Nuclear reactions: proton-proton reaction

Nuclear reactions of interest generally involve several steps. In such a chain,

- The **rate** is controlled by the rate of the slowest reaction in the chain.
- The total energy release is given by the sum of the energies of all the individual steps.

The first, and rate determining step, in the proton-proton chain is,

$$^1H +^1 H \rightarrow^2 H + e^+ + \nu_e$$

where ^2H is a deuteron. This reaction is non resonant and slow because it involves a weak decay. The rate evaluates to,

$$r_{pp} = 1.15 \times 10^9 \ T_9^{-2/3} X^2 \rho^2 e^{-3.380/T_9^{1/3}} \ \mathrm{cm}^{-3} \ \mathrm{s}^{-1}$$

where T_9 is temperature in units of 10^9 K.

The temperature sensitivity is,

$$\nu_{pp} = \frac{11.3}{T_6^{1/3}} - \frac{2}{3}$$

and the lifetime of a proton against destruction is,

$$\tau_p = -\frac{n_p}{\mathrm{d}n_p/\mathrm{d}t} = \frac{n_p}{2r_{pp}}$$

is 6×10^9 yr for Solar central conditions.

To make helium, we need eventually to use 4 protons, two of which need to be converted to neutrons via either positron decays or electron captures. Starting with the fundamental reaction forming a deuteron, there are three main possibilities:

PP-I

$$\begin{aligned} {}^{1}\mathrm{H} + {}^{1}\mathrm{H} &\rightarrow {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\ {}^{2}\mathrm{H} + {}^{1}\mathrm{H} &\rightarrow {}^{3}\mathrm{He} + \gamma \\ {}^{3}\mathrm{He} + {}^{3}\mathrm{He} &\rightarrow {}^{4}\mathrm{He} + {}^{1}\mathrm{H} + {}^{1}\mathrm{H} \end{aligned}$$

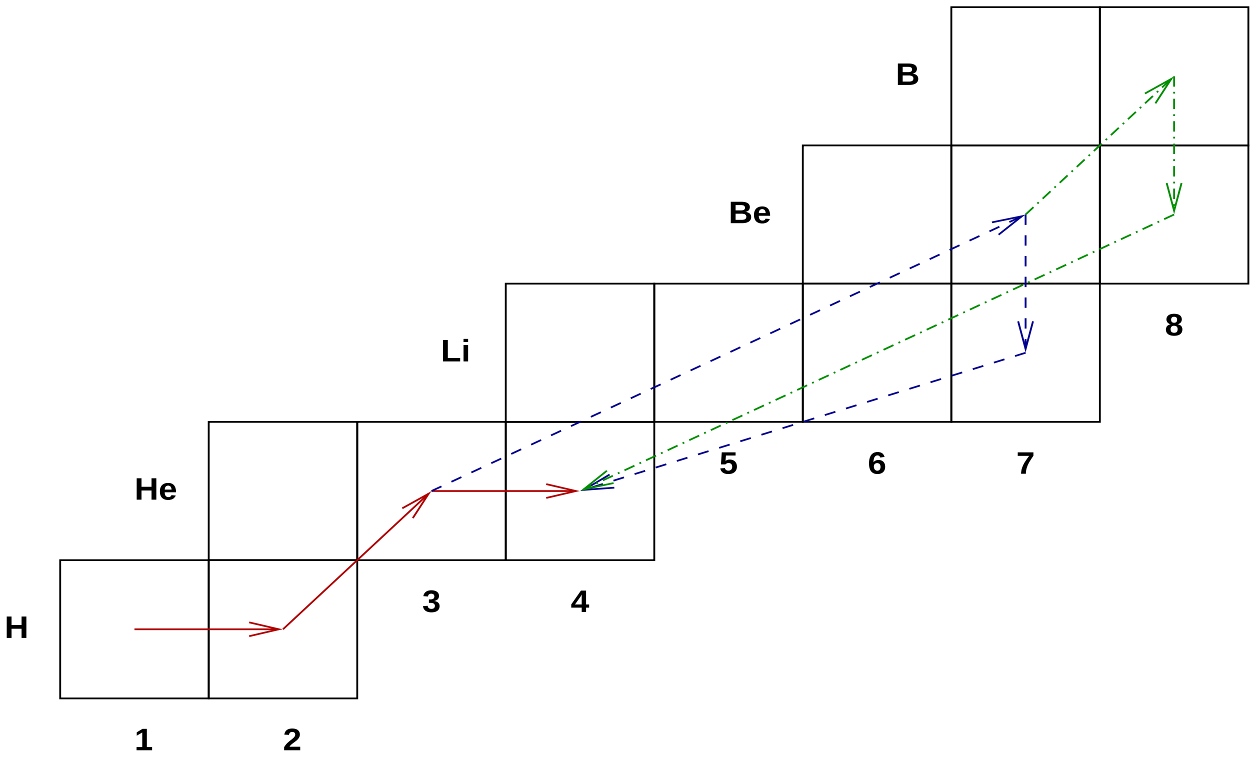
PP-II

$$\begin{aligned} {}^{3}\mathrm{He} + {}^{4}\mathrm{He} &\rightarrow {}^{7}\mathrm{Be} + \gamma \\ {}^{7}\mathrm{Be} + \mathrm{e}^{-} &\rightarrow {}^{7}\mathrm{Li} + \nu_{\mathrm{e}} + \gamma \\ {}^{7}\mathrm{Li} + {}^{1}\mathrm{H} &\rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He} \end{aligned}$$

PP-III

$$\begin{aligned} {}^{7}\mathrm{Be} + {}^{1}\mathrm{H} &\rightarrow {}^{8}\mathrm{B} + \gamma \\ {}^{8}\mathrm{B} &\rightarrow {}^{8}\mathrm{Be} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\ {}^{8}\mathrm{Be} &\rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He} \end{aligned}$$

Graphically,



PP-I is the obvious route, but it requires the somewhat unlikely collision of two relatively short-lived intermediate nuclei. PP-II and PP-III involve splitting a nucleus that is twice as massive as helium.

Relative importance of these chains depends upon the abundance of ^{3}He. Termination via PP-I requires two such nuclei to fuse, so the rate of that chain varies as the square of the ^{3}He number density. Find,

- As T increases, equilibrium abundance of ^{3}He decreases.
- With increasing temperature, importance of PP-I compared to PP-II and PP-III decreases.

Although the rate is determined by the slow initial reaction which is common to all three chains, the exact energy yield depends upon the relative importance of PP-I, PP-II and PP-III (because the fraction of the energy lost to ν_e differs). Numerically though, the leading term is,

$$\epsilon_{\rm eff}(\mathrm{pp-chains}) \approx 2.4 \times 10^4 \, \frac{\rho X^2}{T_9^{2/3}} e^{-3.380/T_9^{1/3}} \ \mathrm{erg\ g^{-1}\ s^{-1}}.$$

Dominant temperature dependence is the exponential arising from $\langle \sigma v \rangle_{pp}$.

The CNO cycle

The other main route to hydrogen burning is the CNO cycle. This involves carbon, nitrogen and oxygen nuclei in a 'catalytic' role. As with the pp reaction, there are several alternative routes.

All basically involve a series of proton captures interspersed with positron decays.

$$\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}$$

Alternative I:

$$
\begin{aligned}
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{16}\mathrm{O} + \gamma \\
{}^{16}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{17}\mathrm{F} + \gamma \\
{}^{17}\mathrm{F} &\rightarrow {}^{17}\mathrm{O} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
{}^{17}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}
\end{aligned}
$$

Alternative II:

$$
\begin{aligned}
{}^{17}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{18}\mathrm{F} + \gamma \\
{}^{18}\mathrm{F} + \mathrm{e}^{-} &\rightarrow {}^{18}\mathrm{O} + \nu_{\mathrm{e}} \\
{}^{18}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{19}\mathrm{F} + \gamma \\
{}^{19}\mathrm{F} + {}^{1}\mathrm{H} &\rightarrow {}^{16}\mathrm{O} + {}^{4}\mathrm{He}
\end{aligned}
$$

Description of the CNO cycle is complicated because the timescale to reach equilibrium abundances of the intermediate nuclei is long. Compared to pp, CNO is favoured,

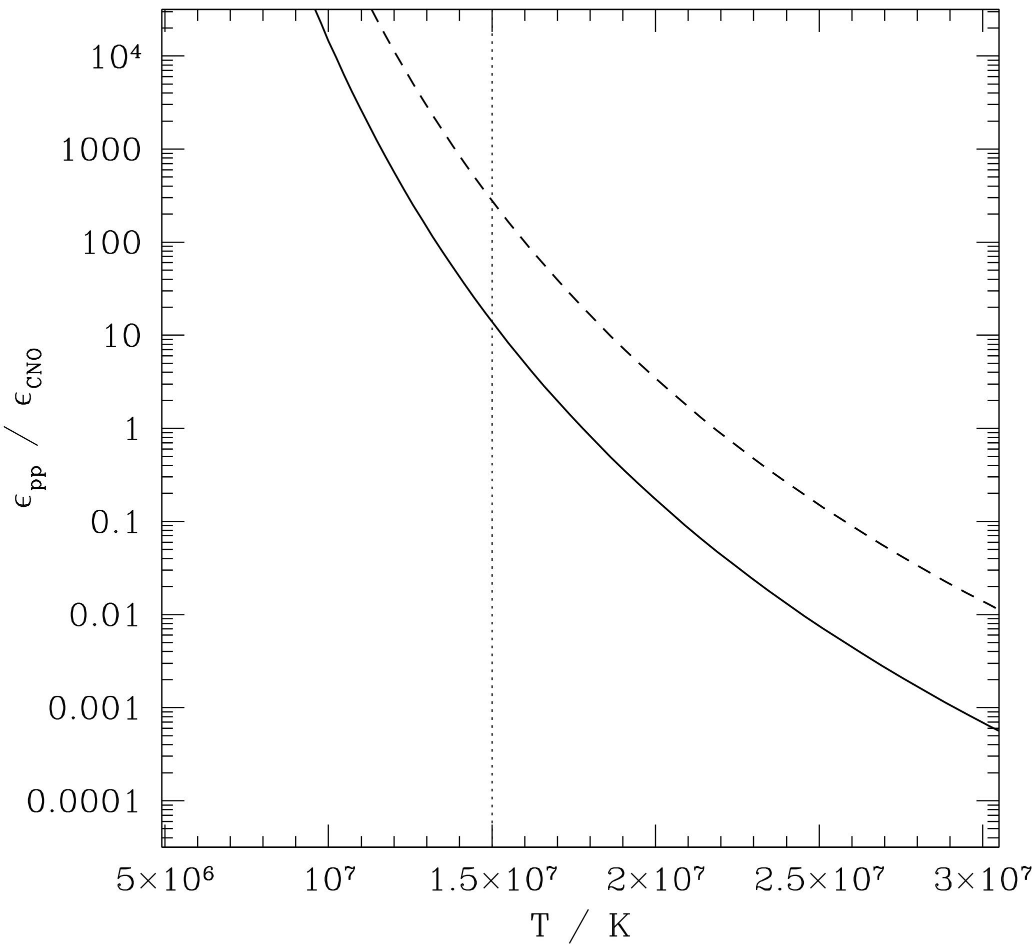
- At higher T, because heavier nuclei are involved.
- At higher metallicity Z.

An approximate rate of energy generation for the CNO cycle is,

$$\epsilon_{\mathrm{CNO}} \approx 4.4 \times 10^{25}\ \frac{\rho X Z}{T_9^{2/3}} e^{-15.228/T_9^{1/3}}\ \mathrm{erg\ g^{-1} s^{-1}}.$$

$\nu \approx 18$ for CNO at $T = 2 \times 10^7$ K.

Relative importance of pp vs CNO cycles:



Solid curve: $X = 0.7$, $Z = 0.02$. Dashed curve: $X = 0.7$, $Z = 0.001$.