Subsequent nuclear reactions

Helium burning yields a mix of carbon, oxygen and neon. If temperatures rise to $\sim 10^9$ K, futher nuclear reactions can occur. Since the binding energies per nucleon of these relatively heavy nuclei are comparable, expect a wide range of possible reactions.

- Important carbon burning reactions:

$$\begin{aligned} {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{20}\mathrm{Ne} + \alpha \\ {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{23}\mathrm{Na} + \mathrm{p}. \end{aligned}$$

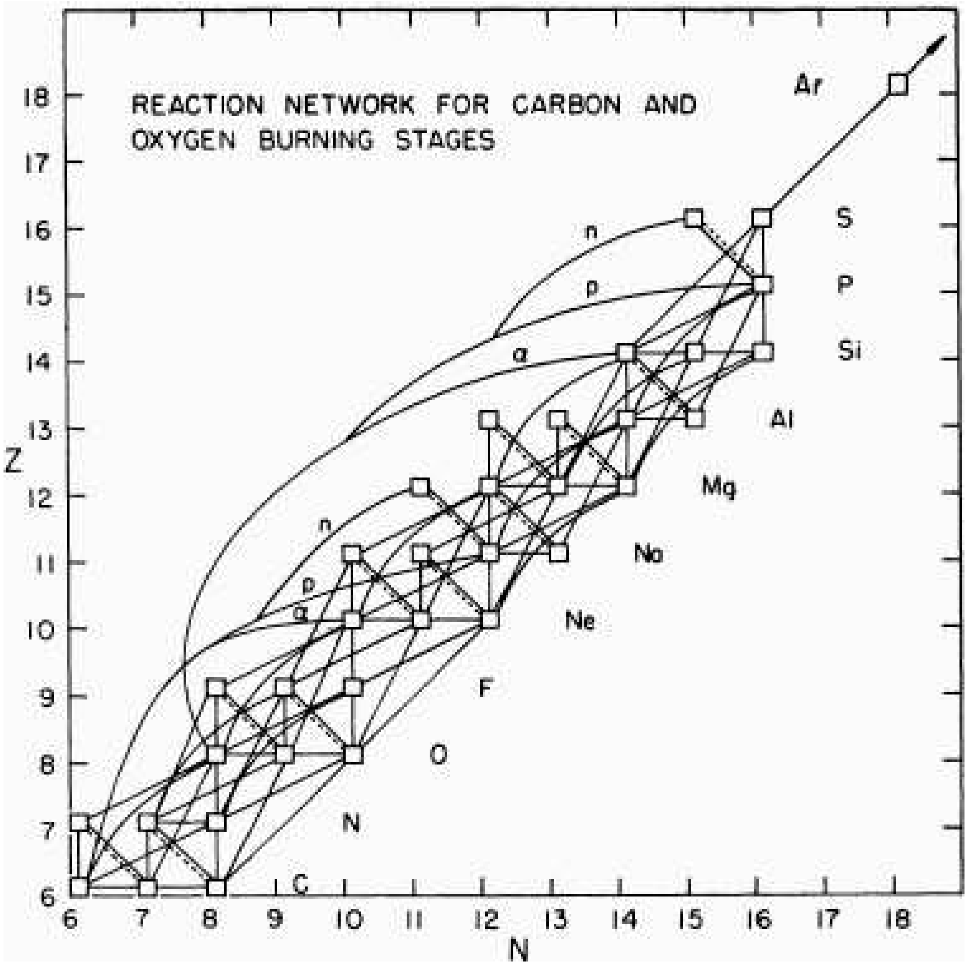
- Most common oxygen burning reaction:

$${}^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{31}\mathrm{P} + \mathrm{p}.$$

Reactions involving neon occur at intermediate temperature. These involve an initial photodisintegration of the nucleus by energetic photons,

$$\begin{aligned} {}^{20}\mathrm{Ne} + \gamma &\rightarrow {}^{16}\mathrm{O} + \alpha \\ {}^{20}\mathrm{Ne} + \alpha &\rightarrow {}^{24}\mathrm{Mg} + \gamma \\ {}^{24}\mathrm{Mg} + \alpha &\rightarrow {}^{28}\mathrm{Si} + \gamma \end{aligned}$$

Example of a full reaction network for carbon burning (from Arnett & Truran 1969):



Subsequent reactions are even more complicated. General points:

- Si burning starts around 3×10^9 K.
- Over long timescales, result is formation of ^{56}Fe.
- Shorter intervals (eg in supernovae) favour production of ^{56}Ni.

Summary of results so far

Equations

(1) Mass continuity

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

(2) Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

(3) Energy generation

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

(4) Energy transport

$$\frac{dT}{dr} = -\frac{3\kappa_R \rho L_r}{16\pi a c r^2 T^3} \quad \text{(radiative)}$$

$$\frac{dT}{dr} = \left(1 - \frac{1}{\Gamma_2}\right) \frac{T}{P}\frac{dP}{dr} - \Delta\nabla T \quad \text{(convective)}$$

Microphysics

Equation of state for non-degenerate matter:

$$P = \left(\frac{\mathcal{R}}{\mu}\right)\rho T + \frac{1}{3}aT^4$$

Use Saha's equation to get μ, but remember pressure ionization. When fully ionized, $\mu = 4/(3+5X)$.

Opacity (Rosseland mean):

$$\kappa_R = \kappa_0 \rho^\alpha T^\beta$$

with $\alpha = 1$, $\beta = -3.5$ for Kramer's opacity, and $\alpha = \beta = 0$ for electron scattering.

Energy generation:

$$\epsilon = \epsilon_0 \rho T^\nu$$

with $\nu \sim 4$ for PP hydrogen burning and $\nu \sim 18$ for CNO hydrogen burning.

Mixing length theory:

$$\Delta \nabla T \simeq 0$$

in stellar interiors.

Boundary conditions

Consider solving the structure equations for a given mass M and composition X, Y, Z.

Central conditions

At $r = 0$:

$$\begin{aligned} m &= 0 \\ L_r &= 0 \end{aligned}$$

The structure equations contain indeterminate ratios at the origin. The boundary conditions plus the requirement that T, ρ and P have zero gradients at $r = 0$ imply,

$$\begin{aligned} m &\rightarrow \frac{4}{3}\pi r^3 \rho_c \\ P &\rightarrow P_c - \frac{2}{3}\pi G \rho_c^2 r^2 \\ L_r &\rightarrow \frac{4}{3}\pi r^3 \rho_c \epsilon_c \end{aligned}$$

where ρ_c denotes the central value etc.

The temperature gradient is,

$$T \to T_c - \frac{1}{8ac}\frac{\kappa_c \rho_c^2 \epsilon_c}{T_c^3} r^2$$

in the radiative case, and

$$T \to T_c - \frac{2\pi}{3} G \nabla_{\mathrm{ad}} \frac{\rho_c^2 T_c}{P_c} r^2$$

for adiabatic convection.

These expansions allow an integration to start at finite (small) radius.

Surface conditions

Consider the pressure scale height at the surface of the Sun,

$$H_p = \frac{P}{\left|\frac{dP}{dr}\right|}.$$

Using hydrostatic equilibrium and assuming gas pressure (take $\mu = 1$ for now), find,

$$H_p \simeq 2 \times 10^7 \ \mathrm{cm} \sim 3 \times 10^{-4} \ R_\odot$$

i.e. the surface of the Sun is quite sharply defined in terms of pressure.

Simplest boundary conditions at $r = R$,

$$\begin{aligned} m &= M \\ P &= 0 \\ T &= 0. \end{aligned}$$

Can demonstrate that these boundary conditions are a reasonable approximation in some cases. e.g. for a radiative atmosphere with Kramer's opacity we showed (Problem set 5) that,

$$P^2 - P_s^2 = J(T^{8.5} - T_e^{8.5})$$

where J is a constant and I have restored the surface integration constants ignored in the problem. The very strong dependence of P on T means that the detailed surface conditions only matter within a fraction of a scale height of the surface.

More work is needed if the outer layers are convective. Assuming a perfect gas,

$$\frac{d \ln P}{d \ln T} = \frac{5}{2}$$

so,

$$P = CT^{5/2}.$$

The constant C determines the entropy of the convective region, and is not determined by zero boundary conditions.

Better boundary conditions can be derived by considering conditions at the photosphere. Stellar atmosphere theory defines this as the level where the optical depth to infinity is 2/3,

$$\tau = \int_R^\infty \kappa\rho dr = \bar{\kappa} \int_R^\infty \rho dr = \frac{2}{3}$$

where $\bar{\kappa}$ is a mean opacity averaged over the stellar atmosphere. In hydrostatic equilibrium,

$$P_{r=R} = \int_R^\infty \frac{Gm}{r^2}\rho dr \simeq \frac{GM}{R^2} \int_R^\infty \rho dr.$$

Eliminating the integral over density we obtain,

$$P_{r=R} = \frac{GM}{R^2}\frac{2}{3\bar{\kappa}}.$$

By definition, the effective temperature is,

$$L = 4\pi R^2 \sigma T_e^4.$$

These equations provide two relations between the surface values of the functions P, T, L_r, which can replace the zero boundary conditions.

Note however that at $\tau \sim 1$, all simple approximations to the radiative transfer break down. Accurate boundary conditions really require solution of a stellar atmosphere problem.