Chemical evolution

Observation of spiral and irregular galaxies show that the fraction of heavy elements varies with the fraction of the total mass which is in the form of gas:
Simplest model to understand these observations is the **one-zone** model (*Sparke & Gallagher 4.3*).

Consider the history of an annulus of a spiral galaxy at some radius $R$. Make several simplifying assumptions:

- No material (gas, stars) enters or leaves the annulus
- Initially the annulus contains only gas, with no heavy elements (i.e. just hydrogen, helium)
- As stars are formed, massive stars explode `instantaneously` as supernovae, returning enriched gas to the ISM
- Turbulent motions keep the gas well mixed, so it has a single well-defined composition

**How does the metal fraction of the gas evolve with time?**
Let mass of interstellar gas in annulus be $M_g$
Mass of heavy elements in gas $M_h$

Define \textbf{metallicity}: $Z = \frac{M_h}{M_g}$

Suppose the mass of stars at this time is $M_s$
Imagine forming new stars, with mass $\lesssim M_s$

Of these:
- Stars with mass $M > 8$ Solar masses explode rapidly as supernovae, returning metals to the ISM
- Lower mass stars, with mass $\lesssim M_s$, remain

The mass of heavy elements produced by this episode of star formation is $p \lesssim M_s$, defining the \textbf{yield} $p$.  

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Total change in the mass of heavy elements due to star formation is then:

\[ \Delta M_h = p \Delta M_s \Delta Z \Delta M_s = (p \Delta Z) \Delta M_s \]

existing metals locked up in low mass stars

Corresponding change in metallicity is:

\[ \Delta Z = \frac{\Delta M_h}{\Delta M_g} = \frac{\Delta M_h}{M_g} \frac{M_h}{M_g^2} \Delta M_g = \frac{1}{M_g} \left( \Delta M_h \Delta Z \Delta M_g \right) \]

By conservation of mass:

\[ \Delta M_s = \Delta M_g \]
Combining the two previous equations get:

$$\ddot{Z} = \dddot{p} \frac{M_g}{M_g}$$

Can write this as a differential equation:

$$\frac{dM_g}{dZ} = \dddot{M_g} \frac{p}{\ddot{p}}$$

If $p$ is a constant (ie does not vary between subsequent generations of stars), then this integrates immediately to give:

$$Z(t) = \ddot{p} \ln \left( \frac{M_g(t)}{M_g(0)} \right)$$

(assuming assumption that $Z(t=0)$ was zero). Relation of this form is very roughly what is observed in some galaxies.