Accretion Disks

Luminosity of AGN derives from gravitational potential energy of gas spiraling inward through an accretion disk. Derive structure of the disk, and characteristic temperatures of the gas.

First consider vertical structure:

Gravitational acceleration in vertical direction:

\[ g = \frac{GM}{r^2} \cos \theta = \frac{GM}{r^2} \frac{z}{r} = \frac{GM}{R^3} z \quad (z \ll R) \]
If the gas is supported against gravity by a pressure gradient, force balance in the vertical direction gives:

\[ \frac{dP}{dz} = -\rho g \]

Assume the disk is isothermal in the vertical direction with sound speed \( c_s \). The pressure is then:

\[ P = \rho c_s^2 \]

Solve for the vertical structure:

\[ c_s^2 \frac{d\rho}{dz} = \rho \frac{GM}{R^3} z = \frac{2}{\rho} \]

\( \rho \) is angular velocity in disk

\[ \frac{d\rho}{\rho} = \frac{2}{c_s^2} \rho R dz \]

\[ \rho = \rho_{z=0} e^{\frac{2z^2}{2c_s^2}} \]

\( \rho_{z=0} \) is density in disk midplane
Rewrite this equation as: \( \square = \square_{z=0} e^{z^2/h^2} \)

…where \( h \) is the vertical scale height of the disk. Since \( h = v_f / r \), can write \( h \) as:

\[
  h^2 \equiv \frac{2c_s^2}{\square^2} = \frac{2c_s^2 R^2}{v_f^2}
\]

\[
  \square \ \frac{h}{R} \ \frac{c_s}{v_f}
\]

The thickness of the disk as a fraction of the radius is given by the ratio of the sound speed to the orbital velocity.

A disk for which \( (h / R) \ll 1 \) is described as a geometrically thin disk. Structure of thin disks is relatively simple because radial pressure forces can be neglected - i.e. \( v_f \) for the gas is the same as a particle orbiting at the same radius.
Angular momentum transport

If the disk is thin, then orbital velocity of the gas is Keplerian:

\[ v_f = \sqrt{\frac{GM}{R}} \]

Specific angular momentum \( v_f R \) is:

\[ l = \sqrt{GMR} \]

i.e. increasing outwards. Gas at large \( R \) has too much angular momentum to be accreted by the black hole.

To flow inwards, gas must lose angular momentum, either:

- By **redistributing** the angular momentum within the disk (gas at small \( R \) loses angular momentum to gas further out and flows inward)
- By loss of angular momentum from the entire system. e.g. a wind from the disk could take away angular momentum allowing inflow
Redistribution of angular momentum within a thin disk is a *diffusive* process - a narrow ring of gas spreads out under the action of the disk *viscosity*:

With increasing time:
- **Mass** all flows inward to small $R$ and is accreted
- **Angular momentum** is carried out to very large $R$ by a vanishingly small fraction of the mass.
Consider gas flowing inward through a thin disk. Easy to estimate the radial distribution of temperature.

Potential energy per unit mass at radius \( R \) in the disk is:

\[
E = \frac{GM}{R} \quad \Rightarrow \quad \frac{dE}{dR} = \frac{GM}{R^2}
\]

Suppose mass \( dM \) flows inward distance \( dR \). Change in potential energy is:

\[
\Delta E = \frac{GM}{R^2} dMdR
\]

Half of this energy goes into increased kinetic energy of the gas. If the other half is radiated, luminosity is:

\[
L = \frac{GMM}{2R^2} dR
\]
Divide by the radiating area, $2 \times 2 \pi R \times dR$ to get luminosity per unit area. Equate this to the rate of energy loss via blackbody radiation:

$$\frac{G M \dot{M}}{8 \pi R^3} = \sigma T^4$$

$\sigma$ is Stefan-Boltzmann constant

Gives the radial temperature distribution as:

$$T = \sqrt[4]{\frac{G M \dot{M}}{8 \pi R^3}}$$

Correct dependence on mass, accretion rate, and radius, but wrong prefactor. Need to account for:

- Radial energy flux through the disk (transport of angular momentum also means transport of energy)
- Boundary conditions at the inner edge of the disk
Correcting for this, radial distribution of temperature is:

\[
T(R) = \sqrt[4]{\frac{3G\dot{M}}{8\pi R^3}} \sqrt{\frac{R_{in}}{R}}^{1/4}
\]

…where \( R_{in} \) is the radius of the disk inner edge. For large radii \( R \gg R_{in} \), we can simplify the expression to:

\[
T(R) = \sqrt[4]{\frac{3G\dot{M}}{8\pi R_{s}^3}} \sqrt{\frac{R}{R_{s}}}^{3/4}
\]

with \( R_{s} = \frac{2GM}{c^2} \) the Schwarzschild radius as before. For a hole accreting at the Eddington limit:

- Accretion rate scales linearly with mass
- Schwarzschild radius also increases linearly with mass

Temperature at fixed number of \( R_{s} \) decreases as \( M^{-1/4} \) - disks around more massive black holes are cooler.
For a supermassive black hole, rewrite the temperature as:

\[ T(R) = 6.3 \times 10^5 \left( \frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left( \frac{M}{10^8 M_{\text{sun}}} \right)^{1/4} \left( \frac{R}{R_s} \right)^{3/4} \text{ K} \]

Accretion rate at the Eddington limiting luminosity (assuming \( h = 0.1 \))

A thermal spectrum at temperature \( T \) peaks at a frequency:

\[ h n_{\text{max}} = 2.8kT \]

An inner disk temperature of \( \sim 10^5 \) K corresponds to strong emission at frequencies of \( \sim 10^{16} \) Hz. Wavelength \( \sim 50 \) nm.

Expect disk emission in AGN accreting at close to the Eddington limit to be strong in the ultraviolet - origin of the broad peak in quasar SEDs in the blue and UV.
Disk has annuli at many different temperatures - spectrum is weighted sum of many blackbody spectra.

- **Flat** - $\theta^{1/3}$ - region

Consistent with the broad spectral energy distribution of AGN in the optical and UV regions of the spectrum.