Evidence for the cosmological constant

Evolution of the scale factor in flat universes with matter plus cosmological constant

\( \Lambda > 0 \)

\( \Lambda = 0 \)

Unique signature of positive \( \Lambda \) is an acceleration of the expansion of the Universe.

Need to measure \( \frac{d^2a}{dt^2} \)

Difficult because we need to measure a second order effect i.e. must look at distant objects to `see’ curvature in \( a(t) \).

But not too distant - at early times \( \Lambda \) has little effect on the expansion. Turns out that \( z \sim 1 \) is about optimal…

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Deceleration parameter

Expand $a(t)$, the function describing the evolution of the scale factor with time, in a power series about $t = t_0$:

$$a(t) = a(t_0) + (t - t_0) \frac{da}{dt}igr|_{t=t_0} + \frac{1}{2} (t - t_0)^2 \frac{d^2a}{dt^2}igr|_{t=t_0} + \ldots$$

$$= a(t_0) + (t - t_0) \frac{\dot{a}(t_0)}{a(t_0)} + \frac{1}{2} (t - t_0)^2 \frac{\ddot{a}(t_0)}{a(t_0)} + \ldots$$

$$= a(t_0) + H_0 (t - t_0) \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots$$

...which defines the deceleration parameter $q_0$:

$$q_0 = \frac{\ddot{a}(t_0) a(t_0)}{\dot{a}^2(t_0)} = \frac{\ddot{a}(t_0)}{a(t_0) H_0^2}$$

positive $q_0$ means the expansion is decelerating

Note: as with any truncated power series, this description becomes inadequate at large enough $(t-t_0)$...
Can use the Friedmann equations to evaluate $q_0$ for various universes:

**Matter dominated universes**

For $\ddot{a} = 0$ models we obtained:

$$\ddot{a}(t_0) = \frac{4G}{3} \ddot{a}(t_0) a(t_0)$$

$$q_0 = \frac{\ddot{a}(t_0)}{a(t_0) H_0^2} = \frac{4G}{3H_0^2} \ddot{a}(t_0) a(t_0) = \frac{1}{2} \frac{\ddot{a}}{\dot{a}}$$

**Flat matter-dominated** universe has $q_0 = 0.5$

**Open** universes have $0 < q_0 < 0.5$

**Cosmological constant dominated universes**

From Friedmann equations, or from solution

$$a = a_0 e^{\frac{\ddot{a}}{3} t}$$

…find that $q_0 = -1$ in limit where positive $\ddot{a}$ completely dominates the evolution of the scale factor
Observational tests of the deceleration parameter

Most useful is the magnitude - redshift relation for a population of standard candles (in practice supernovae). At small $z$ this is just the linear Hubble diagram:

$\log z$

$m$

Higher redshift behavior will vary depending on $q_0$

Suppose the sources emit radiation at time $t_e$ ($t_e < t_0$). Need to know:

- What redshift corresponds to that time
- What flux (and therefore apparent magnitude) the sources will have when radiation reaches observer
Cosmological redshift

Suppose that radiation is emitted at time \( t_e \), when the Universe has scale factor \( a(t_e) \), and received at time \( t_0 \). Redshift is:

\[
z = \frac{a(t_0)}{a(t_e)} - 1
\]

This is quite intuitive - wavelength of radiation expands along with expansion of the Universe.

This allows us to replace \( a(t_e) / a(t_0) \) in the power series expansion of \( a(t) \) with the observable quantity \( z \).
Luminosity distance

In a static, Euclidean universe, flux falls off with distance as:

\[ f = \frac{L}{4 \pi D^2} \]

This needs to be modified by two factors in an expanding universe, since:

- Photons are redshifted

\[ E_0 = \frac{hc}{\lambda_0} = \frac{hc}{\lambda_e(1 + z)} = \frac{E_e}{1 + z} \]

- Time dilation means that the rate at which photons arrive is reduced by a further factor of \((1+z)\)

Result is an expression for the flux from a source at comoving distance \(R_e\) and redshift \(z\):

\[ f = \frac{L}{4 \pi R_e^2 a^2(t_0)(1 + z)^2} \]
**Observational quantities** are the measured flux $f$, and the intrinsic luminosity of the source $L$ (assuming that somehow this can be deduced independently). Define the **luminosity distance**:

$$d_L(z) = \left( \frac{L}{4\pi f} \right)^{1/2}$$

…which is just the `ordinary' distance for small $z$

The luminosity distance can be expressed as:

$$d_L(z) = \frac{1}{H_0} z + \frac{1}{2} (1 - q_0) z^2 + \ldots$$

If we can measure luminosity distances accurately (i.e. we have a `standard candle') then:

- plot of $d_L(z)$ vs $z$ gives value of the Hubble constant
- deviations from a straight line fit give $q_0$
Type 1a supernovae appear to be good enough standard candles to make a direct measurement of $q_0$ feasible.

Key discovery: although there is some variation in absolute magnitude for Type 1a SN, most of the dispersion can be removed if the rate of decay of the luminosity is measured...

Peak luminosity vs rate of decline from Phillips (1993)

Most luminous SN are also slowest to decline

**Fig. 1.** Decline rate–peak luminosity relation for the nine best-observed SN Ia’s. Absolute magnitudes in $B$, $V$, and $I$ are plotted vs. $\Delta m_{15}(B)$, which measures the amount in magnitudes that the $B$ light curve drops during the first 15 days following maximum.
Hubble diagram for Type 1a supernovae

In 1998, two groups published evidence in favor of a positive \( \Omega \) from observations of Type 1a supernovae:
Favored flat model has cosmological constant dominating the current evolution of the Universe.

\[ W = 0.7 \]

Evidence for an accelerating Universe.
Supernova measurements on their own actually define an allowed region in the plane of $\Omega_L$ vs $\Omega_{\text{matter}}$

Need additional constraints (e.g. flatness) to pin down actual value of $\Omega_L$

Example of degeneracy, distinct Universes produce identical results for this cosmological test

Need complementary tests to identify which Universe we live in…