that stars form from molecular clouds with characteristic sizes $\gg R_\odot$? Thought that there is angular momentum loss both during star formation and via stellar winds throughout the star's lifetime.
- Possibility of *mixing* due to rotationally induced circulation.
- Role of rotation in generating magnetic fields.

Discuss here effects of a constant (in time) rotation profile, but redistribution of angular momentum also very important (and uncertain).

Empirically (e.g. Kawaler 1987), the Sun's slow rotation is typical for low mass stars on the main sequence. Stars with spectral types earlier that around F2 rotate much more rapidly:



$v_{\rm crit}$ line is a measure of the break up velocity.

Conservative rotation

Simplest case to consider is that of *conservative* rotation. Assume that the centifugal acceleration can be derived from a potential,

$$\Omega^2 r \mathbf{e}_r = -\nabla V,$$

where Ω is the angular velocity, $\mathbf{e}_r$ is a unit vector perpendicular to the axis of rotation, and r is the distance from the axis.

Note: for this discussion we use *cylindrical* polar co-ordinates (r, ϕ, z) exclusively.

An angular velocity distribution is conservative if Ω is only a function of r – ie the rotation rate is constant on cylinders. Solid-body rotation is a special case.

The centrifugal potential is,

$$V = -\int_0^r \Omega^2 r dr.$$

Combine this with the gravitational potential to form a total potential,

$$\Psi = \Phi + V.$$

In hydrostatic equilibrium, then,

$$\nabla P = -\rho \nabla \Psi.$$

This implies,

- That the vectors ∇P and $-\nabla\Psi$ are parallel, i.e. equipotential surfaces of Ψ coincide with surfaces of constant pressure.
- $\rightarrow P = P(\Psi)$ only.
- Since $\rho = -\mathrm{dP}/\mathrm{d}\Psi$, density is also only a function of Ψ.
- For an ideal gas,
$$\frac{T}{\mu} = \frac{P}{\rho\mathcal{R}}.$$
If $\mu =$ constant, then $T = T(\Psi)$.

Summary: for a chemically homogenous star, rotating conservatively in hydrostatic equilibrium, P, T and ρ are all constant on surfaces of constant total potential.

Likewise, the opacity and the energy generation rate are functions of Ψ only.

Von Zeipel's Theorem

Consider radiative transport in the rotating star. Dropping the assumption of spherical symmetry,

$$\mathbf{F} = -\frac{4ac}{3\kappa\rho}T^3\nabla T,$$

where $\mathbf{F}$ is the vector of the radiative energy flux. Since $T = T(\Psi)$,

$$\mathbf{F} = -\frac{4ac}{3\kappa\rho}T^3\frac{\mathrm{dT}}{\mathrm{d}\Psi}\nabla\Psi.$$

In a stationary state, the divergence of the radiative flux must be balanced by nuclear energy generation,

$$\nabla \cdot \mathbf{F} = \epsilon\rho.$$

This can be written as,

$$-\frac{4ac}{3\kappa\rho}T^3\frac{\mathrm{dT}}{\mathrm{d}\Psi}\nabla^2\Psi - |\nabla\Psi|^2\frac{\mathrm{d}}{\mathrm{d}\Psi}\left(\frac{4ac}{3\kappa\rho}T^3\frac{\mathrm{dT}}{\mathrm{d}\Psi}\right) = \epsilon\rho$$

Splitting the Laplacian into gravitational and rotational terms,

$$-k(\Psi)\left(4\pi G\rho - \frac{1}{r}\frac{\mathrm{d}(r^2\Omega^2)}{\mathrm{d}r}\right) - |\nabla\Psi|^2\frac{\mathrm{d}k(\Psi)}{\mathrm{d}\Psi} = \epsilon\rho.$$

where k is a function incorporating all the terms in the previous expression that are just functions of Ψ.

Easy to construct an example that demonstrates that this equation cannot generally be satisfied:

- Terms $4\pi G\rho k(\Psi)$ and $\epsilon\rho$ are constant on equipotential surfaces.
- For solid-body rotation,

$$\frac{1}{r}\frac{\mathrm{d}(r^2\Omega^2)}{\mathrm{d}r} = 2\Omega^2$$

  i.e. a constant *everywhere*.

- $\nabla\Psi = -\mathbf{g}_{\mathrm{eff}}$, the effective gravitational acceleration including both gravity and centrifugal forces. This is **not constant** on equipotential surfaces (effective gravity is smaller at the equator than at the poles).

Since three constant terms (on an equipotential surface) cannot everywhere balance a variable term, conclude that a conservatively rotating star cannot be in a steady-state thermal equilibrium.

Note:

- A similar result can be proved for non-conservative rotation.
- As for convection, gradients in μ are stabilizing. If μ develops a non-spherical distribution, possible to set up a rotating steady-state.

Meridional circulation

Since a steady state thermal equilibrium cannot be achieved, expect:

- Some regions of the star will tend to cool off, while others will heat up.
- $\rightarrow$ Buoyancy forces.
- $\rightarrow$ Meridional circulation in addition to rotation.

Fairly straightforward to show that if the rotation of the star is parameterized by χ, where,

$$\chi = \frac{\Omega^2}{2\pi G\rho},$$

then the circulation timescale is,

$$\tau_{\rm circ} \approx \frac{GM^2}{LR}\frac{1}{\chi} \approx \frac{\tau_{\rm KH}}{\chi}.$$

This is the *Eddington-Sweet* timescale. For the Sun, $P = 25$ days, $\rho = 1.4$ g cm^{-3}, so $\chi \sim 10^{-5}$. Conclude,

$$\tau_{\rm circ} \sim 10^{12} \text{ yr}$$

so irrespective of composition do not expect the Sun to be mixed as a result of rotation. However, layers in the Sun, and more rapidly rotating massive stars, could be vulnerable to mixing.

Lithium depletion

The surface abundance of lithium provides one test of the importance of mixing. Lithium is weakly bound and can be destroyed by the reaction,

$$^7\mathrm{Li} + \mathrm{p} \rightarrow ^4 \mathrm{He} + \alpha$$

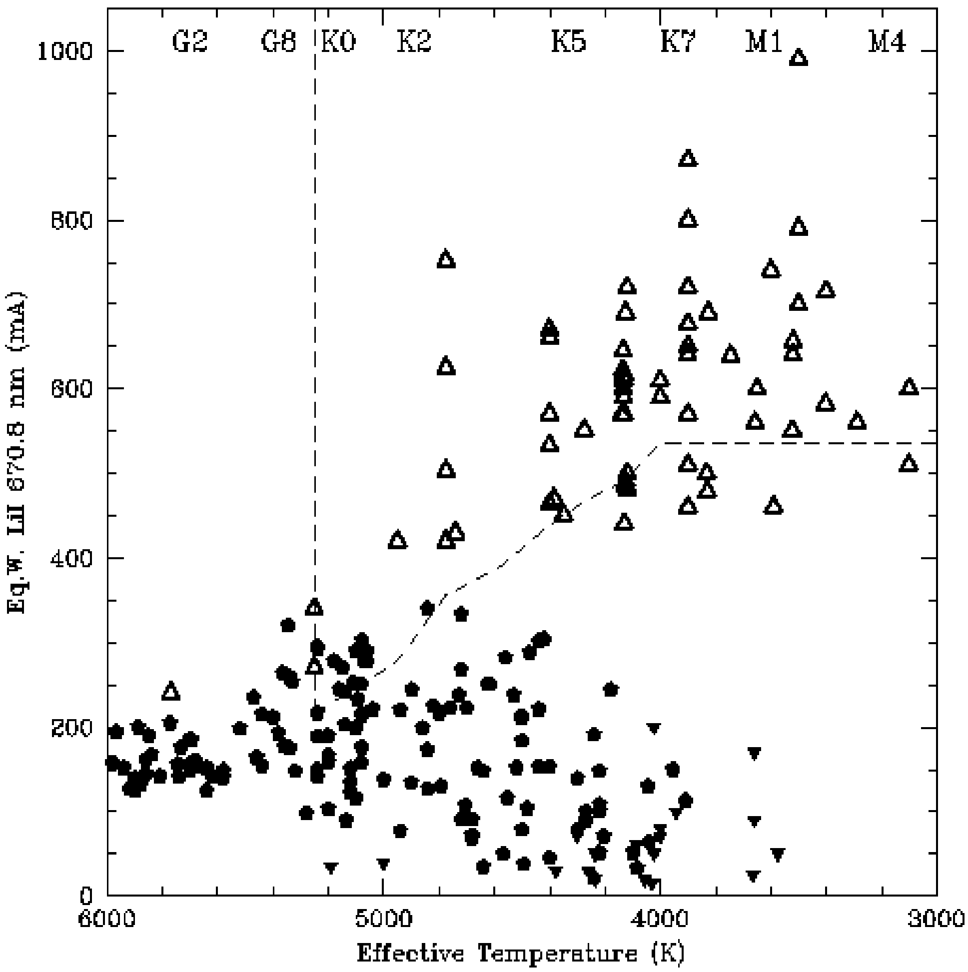
at temperatures above 2.5×10^6 K. Expect lithium depletion only if the bottom of the surface convection zone reaches that temperature. Depletion can occur,

- During fully convective pre-main-sequence evolution.
- On the main sequence. For stars with $M \approx M_\odot$, the base of the convection zone is close to the critical temperature, so the lithium abundance is potentially sensitive to any additional mixing there.

Models predict:

- No lithium depletion for $M > 1.7\ M_\odot$.
- Significant depletion for $M \approx M_\odot$.
- Strong depletion for sub-Solar mass stars.

Lithium as an age / mass indicator

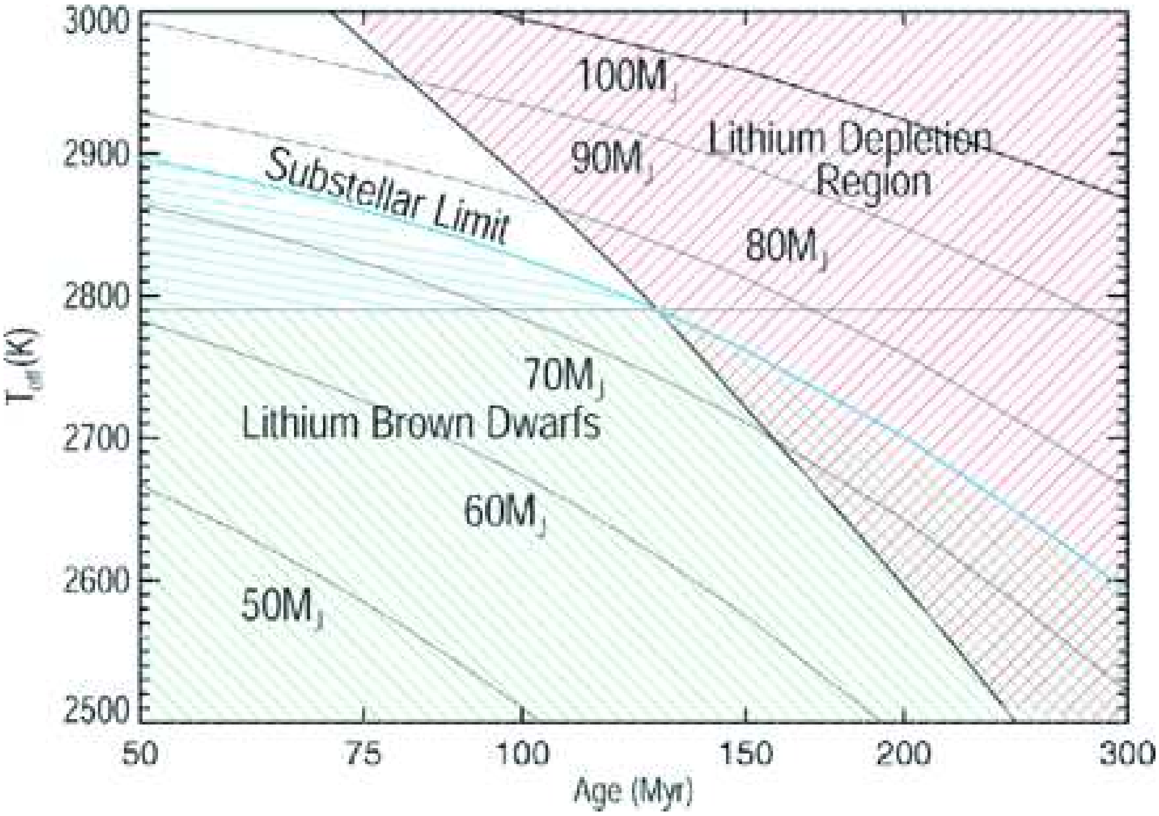


Equivalent widths of Li absorption line for low mass stars. T Tauri stars are shown as open triangles, low mass members of young clusters as filled symbols. Figure from Martin (1997).

If the mass is low enough, the core becomes degenerate before the ignition temperature for Li is reached. Models suggest that this occurs for $M < 0.06\ M_\odot \rightarrow$ presence of lithium at late times provides evidence for brown dwarf nature of candidate substellar objects.

Model tracks

From Baraffe & Chabrier, quoted by Basri (ARAA, 2000).



Lithium depletion region to the right is where more than 99% of the lithium has been destroyed.

At late times this is a clean test except for brown dwarfs very close to the substellar boundary.