Cosmic Microwave Background

Following recombination, photons that were coupled to the matter have had very little subsequent interaction with matter. Now observed as the cosmic microwave background.

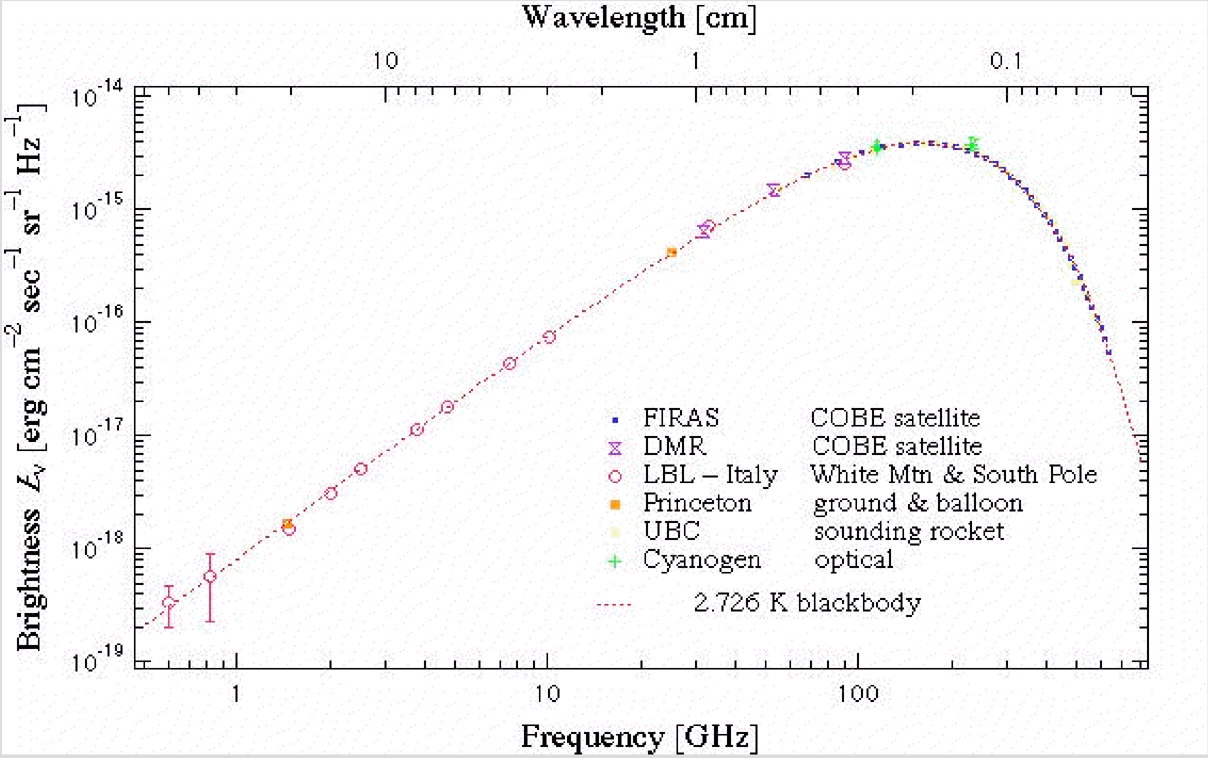
Arguably the most important cosmological probe, because it originates at a time when the Universe was very nearly uniform:

- Fluctuations were **small** - easy to calculate accurately (linear rather than non-linear)
- Numerous complications associated with galaxy and star formation (cooling, magnetic fields, feedback) that influence other observables not yet important.

Basic properties: isotropy, thermal spectrum
Anisotropies: pattern of fluctuations

Basic properties of the CMB

Excellent first approximation: CMB has a **thermal spectrum** with a uniform temperature of T = 2.7 K in all directions.



Thermal spectrum - support for the hot big bang model

Isotropy - evidence that the Universe is homogenous on the largest observable scales

Can show that thermal radiation, filling the Universe, maintains a thermal spectrum as the Universe expands.

Suppose that at recombination the radiation has a thermal spectrum with a temperature T ~ 3000 K. Spectrum is given by the Planck function:

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

At time t, number of photons in volume V(t) with frequencies between ν and $\nu + d\nu$ is:

$$dN(t) = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}V(t)d\nu$$

(since each photon has energy $h\nu$ and the energy density of thermal radiation is $\frac{4\pi}{c}\int B_\nu d\nu$)

Now consider some later time t' > t. If there have been no interactions, the number of photons in the volume remains the same:

$$dN(t) = dN(t')$$

However, the volume has increased with the expansion of the Universe and each photon has been redshifted:

$$V(t') = V(t)\,\frac{a^3(t')}{a^3(t)}$$

$$\nu' = \nu\,\frac{a(t)}{a(t')}$$

$$d\nu' = d\nu\,\frac{a(t)}{a(t')}$$

Substitute for V(t), ν and $d\nu$ in formula for dN(t), and use fact that dN(t') = dN(t)

Obtain:

$$dN(t') = \frac{8\pi\nu'^2}{c^3}\frac{1}{e^{h\nu'/[kT\,a(t')/a(t)]}-1}V(t')\,d\nu'$$

which is a thermal spectrum with a new temperature:

$$T' = T\frac{a(t)}{a(t')}$$

Conclude: radiation preserves its blackbody spectrum as the Universe expands, but the temperature of the blackbody decreases:

$$T \propto a^{-1} \propto (1+z)$$

Recombination happened when T ~ 3000 K, at a redshift z = 1090.

CMB anisotropies

Universe at the time of recombination was not completely uniform - small over (under)-densities were present which eventually grew to form clusters (voids) etc.

In the microwave background sky, fluctuations appear as:

- A **dipole** pattern, with amplitude:

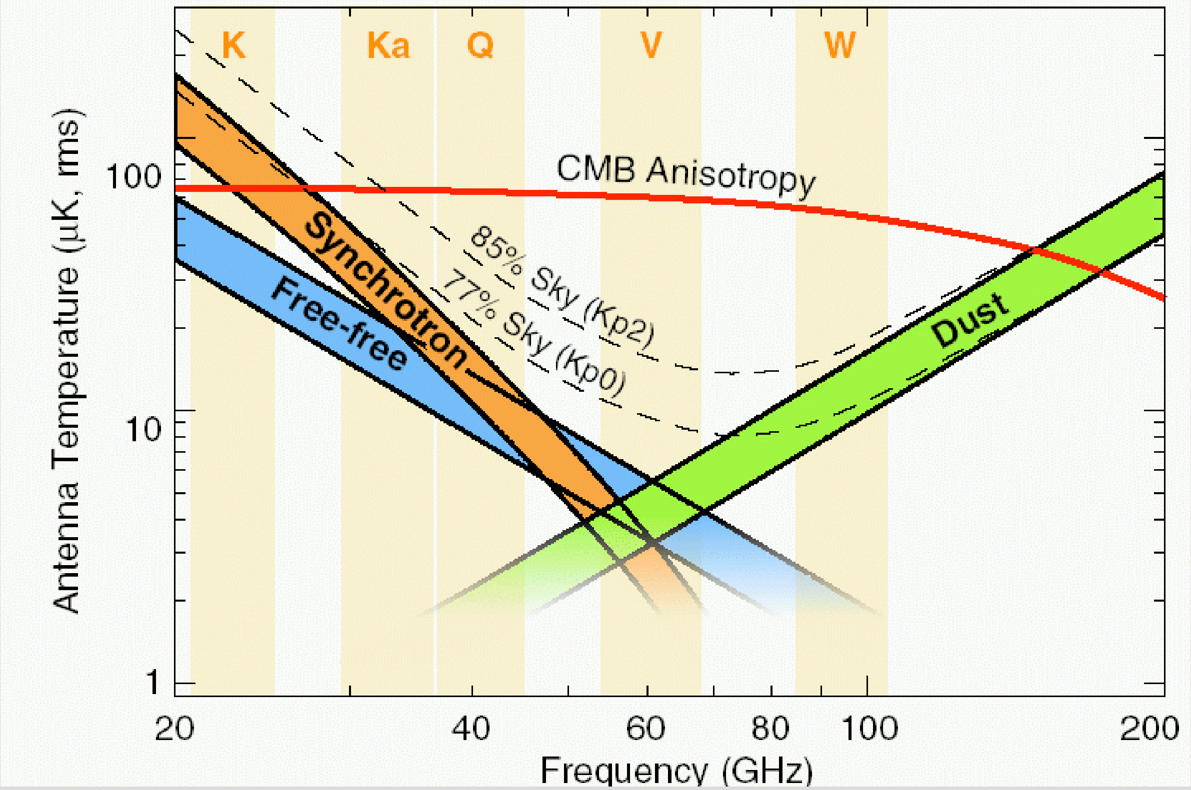
$$\frac{\Delta T}{T} \sim 10^{-3}$$

Origin: Milky Way's velocity relative the CMB frame. Reflects the presence of **local** mass concentrations - clusters, superclusters etc.

- Smaller angular scale anisotropies, with $\Delta T / T \sim 10^{-5}$.
  - originate (mostly) from the epoch of recombination
  - greatest cosmological interest

Experiments detect any cosmic source of microwave radiation - not just cosmic microwave background:

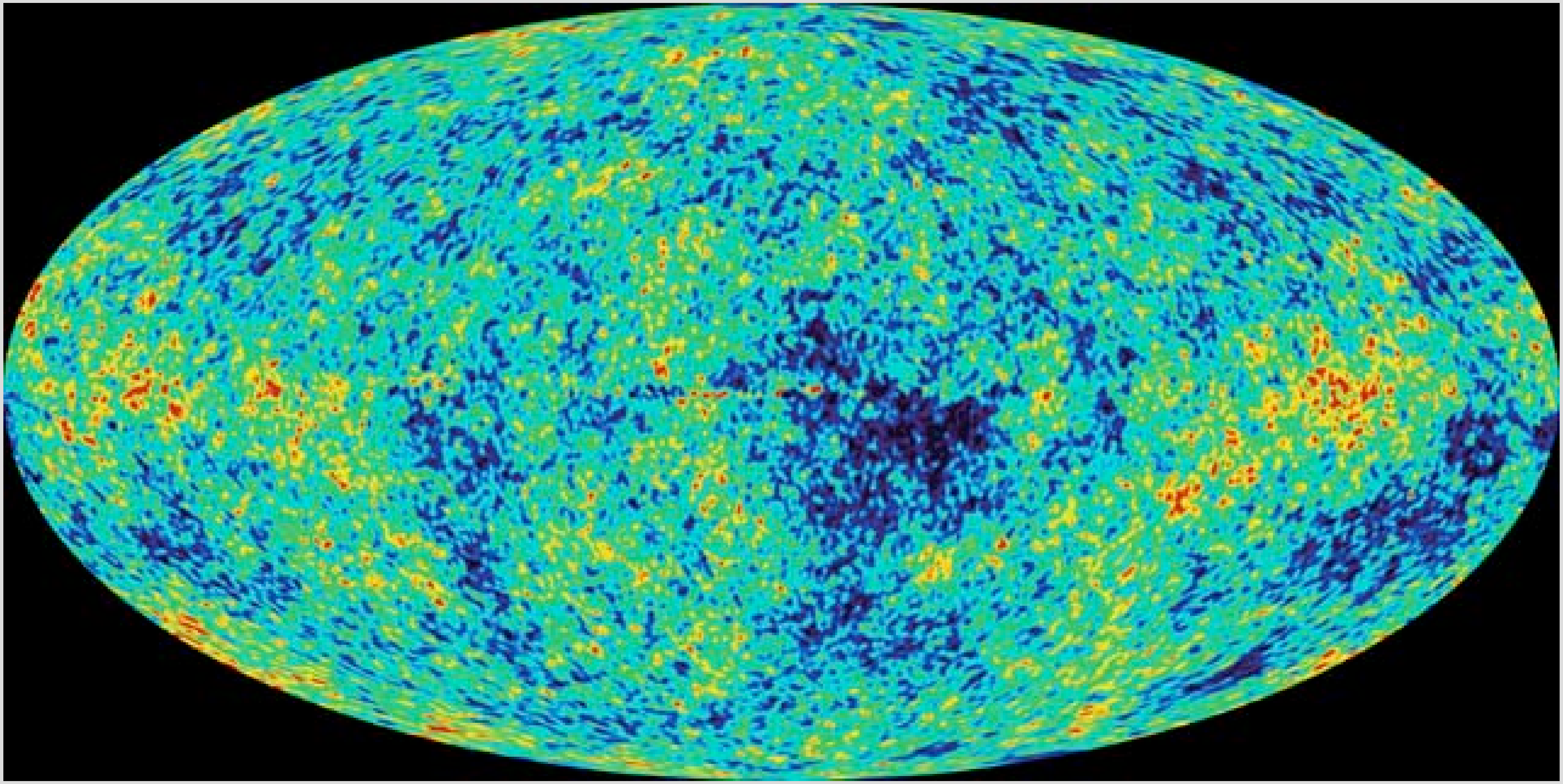
- Low frequencies - free-free / synchrotron emission
- High frequencies - dust



CMB dominates at around 60 GHz

Also different spectra - can be separated given measurements at several different frequencies

Full sky map from WMAP



- Dipole subtracted (recall dipole is much larger than the smaller scale features)
- Galactic foreground emission subtracted as far as possible

Characterizing the microwave background sky

First approximation - actual positions of hot and cold spots in the CMB is `random' - does not contain useful information.

Cosmological information is encoded in the **statistical** properties of the maps:

- What is the characteristic size of hot / cold spots?
  ~ one degree angular scale
- How much anisotropy is there on different spatial scales?
  CMB is a map of temperature fluctuations on a sphere - conventionally described in terms of **spherical harmonics** (Earth's gravitational field is similarly described).

Spherical harmonics

Any quantity which varies with position on the surface on a sphere can be written as the sum of spherical harmonics:

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$

measured anisotropy map as function of spherical polar angles θ and φ

weight - how much of the signal is accounted for by this particular mode

spherical harmonic function

The spherical harmonic functions themselves are just (increasingly complicated) trignometric functions, e.g.:

$$Y_{22}(\theta,\phi) = \sqrt{\frac{5}{96\pi}}\, 3\sin^2\theta\, e^{2i\phi}$$

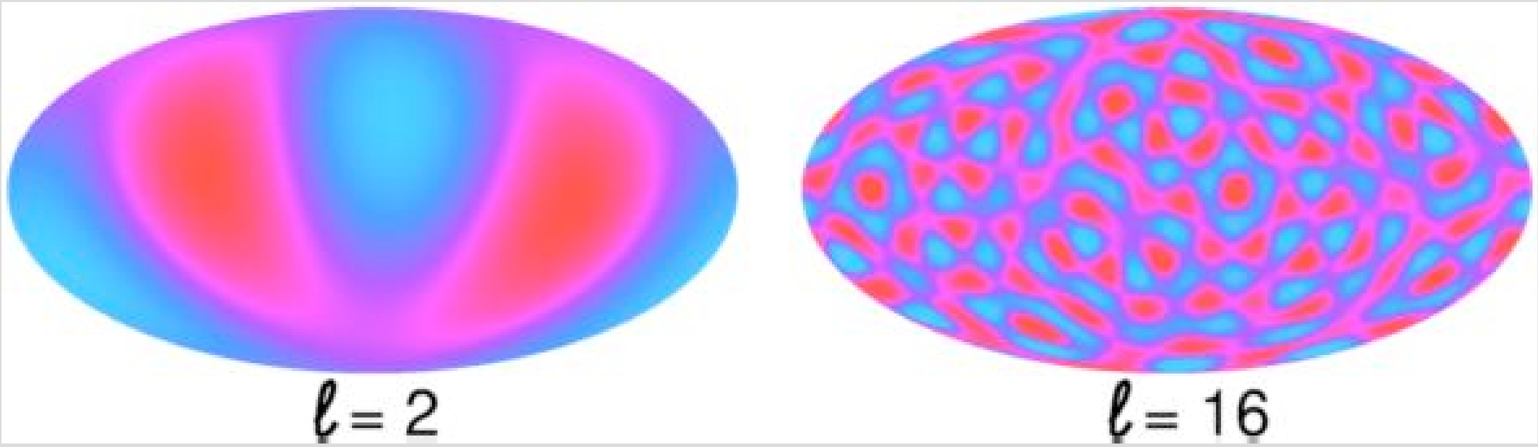
Having decomposed the observed map into spherical harmonics, result is a large set of coefficients a_{lm}. Next compute the average magnitude of these coefficients as a function of l:

$$C_l \equiv \left\langle \left| a_{lm} \right|^2 \right\rangle$$

Plot of C_l as a function of l is described as the angular power spectrum of the microwave background. Each C_l measures how much anisotropy there is on a particular angular scale, given by:

$$\theta \sim \frac{180^\circ}{l}$$

Examples:



Map with C_l nonzero only for $l = 2$

Map with C_l nonzero only for $l = 16$

In currently popular cosmological models based on the theory of inflation, the primary CMB anisotropies are truly random - in the sense that a plot of C_l vs l describes **completely** the cosmological information contained in the original map.

➡ Astonishing result - means that angular power spectrum is basic measurement to compare with theory