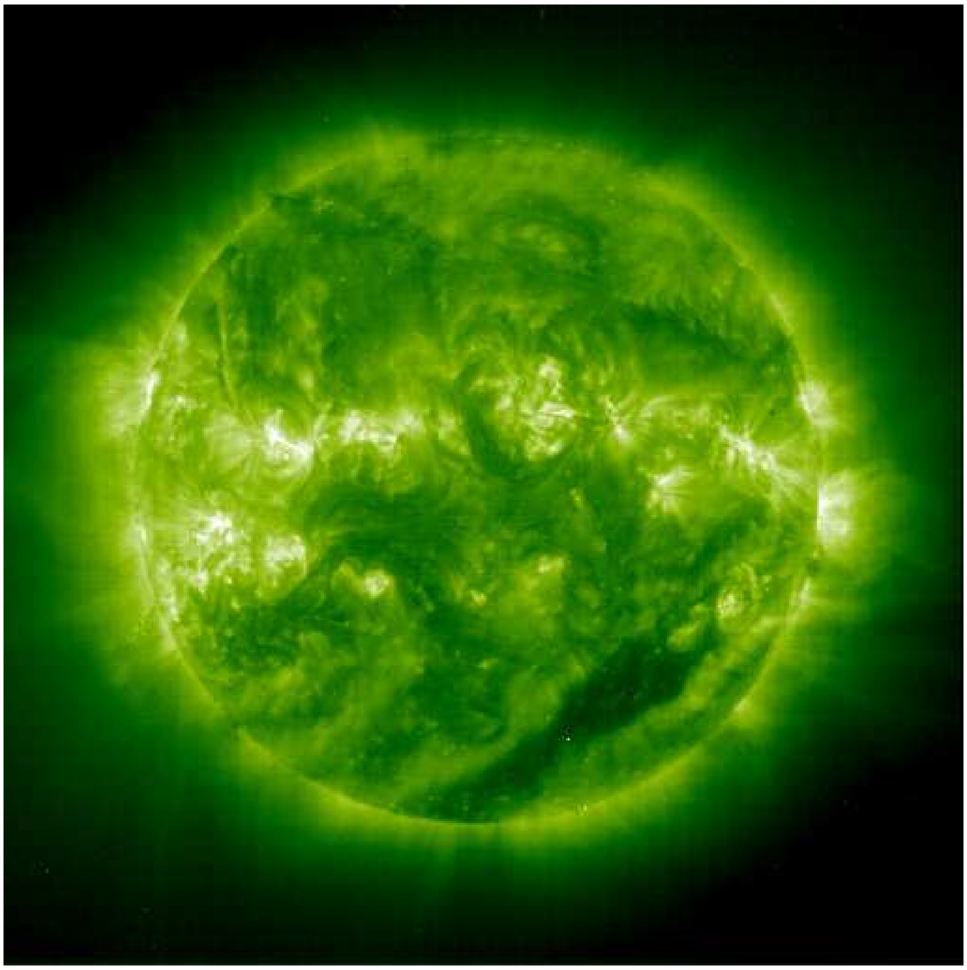
ASTR 5700: STELLAR STRUCTURE AND EVOLUTION

`http://jilawww.colorado.edu/~pja/stars02/`



The Sun in the extreme UV imaged by SOHO

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Aims of the course

(1) Introduce the equations needed to model the internal structure of stars.

(2) Study the microphysics relevant for stars – the equation of state, the opacity, relevant nuclear reactions (overlap with IP1).

(3) Examine the properties of simple models for stars, and consider how real models are computed.

(4) Survey (mostly qualitatively) how stars evolve, and the end-points of stellar evolution (white dwarfs, neutron stars).

(5) Discuss a handful of ongoing research areas in stellar physics.

Books

Stellar Interiors: Physical Principles, Structure and Evolution
C.J. Hansen & S.D. Kawaler

Lots of other good options:

Principles of Stellar Evolution and Nucleosynthesis
D.D. Clayton

Stellar Structure and Evolution
R. Kippenhahn & A. Weigert

Should be on 1 day reserve in Math / Physics library

Assessment

Problem sets on an approximately weekly basis (one or two of which will be primarily computational, the rest analytic) [60 %, lowest grade will be ignored]

3 discussion sessions in class [15 % – 'tickable exercise']

Short written review of a topical subject / paper relevant (broad interpretation of relevant OK) to stellar astrophysics [25 % – due at end of semester]

Check `http://arXiv.org/archive/astro-ph` for ideas

No final exam

Predicted vs measured sound speed in the Sun

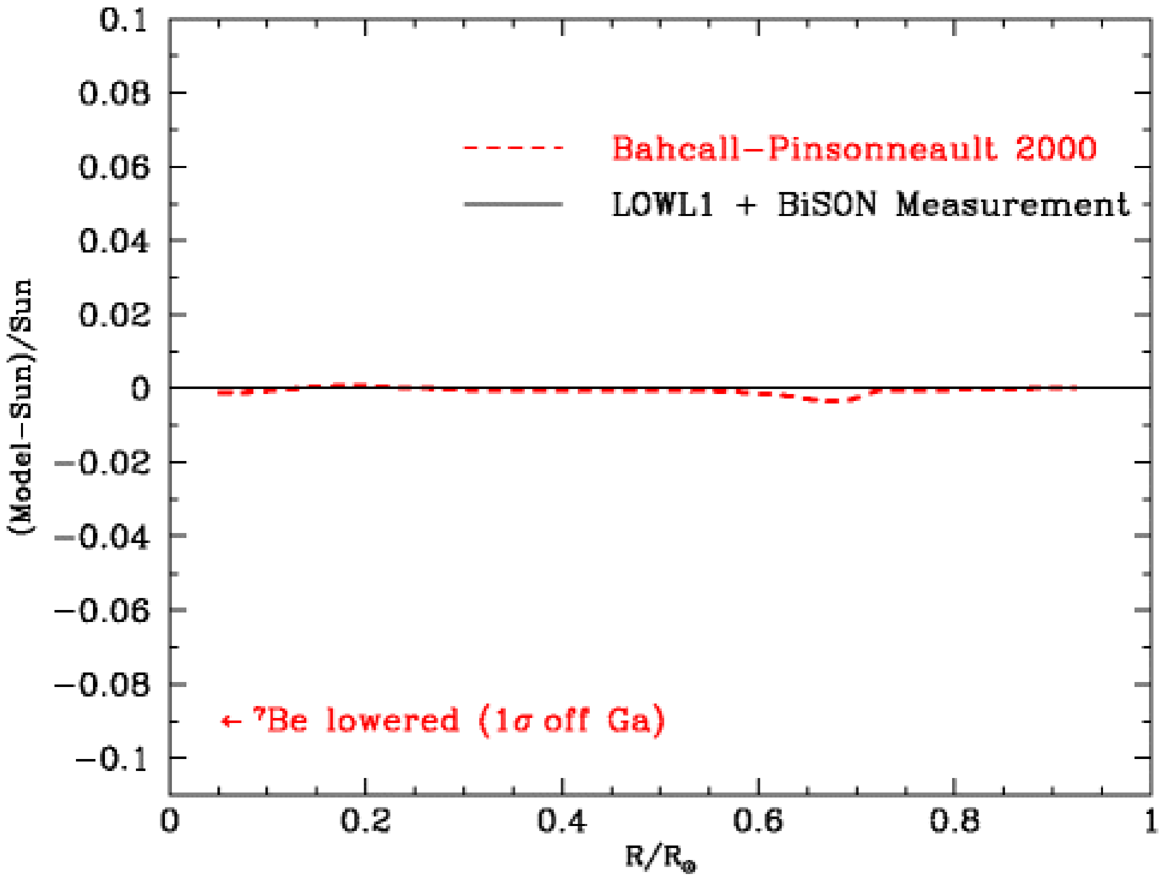

Figure from `http://www.sns.ias.edu/~jnb/`

Differences at the 0.1% level!

Q: Why is the largest deviation at $\approx 0.7\ R_\odot$*?*

The mathematical problem of stellar structure

We want to determine the structure of an isolated mass M of gas with a given composition. Simplest assumptions:

(1) No rotation $\rightarrow$ spherical symmetry

Sun: rotation period $\sim$ *month, c.f. orbital period at surface of a few hours. Good approximation, though even slow rotation in principle has qualitative influence on solutions.*

(2) No magnetic fields

Sun: equipartition field of order 10^2 MG, *c.f. surface fields in Sunspots of order a kG. Magnetic fields thought to be energy reservoir in some neutron stars (magnetars), but even there not important for structure.*

(3) Static

Sun: convection but no large scale variability, Solar wind small. Not valid for pulsating stars.

(4) Newtonian gravity

Sun: $v_{\mathrm{escape}} \approx 600 \ \mathrm{kms}^{-1} \ll c$. *OK except for neutron stars.*

What equations can we use to describe the structure?

Equations of stellar structure

(We will derive these properly later on)

(1) Mass

Treat radius r as independent variable (spherical symmetry).

Let $m(r)$ be mass contained within radius r. Density ρ. Consider shell between r, $r + dr$,

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho$$

(2) Equation of motion

Pressure $P(r)$. In hydrostatic equilibrium, pressure gradient must balance gravity,

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho \frac{Gm}{r^2},$$

with G the Gravitational constant.

This involves the *equation of state*, e.g. for an ideal gas,

$$P = \frac{R}{\mu} \rho T,$$

where R is the gas constant, $T(r)$ is temperature, μ mean molecular weight. More generally, $P = P(\rho, T, \text{composition...})$, $\mu = \mu(\rho, T, \text{composition...})$.

$\rightarrow$ need thermodynamics to compute this.

(3) Energy generation

Let L_r be energy flux through sphere of radius r. Then,

$$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2 \rho \epsilon,$$

where ϵ is the energy generation rate per unit mass.

$\rightarrow$ need $\epsilon(\rho, T, ...)$ from nuclear physics.

(4) Energy Flow

May be via radiation, convection, conduction. For radiation,

$$\frac{\text{Flux}}{\text{Area}} = -\text{conductivity} \times \nabla T$$

$$\frac{L_r}{4\pi r^2} = -\frac{4ac}{3\kappa\rho} T^3 \frac{\mathrm{d}T}{\mathrm{d}r},$$

where a is the radiation density constant, c is the speed of light, and κ the opacity.

$\rightarrow$ need $\kappa(\rho, T, ...)$ from quantum mechanics.

With these approximations, structure is determined by four first-order ODEs + appropriate boundary conditions.

Boundary conditions

At $r = 0$:

$$\begin{aligned} L_r &= 0 \\ m &= 0 \end{aligned}$$

At $r = R$, the 'stellar radius':

$$\begin{aligned} m &= M \\ L_r &= L \\ P &= P_{\text{surface}} \simeq 0 \end{aligned}$$

There appear to be 5 boundary conditions, but R is a priori unknown – i.e. in this formulation R is an eigenvalue which we need to solve for.

Usually, we will write equations with m as the independent variable, e.g.:

$$\frac{\mathrm{d}r}{\mathrm{d}m} = \frac{1}{4\pi r^2 \rho}$$

Then for a star of given mass the location where the boundary conditions are to be applied is fixed.

At $m = 0$, $L_r = 0$, $r = 0$.

At $m = M$, $L_r = L$, $P = P_{\text{surface}} \simeq 0$.

Note: we are glossing over some subtleties in the exterior boundary conditions here...

Solutions

In principle, can try and solve these equations for a mass M of gas of any specified composition. Note however that:

(1) A solution may not exist. For a trivial physical example, we can't make a star out of $10M_\odot$ of iron – with no energy producing nuclear reactions it will promptly collapse to a black hole.

(2) There's no mathematical guarantee that the solution is unique. Physically we don't have to worry about this too much for main sequence stars, though (for example) it's been suggested that very slow accretion could allow brown dwarfs with masses above the usual hydrogen burning limit (Salpeter, 1992, **393**, 258).

Caveat: For *young* stars (a few Myr old or less at a Solar mass) there's no doubt that the accretion history matters for the structure.