ASTR 5820: Project 1

(Tuesday Nov 4th)

The surface density of an actively accreting protoplanetary disk evolves in time according to the diffusive equation,

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(\nu \Sigma R^{1/2}\right)\right]$$

where ν is the viscosity. As we noted previously, there are only very limited circumstances in which this equation has analytic solutions. The aim here is to solve the equation numerically and study how the disk evolution differs if the gas orbits a binary star (or a star plus a massive planet).

1. Write a program to solve the evolution equation numerically on a spatial grid extending between $R = R_{in}$ and $R = R_{out}$, with boundary conditions $\Sigma = 0$ at both boundaries. A simple scheme is to first transform the equation into a simpler form following the approach of Pringle, Verbunt & Wade (1986), and then use the explicit scheme suggested for diffusive initial value problems in Numerical Recipes. Take care to use a stable time step! Verify that your code works properly by comparing against an analytic solution (either the Green's function solution or the self-similar solution, though in the latter case be warned that you will need a large R_{out}).

2. Set up as an initial condition ($t = 0$) a protoplanetary disk model,

$$\Sigma = 4000\left(\frac{r}{1\ \mathrm{AU}}\right)^{-1}\ \mathrm{g cm^{-2}}$$

between 0.1 au and 30 au. I suggest using a spatial grid that extends between 0.1 au and 100 au – 100 grid points should be fine if you use the non-uniform grid suggested in Pringle et al. Set the density in the region 30 au < R < 100 au to some small value. Assume that the above surface density profile is (close to) a steady state at small radius, ie that,

$$\nu \propto R$$

and set the constant of proportionality such that the initial accretion rate decays by an order of magnitude in 2 Myr.

Determine this constant and plot how the surface density profile evolves with time.

3. Now consider a disk surrounding a binary star. To a first approximation, you may consider that the effect of the binary is to prevent gas accreting interior to some radius. Accordingly, you can represent the effect of a binary by modifying your code so that the inner boundary condition is $v_r = 0$, rather than $\Sigma = 0$.

Calculate the evolution of a disk with constant viscosity around a binary star, starting from an initial condition of a narrow ring of gas at $R = 2\ R_{in}$. Plot how the surface density at the innermost grid point varies with time.

Interpret physically what is going on in the evolution of a disk around a binary.