ASTR 3830: Problem Set 2
Due in class Tuesday February 20th

1. The area inside the Einstein ring is given by $A = \pi d_L^2 \theta_E^2$, where d_L is the distance to the lens and θ_E is the angular size of the Einstein ring. Assume that the distance to the **sources**, d_S, is fixed, and write $d_L = x d_S$:

Sketch how A varies with x for $0 < x < 1$ (don't forget that θ_E depends on both d_L and d_{LS}).

By differentiating and setting the resultant expression to zero, find the value of x that maximizes the physical area inside the Einstein ring.

(This proves an important result for the study of gravitational lensing, which we'll return to later in the semester.)

2. Assume that early in the history of the Milky Way, a short burst of star formation formed a population of stars in the halo with a Salpeter mass function,

$$\xi(M) = \xi_0 M^{-2.35},$$

with a lower mass cutoff at 1 Solar mass. By now, all these stars have died. If every star with a mass less than 8 Solar masses leaves a 0.6 Solar mass white dwarf, compute the **fraction** of the total mass that was originally in stars that is now locked up in white dwarfs.

3. The mass density in the form of stars near the Galactic Center follows a power-law in radius r:
$$\rho = \rho_0 r^{-7/4}$$
where ρ_0 is a constant.

(a) If the total mass of stars within 2 pc of the center is 4.5×10^6 Solar masses, calculate the constant ρ_0 [this requires integrating the above density over spherical shells between $r = 0$ and $r = 2$ pc. For the latter parts of the question, it will help to work this out in c.g.s. units.]

(b) Close enough to the central black hole, the velocity of stars will be dominated by the gravity of the hole. If the hole has mass M_{BH}, write down an expression for the Keplerian orbital velocity as a function of radius.

(c) Consider a massive star of mass M_* orbiting at radius r from the black hole. The star suffers dynamical friction due to its motion through the background of much lower mass stars. Show that the dynamical friction time:

$$t_{friction} = \frac{V}{|dV/dt|}$$

scales as $r^{1/4}$ if the star is close enough to the black hole that the velocity is described by the formula of part (b). Calculate $t_{friction}$ for a star of mass 10 Solar masses at $r = 1$ pc. Assume that $\ln \Lambda = 10$, and take the mass of the central black hole to be 4.5×10^6 Solar masses [hint: remember that the product *nm* in the dynamical friction formula is just the mass density r, which you can calculate using the result from the first part. If you get a crazy answer – seconds or 10^{100} years – it's probably a sign that your units aren't consistent.]