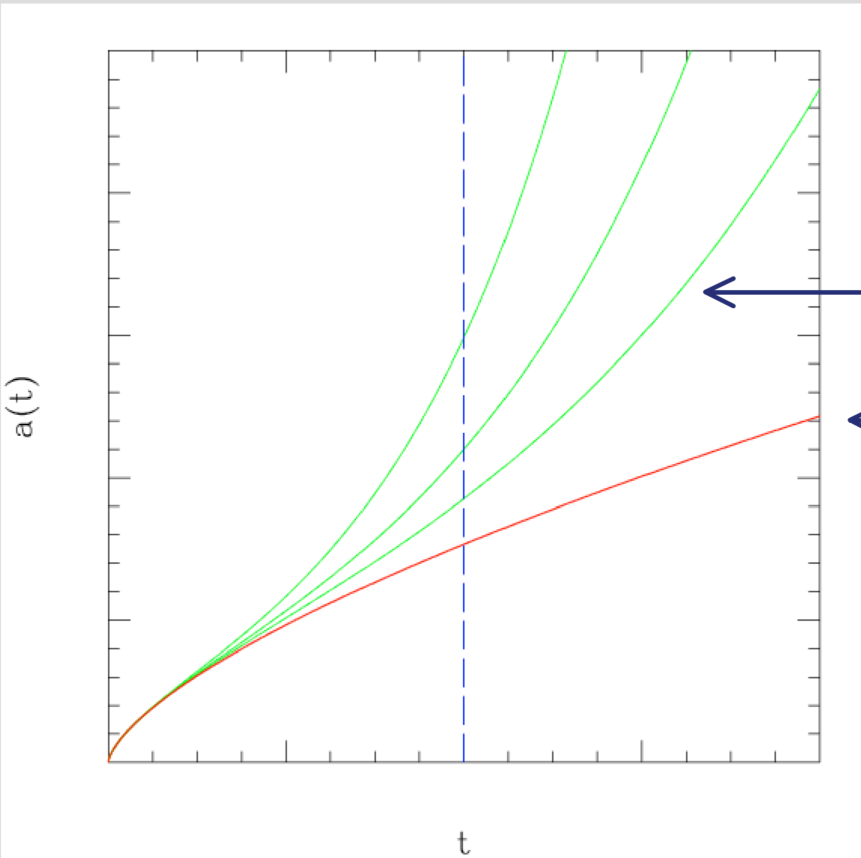
Evidence for the cosmological constant



Evolution of the scale factor in flat universes with matter plus cosmological constant

$\Lambda > 0$

$\Lambda = 0$

Unique signature of positive Λ is an **acceleration** of the expansion of the Universe.

Need to measure $\dfrac{d^2a}{dt^2}$

Difficult because we need to measure a second order effect i.e. must look at distant objects to `see' curvature in a(t).

But not **too distant** - at early times Λ has little effect on the expansion. Turns out that z ~ 1 is about optimal…

Deceleration parameter

Expand a(t), the function describing the evolution of the scale factor with time, in a power series about $t = t_0$:

$$a(t) = a(t_0) + (t - t_0)\left.\frac{da}{dt}\right|_{t=t_0} + \frac{1}{2}(t - t_0)^2 \left.\frac{d^2 a}{dt^2}\right|_{t=t_0} + \ldots$$

$$= a(t_0)\left[1 + (t - t_0)\frac{\dot{a}(t_0)}{a(t_0)} + \frac{1}{2}(t - t_0)^2 \frac{\ddot{a}(t_0)}{a(t_0)} + \ldots\right]$$

$$= a(t_0)\left[1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots\right]$$

…which defines the deceleration parameter q_0:

$$q_0 \equiv -\frac{\ddot{a}(t_0) a(t_0)}{\dot{a}^2(t_0)} = -\frac{\ddot{a}(t_0)}{a(t_0) H_0^2}$$

positive q_0 means the expansion is decelerating

Note: as with any truncated power series, this description becomes inadequate at large enough $(t-t_0)$…

Can use the Friedmann equations to evaluate q_0 for various universes:

<u>Matter dominated universes</u>

For Λ = 0 models we obtained:

$$\ddot{a}(t_0) = -\frac{4\pi G}{3}\rho_m(t_0)a(t_0) \quad \Rightarrow \quad q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)H_0^2} = \frac{4\pi G}{3H_0^2}\rho_m(t_0) = \frac{1}{2}\frac{\rho_m}{\rho_c}$$

Flat matter-dominated universe has $q_0 = 0.5$
Open universes have $0 < q_0 < 0.5$

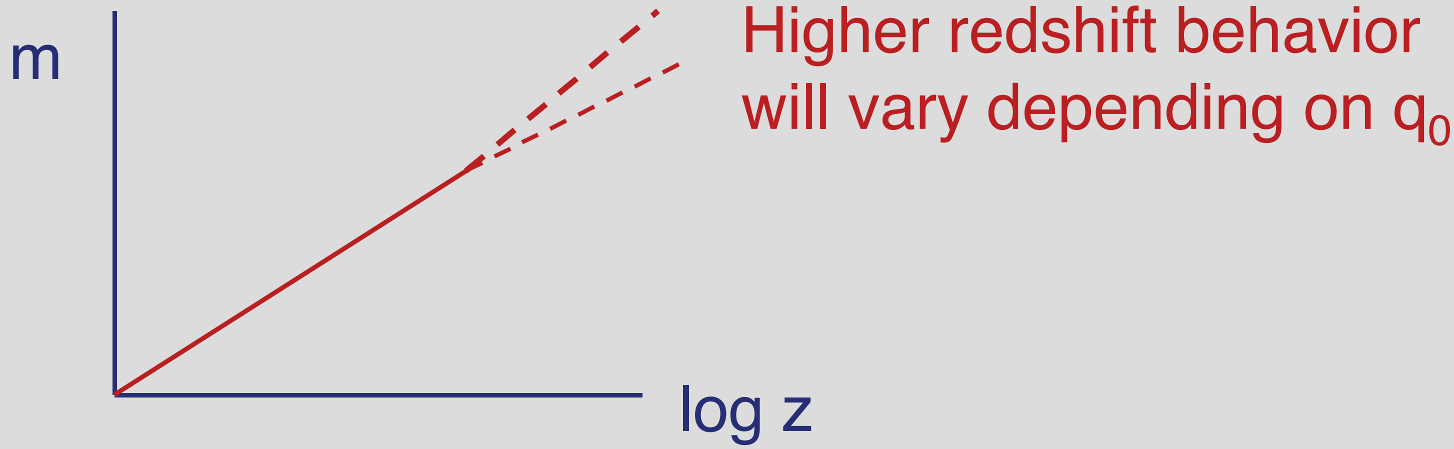
<u>Cosmological constant dominated universes</u>

From Friedmann equations, or from solution $a = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$

…find that $q_0 = -1$ in limit where positive Λ completely dominates the evolution of the scale factor

Observational tests of the deceleration parameter

Most useful is the magnitude - redshift relation for a population of standard candles (in practice supernovae). At small z this is just the linear Hubble diagram:



Suppose the sources emit radiation at time t_e ($t_e < t_0$).
Need to know:

- What redshift corresponds to that time
- What flux (and therefore apparent magnitude) the sources will have when radiation reaches observer

Cosmological redshift

Suppose that radiation is emitted at time t_e, when the Universe has scale factor $a(t_e)$, and received at time t_0. Redshift is:

$$z = \frac{\lambda_0}{\lambda_e} - 1 = \frac{a(t_0)}{a(t_e)} - 1$$

observational definition of the redshift

relation to cosmology

This is quite intuitive - wavelength of radiation expands along with expansion of the Universe.

This allows us to replace $a(t_e)$ / $a(t_0)$ in the power series expansion of $a(t)$ with the observable quantity z.

Luminosity distance

In a static, Euclidean universe, flux falls off with distance as:

$$f = \frac{L}{4\pi D^2}$$

This needs to be modified by two factors in an expanding universe, since:

- Photons are **redshifted** $$E_0 = \frac{hc}{\lambda_0} = \frac{hc}{\lambda_e(1+z)} = \frac{E_e}{1+z}$$

- Time dilation means that the **rate** at which photons arrive is reduced by a further factor of (1+z)

Result is an expression for the flux from a source at comoving distance R_e and redshift z:

$$f = \frac{L}{4\pi R_e^2 a^2(t_0)(1+z)^2}$$

Observational quantities are the measured flux f, and the intrinsic luminosity of the source L (assuming that somehow this can be deduced independently). Define the **luminosity distance**:

$$d_L(z) \equiv \left(\frac{L}{4\pi f} \right)^{1/2}$$

…which is just the `ordinary' distance for small z

The luminosity distance can be expressed as:

$$d_L(z) = \frac{1}{H_0} \left[z + \frac{1}{2}(1 - q_0) z^2 + \ldots \right]$$

If we can measure luminosity distances accurately (i.e. we have a `standard candle') then:

- plot of $d_L(z)$ vs z gives value of the Hubble constant
- deviations from a straight line fit give q_0

Type 1a supernovae appear to be good enough standard candles to make a direct measurement of q_0 feasible.

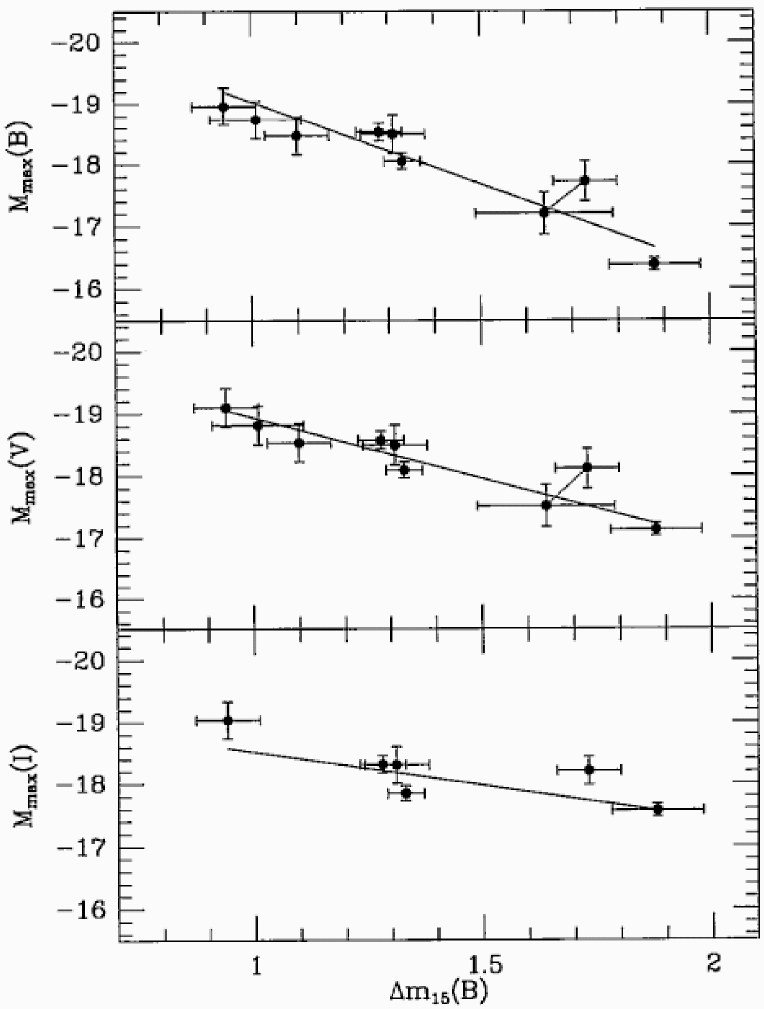


FIG. 1.—Decline rate–peak luminosity relation for the nine best-observed SN Ia's. Absolute magnitudes in *B*, *V*, and *I* are plotted vs. $\Delta m_{15}(B)$, which measures the amount in magnitudes that the *B* light curve drops during the first 15 days following maximum.

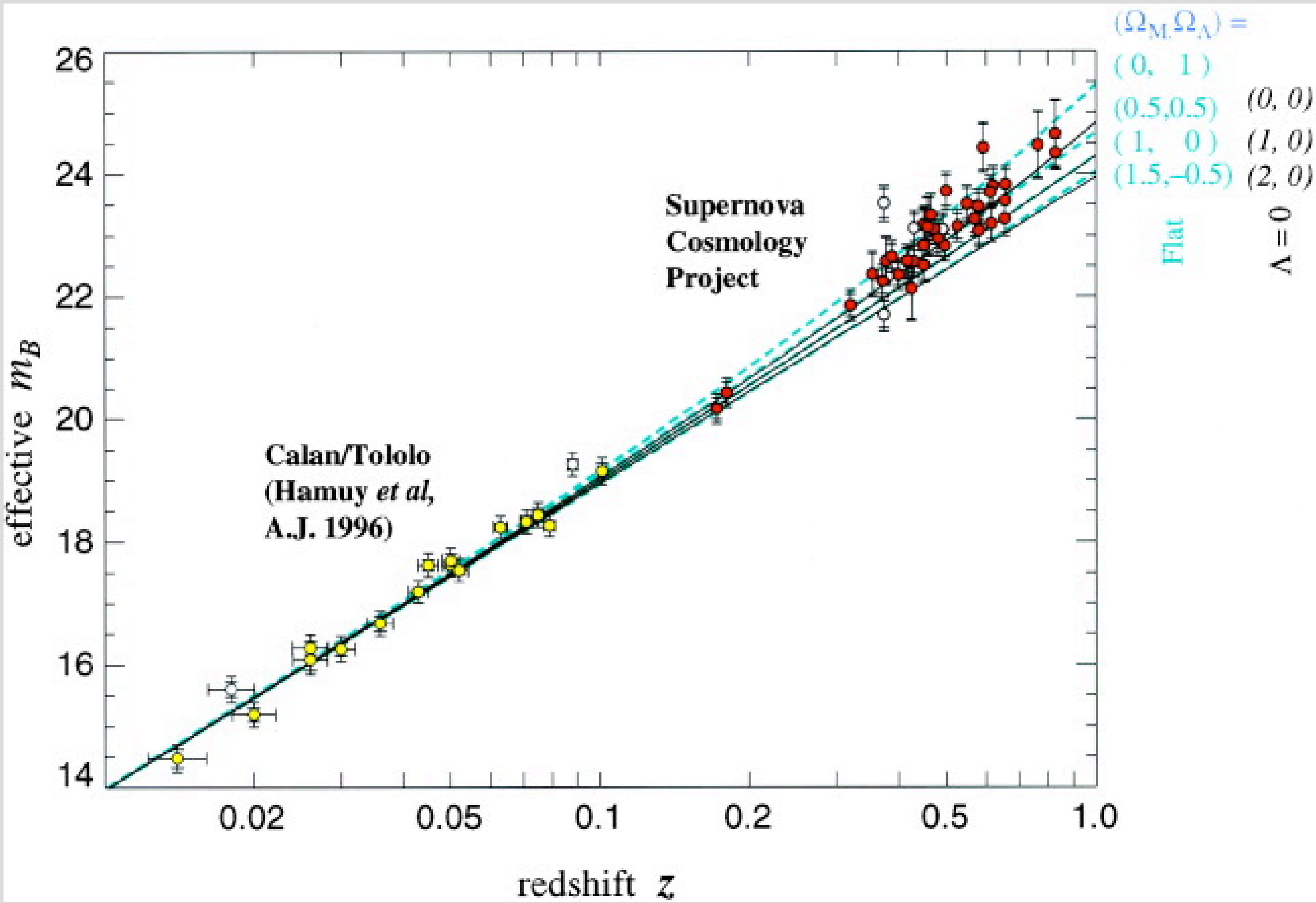
Key discovery: although there is some variation in absolute magnitude for Type 1a SN, **most** of the dispersion can be removed if the rate of decay of the luminosity is measured…

Peak luminosity vs **rate of decline** from Phillips (1993)

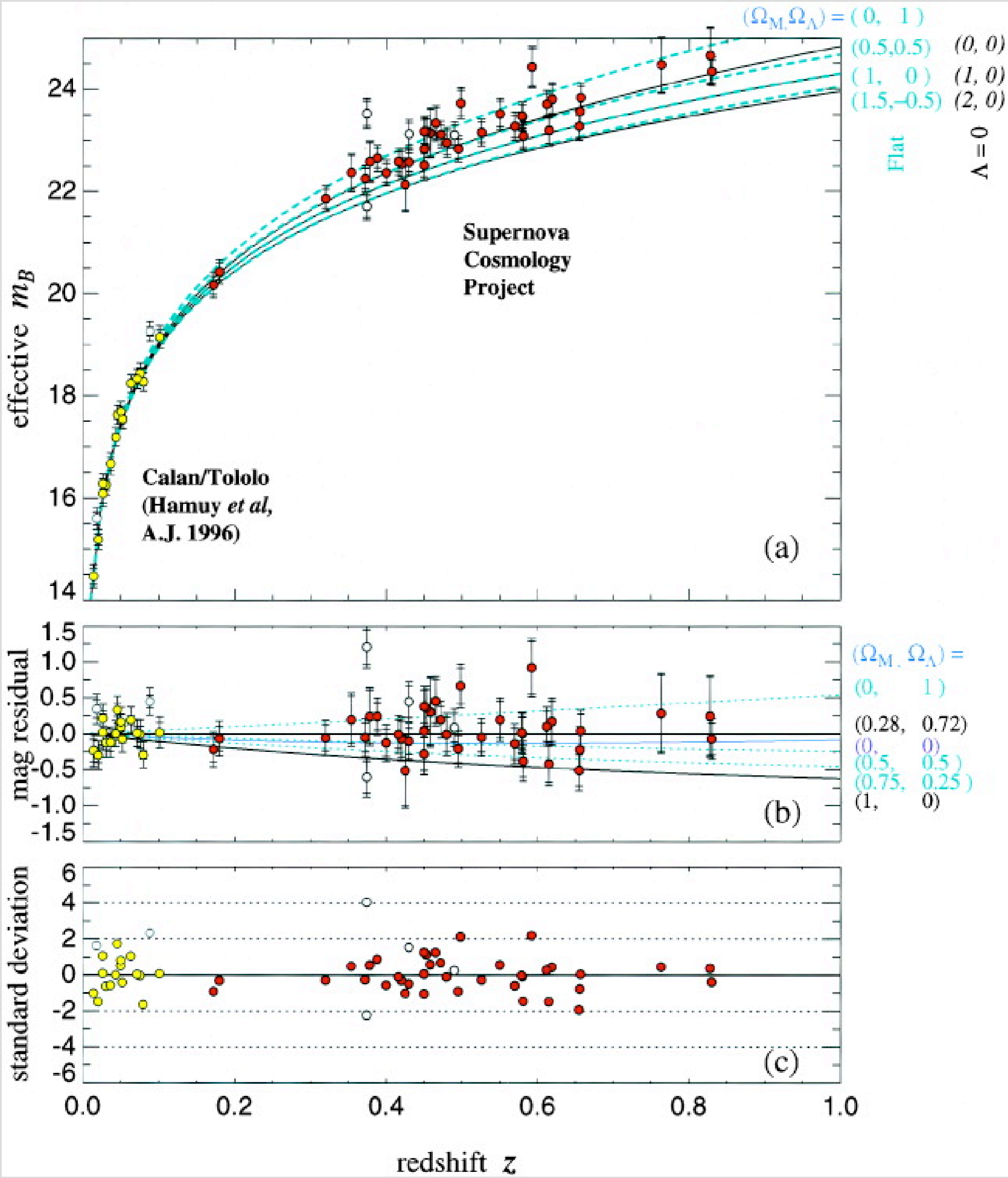
Most luminous SN are also slowest to decline

Hubble diagram for Type 1a supernovae

In 1998, two groups published evidence in favor of a positive from observations of Type 1a supernovae:

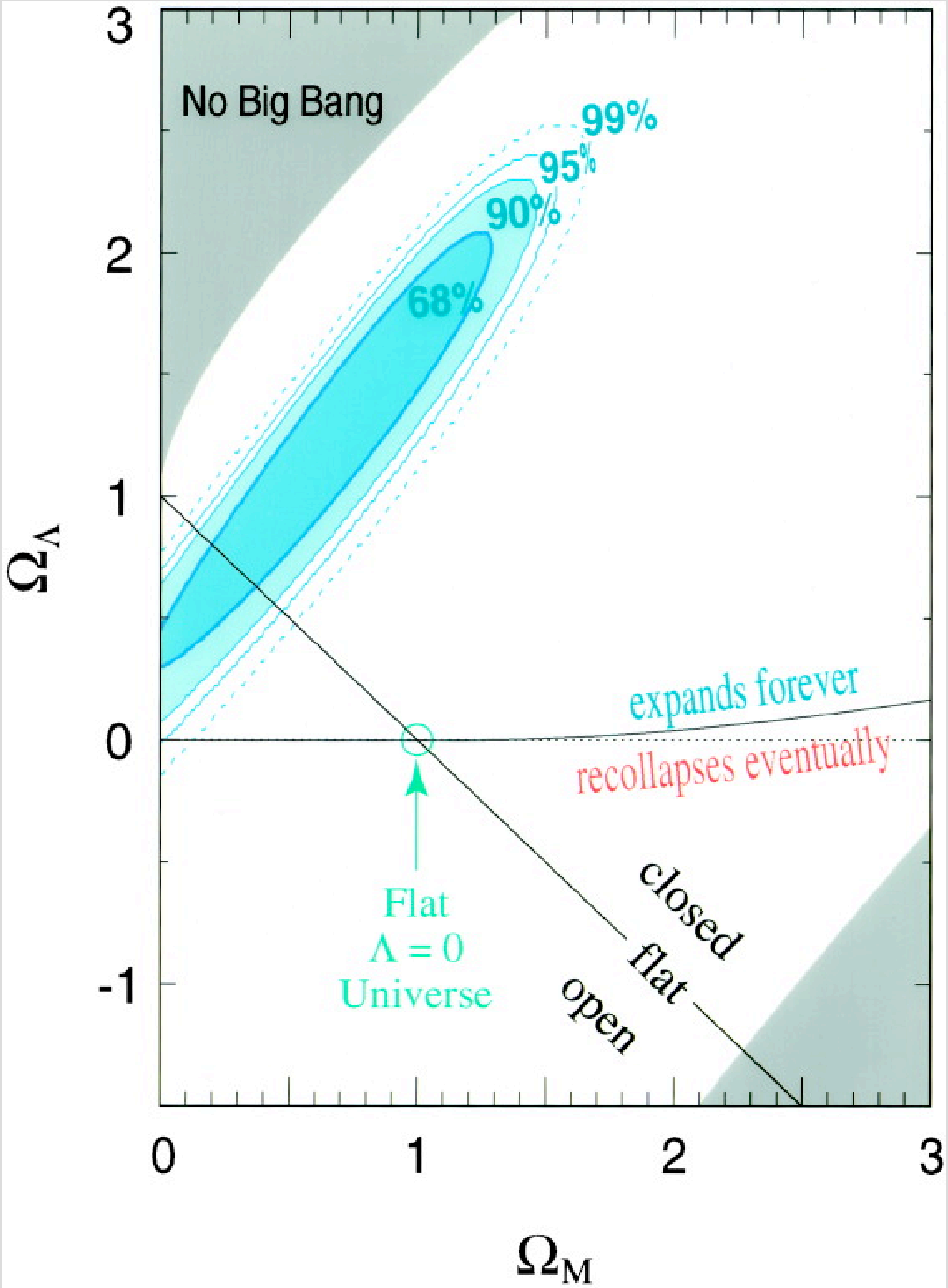


Favored flat model has cosmological constant dominating the current evolution of the Universe



$\Omega_\Lambda = 0.7$

Evidence for an accelerating Universe



Supernova measurements on their own actually define an allowed region in the plane of Ω_Λ vs Ω_{matter}

Need additional constraints (e.g. flatness) to pin down actual value of Ω_Λ

Example of **degeneracy**, distinct Universes produce identical results for this cosmological test

Need **complementary** tests to identify which Universe we live in…