ASTR 5820: Problem Set #3

(due Tuesday October 7th)

1. In our derivation of the vertical density profile of protoplanetary disks, we assumed that the only contribution to the gravitational potential was that of the central star. This question explores when this assumption is violated.

    (a) Consider a thin, infinite sheet of matter with uniform surface density Σ, which occupies the z = 0 plane. Calculate the gravitational acceleration g_z^{disk} at a height z above this plane (this is easy if you apply Gauss' theorem to the Poisson equation, but you can do it however you want).

    (b) Now consider a disk with local surface density Σ at radius R from a star of mass M_*. By equating the vertical gravitational acceleration due to the star with that from the disk itself at z = h (where h is the scale height that the disk would have if it were of negligible mass), find the maximum ratio of disk mass to stellar mass M_{disk} / M_* for which the influence of the disk self-gravity is negligible. Your answer should be a function of the local aspect ratio of the disk (h / R) only.

2. Ionization due to the decay of radioactive elements provides a source of non-thermal ionization within the disk. Consider a species whose decay, with rate constant λ, yields an energy E. The fractional abundance of the species relative to hydrogen is f. If it takes, on average, an amount of energy $\Delta\varepsilon$ to generate an ion pair in hydrogen, write down an expression for the ionization rate (units s^{-1}) ζ due to this process.

    Now assume that ionization due to radioactive decay is balanced against dielectronic recombination:

    $$e^- + H_2^+ \rightarrow H + H$$

    with a rate coefficient:

    $$\beta = 3 \times 10^{-6} T^{-1/2} \text{ cm}^3 \text{ s}^{-1}$$

    Find an expression for the equilibrium electron abundance x in gas with a neutral density n.

    Evaluate x for gas at $n = 10^{15}$ cm^{-3} assuming that radioactive decay of ^{26}Al dominates the ionization (E = 3 MeV, $\lambda = 3 \times 10^{-14}$ s^{-1}, $f = 10^{-10}$, $\Delta\varepsilon = 36$ eV).