ASTR 3830: Problem Set 2

(due in class Friday February 9th)

1. The area inside the Einstein ring is given by $A = \pi d_L^2 \theta_E^2$, where d_L is the distance to the lens and θ_E is the angular size of the Einstein ring. Assume that the distance to the **sources**, d_S, is fixed, and write $d_L = x d_S$:

Sketch how A varies with x for $0 < x < 1$ (don't forget that θ_E depends on both d_L and d_{LS}).

By differentiating and setting the resultant expression to zero, find the value of x that maximizes the physical area inside the Einstein ring.

(this proves one of the most important results for the study of gravitational lensing, which we'll return to later in the semester)

2. Assume that early in the history of the Milky Way, a short burst of star formation formed a population of stars in the halo with a Salpeter mass function,

$$\xi(M) = \xi_0 M^{-2.35},$$

with a lower mass cutoff at 1 Solar mass. By now, all these stars have died. If every star with a mass less than 8 Solar masses leaves a 0.6 Solar mass white dwarf, compute the **fraction** of the total mass that was originally in stars that is now locked up in white dwarfs.

(hint: to do this, first find the total mass in stars by integrating the IMF. This will depend on ξ_0. Then find the mass that ends up in white dwarfs, remembering that since each low mass star yields the same mass white dwarf, it's the number of such stars rather than their integrated mass that must be computed.)

3. Suppose that the rate of star formation in the Galaxy is decreasing with time according to the expression $S(t) = k \exp[-t/t_*]$, where k and t_* are constants and S(t) is the star formation rate in units of Solar masses per year.

If the age of the Galaxy is 10 Gyr, find the fraction of all the 3 Solar mass stars ever made that are still on the main sequence. Assume that $t_* = 3$ Gyr, and that the main sequence lifetime of a 3 Solar mass star is 350 Myr.