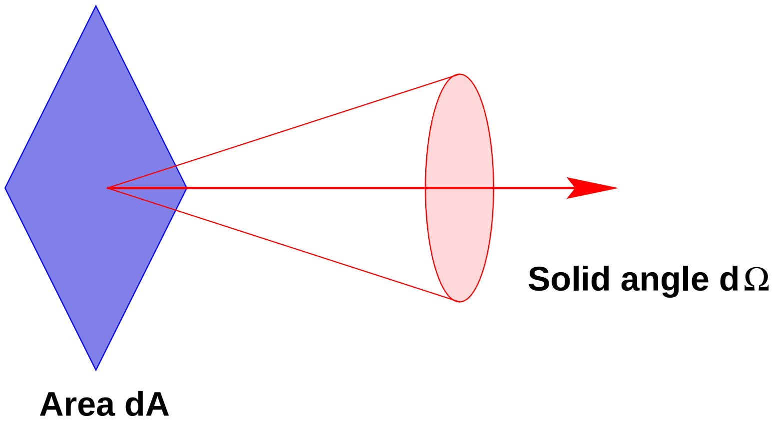
Review of radiative transfer

For scales $L \gg \lambda$, radiation travels in straight lines (called rays) in free space and in uniform media.

Construct an area dA normal to a ray, and consider all rays passing through dA whose directions lie within a small solid angle $d\Omega$.



The energy passing through dA in time dt within frequency range $d\nu$ is:

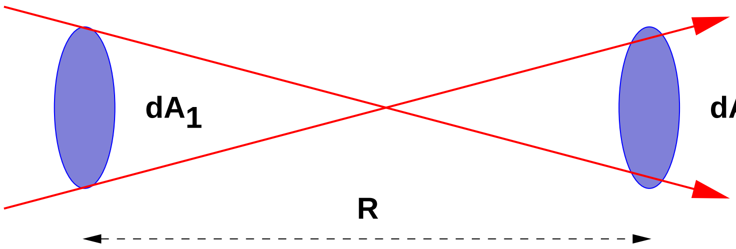
$$dE = I_\nu \; dA \; d\Omega \; d\nu \; dt$$

which defines the **specific intensity** or **brightness** I_ν.

I_ν has units erg s^{-1} cm^{-2} steradian^{-1} Hz^{-1}. It depends upon location, direction, and frequency.

The amount of energy passing through the surface is given by $F \; dA \; dt$ where F is the energy flux with units erg s^{-1}cm^{-2}.

Specific intensity is constant along a ray in free space



Consider areas dA_1 and dA_2 normal to a ray. Energy dE is carried through *both* areas by those rays that pass through them both.

$$dE = I_{\nu_1}\, dA_1\, d\Omega_1\, d\nu_1\, dt = I_{\nu_2}\, dA_2\, d\Omega_2\, d\nu_2\, dt$$

where $d\Omega_1$ is the solid angle subtended by dA_2 at dA_1 etc. Using $d\Omega_1 = dA_2/R^2$, $d\Omega_2 = dA_1/R^2$, and $d\nu_1 = d\nu_2$, we have,

$$I_{\nu_1} = I_{\nu_2}$$

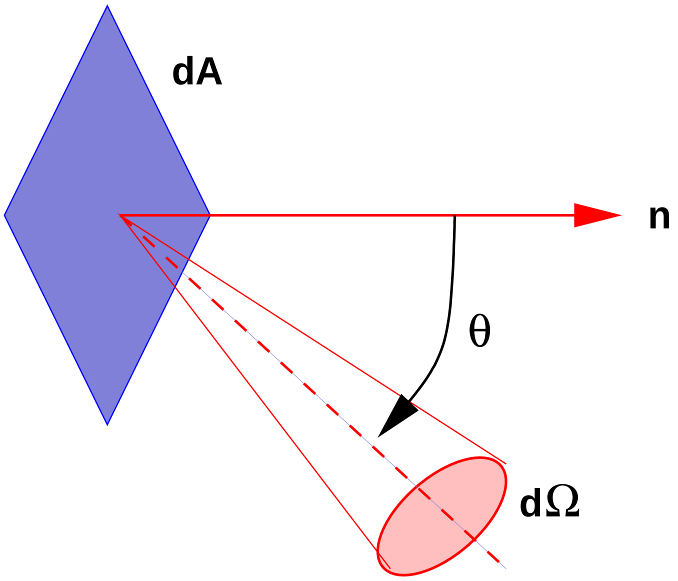
ie specific intensity is constant along a ray in free space. If we denote distance along a ray by s,

$$\frac{dI_\nu}{ds} = 0$$

where ds is a differential element of length along the ray.

Net flux and momentum flux

For an element at an arbitrary angle, the amount of flux is reduced because the effective area is smaller:



$$
\begin{aligned}
dF_\nu &= I_\nu \cos\theta d\Omega \\
F_\nu(\mathbf{n}) &= \int I_\nu \cos\theta d\Omega
\end{aligned}
$$

$F_\nu(\mathbf{n})$ is the **net flux** in the direction of $\mathbf{n}$. For an isotropic radiation field $F_\nu(\mathbf{n}) = \mathbf{0}$.

The momentum of a photon is E/c. The momentum flux in the direction of $\mathbf{n}$ is then:

$$p_\nu(\mathbf{n}) = \frac{1}{c}\int I_\nu \cos^2\theta d\Omega$$

where one factor of $\cos\theta$ comes from the number of photons, the other as we add up only the normal component of the momentum. F_ν and p_ν are described as *moments* of the intensity.

We can also define the **mean intensity**, J_ν:

$$J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega$$

...the average of the specific intensity over all angles.

Emission

Define the spontaneous **emission coefficient** j. This is the energy emitted per unit time per unit solid angle and per unit volume,

$$dE = j\ dV\ d\Omega\ dt.$$

Likewise define a monochromatic version j_ν.

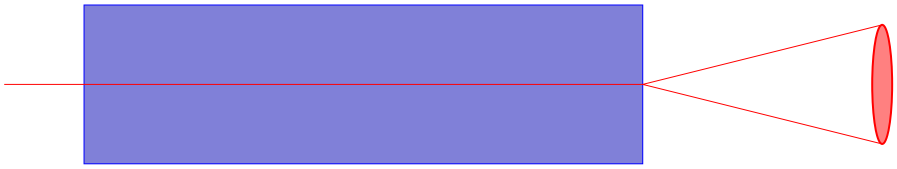
In travelling ds, a beam of radiation sweeps out a volume $dV = dAds$. The change in the intensity due to spontaneous emission is,

$$dI_\nu = j_\nu ds$$

Note: we restrict ourselves to *spontaneous* emission. Stimulated emission depends upon I_ν and is more conveniently treated as 'negative absorption'.

Absorption

Consider a beam passing through an absorbing medium,



Define the **absorption coefficient**, α_ν, by

$$dI_\nu = -\alpha_\nu I_\nu ds$$

ie the fractional loss in intensity in travelling a distance ds is $\alpha_\nu ds$ (convention: positive α_ν means energy *loss*).

Suppose the absorption is due to particles with number per unit volume n. Each presents an effective absorbing area (or **cross section**) to the radiation σ_ν (units, cm^2). Then,

$$\alpha_\nu = n\sigma_\nu$$

Finally, define the **mass absorption coefficient** (or, **opacity** coefficient) κ_ν,

$$\alpha_\nu = \rho\kappa_\nu$$

κ_ν has units cm^{2}g^{-1}.

Combining the effects of absorption and emission,

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

An ODE along a straight line.

If we define the optical depth τ via,

$$d\tau_\nu = \alpha_\nu ds$$

and divide through by the absorption coefficent, we can write the equation of radiative transfer in the simple form,

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

where we have defined the **source function** S_ν, as the ratio between the emission and absorption coefficients,

$$S_\nu \equiv \frac{j_\nu}{\alpha_\nu}.$$

In stellar interiors, need to consider consequences of S_ν being very close to, but not exactly equal to, the Planck function.