Problem Set 1 (due 17th September)

(1) A planet of radius $R_p$ orbits a star of radius $R_\ast$ in a circular orbit at radius $a$. If the orbit plane is randomly distributed relative to the observer’s line of sight, show that the probability that transits will be observed is,

$$p_{\text{trans}} = \frac{R_\ast + R_p}{a}.$$

If the orbital period of the planet is $P$, show that for a system viewed at an inclination angle $i$ the duration of the transit is,

$$t_{\text{trans}} \simeq \frac{P}{\pi} \sqrt{\left(\frac{R_\ast}{a}\right)^2 - \cos^2 i}.$$

[Note that $i = 90^\circ$ corresponds to a perfectly edge-on system.]

(2) Suppose that the planet, of mass $M_p$, orbital radius $a_p$, and orbital period $P$, is itself orbited by a moon. The moon has mass $M_s$ and orbital radius $a_s$. Find an expression for the magnitude of the variations in the time of transit, $\tau$, that result.

How precisely would transit times have to be measured to detect the presence of a moon similar to Jupiter’s Ganymede orbiting a Jupiter mass planet at 0.1 AU about a Solar-mass star?

(3) Consider an Earth-like planet orbiting a Solar-type star at 1 AU. Suppose that the star is a 12th magnitude object in the visible, and that the only noise source that affects the measurement of the light curve is shot noise (i.e. statistical fluctuations in the number of photons received by a detector due to counting statistics). For a 100% efficient telescope and detector system, estimate the minimum size of telescope required to detect a single transit at $5\sigma$ statistical significance.

[Possibly useful astronomy for this question includes: the relation between magnitudes and fluxes is $m = -2.5 \log_{10} F + \text{constant}$, the Sun has an apparent magnitude of $m = -26.8$ and a luminosity of $3.83 \times 10^{33}$ erg s$^{-1}$, “visible” light has a wavelength of around 550 nm.]