Radiation dominated phase

Recall that solution for matter dominated universe relied on the matter density scaling as $\rho_m \sim a^{-3}$.

For radiation, energy of each photon is redshifted as the universe expands. Energy density in radiation therefore scales as $\rho_r \sim a^{-4}$.

At sufficiently early epochs, radiation rather than matter dominates the evolution of the scale factor.

At very high temperatures, other relativistic particles will also be present - contribute to the evolution in same way as photons.
If radiation dominates, easy to see that Friedmann equation takes the form:

\[ \dot{a}^2 = \frac{B^2}{a^2} \quad \text{...where B is a constant} \]

Solving this equation:

\[ ada = Bdt \]

\[ a = \sqrt{2B} \, t^{1/2} \quad \text{...compared to the } t^{2/3} \text{ expansion predicted for the matter dominated case.} \]
Ignoring (for now) the very early Universe, two critical events during the radiation dominated phase:

- **Nucleosynthesis** - formation of light elements from initial mix of neutrons and protons. Occurs when the Universe is a few minutes old, particles have energies corresponding to nuclear reactions (MeV).
- **Recombination** - formation of neutral hydrogen from protons and electrons. Greatly reduces the opacity of the Universe, allowing photons to decouple from matter - forming today the microwave background.

Important for both phases: Universe contains many more photons than baryons (by factor of \(\sim 10^{10}\)). Presumably a consequence of physics in the early Universe.
Basic picture of nucleosynthesis

When the Universe is hot enough, equilibrium between protons and neutrons, maintained by reactions:

\[ p + e^{-} \rightarrow n + \bar{\nu}_{e} \]
\[ n + e^{+} \rightarrow p + \bar{\nu}_{e} \]
\[ n \rightarrow p + e^{-} + \bar{\nu}_{e} \]

As the Universe cools, protons become favored over neutrons because they are lighter. Equilibrium ratio is:

\[ \frac{n_n}{n_p} = e^{\frac{mc^2}{kT}} = e^{\frac{1.5 \times 10^{10} K}{T}} \]

OK until \( T \sim 10^{10} \) K, when nuclear reaction rates become too slow to maintain equilibrium. Occurs at \( t \sim 1 \) s.
Thereafter, no new neutrons are formed, and existing ones decay with half life of \(\sim 10\) minutes. Sets the number of neutrons available for nucleosynthesis - almost all of which eventually end up in \(^4\text{He}\), with mass fraction:

\[
Y = \frac{2n_n}{n_n + n_p}
\]

Substituting the value \(n_n / n_p = 0.2\) at the `freeze-out’ temperature gives very roughly the right abundance of \(^4\text{He}\) - \(Y = 0.3\). Actual value is lower, partly because the deuterium forming reaction:

\[
n + p \rightarrow d + \gamma
\]

is initially suppressed by the high photon to baryon ratio.
Detailed calculations of primordial nucleosynthesis depend upon:

- Number of particle species contributing to the energy density at $t \sim$ minutes (this number is known from accelerator measurements)
- Number of baryons relative to the number of photons, equivalently $\frac{B}{h^2}$ - only free parameter in standard theory of big bang nucleosynthesis

Why no heavy elements?

- No stable nuclide with mass 5
- Jumping the gap requires e.g. $d + ^3He$, but these reactions are slow because both nuclei are positively charged.
Basically: the higher the density of nucleons, the faster they burn to form (eventually) $^4$He.

High baryon density favors:
- Slightly more $^4$He
- Much less deuterium and $^3$He

Also some $^7$Li and very small amounts of other nuclei.
Comparison with observations

Nucleosynthesis predicts the **primordial** abundance of a handful of light elements. Observational challenge is to measure these abundances in conditions as close as possible to primordial:

**$^4\text{He}$**: advantage - produced in large quantities so takes a significant amount of star formation to change the primordial abundance. **But** - only weakly sensitive to baryon density.

**Deuterium**: very fragile nucleus so burnt at low temperatures during pre-main sequence nuclear burning. Not produced in stellar evolution. Deuterium abundance in the most metal-poor gas ought to be good measure of primordial abundance.
\(^3\text{He}\): more complex evolution - deuterium burns to \(^3\text{He}\) in stars, but \(^3\text{He}\) is then destroyed by subsequent nuclear reactions in the core.

\(^7\text{Li}\): also complex - can be produced via cosmic-ray reactions with other nuclei in significant amounts as well as affected by stellar processing. Abundance in very metal poor halo stars best measure of primordial value.

Measurement of one of these abundances:
- Fixes single free parameter (baryon density)
- Can be compared to microwave background result

Two or more abundances permit check of consistency of theory…
Primordial helium abundance

Measure the gas phase abundance of $^4$He in metal poor extragalactic H II regions and extrapolate to zero metallicity:

Olive et al. 1997

$^4$He

Gives $Y = 0.234$, but note substantial scatter in individual measurements…
Primordial deuterium abundance

Spectra of quasars show very large number of absorption lines blueward of Lyman alpha:

Lyman-\(\alpha\) forest - absorption due to hydrogen along the line of sight between us and the quasar. Not pristine gas, but can have metallicity \(\sim 10^{-2}\) Solar - i.e. not much stellar processing.

KECK HIRES spectrum
Almost all of the absorption is due to hydrogen (also some due to carbon etc). Very small amount is due to deuterium. This can be measured because the absorption due to D is offset $\sim 0.1$ nm to the blue compared to hydrogen:

Energy levels of hydrogenlike atoms

$$E_n = \frac{\hbar}{2} c^2 \left( \frac{Z\hbar}{n^2} \right)^2$$

reduced mass

$$\hbar = \frac{m_e M}{m_e + M}$$

Principle of the measurement: look for an **extremely strong** (saturated) hydrogen absorber, then measure depth of the weak corresponding deuterium feature in the blue wing of the spectral line.

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Example from Burles & Tytler (1998)

Recent measurements give: \[
\frac{D}{H} = 2.74 \times 10^5
\]
Observed D/H ratio measurement from microwave background. Measurement of baryon density at t ~ few minutes agrees with that when t is 0.4 Myr - very good test of basic consistency of cosmology.

`Low' $^4$He result is not in such good agreement...

Observed D/H ratio measurement from microwave background. Measurement of baryon density at t ~ few minutes agrees with that when t is 0.4 Myr - very good test of basic consistency of cosmology...