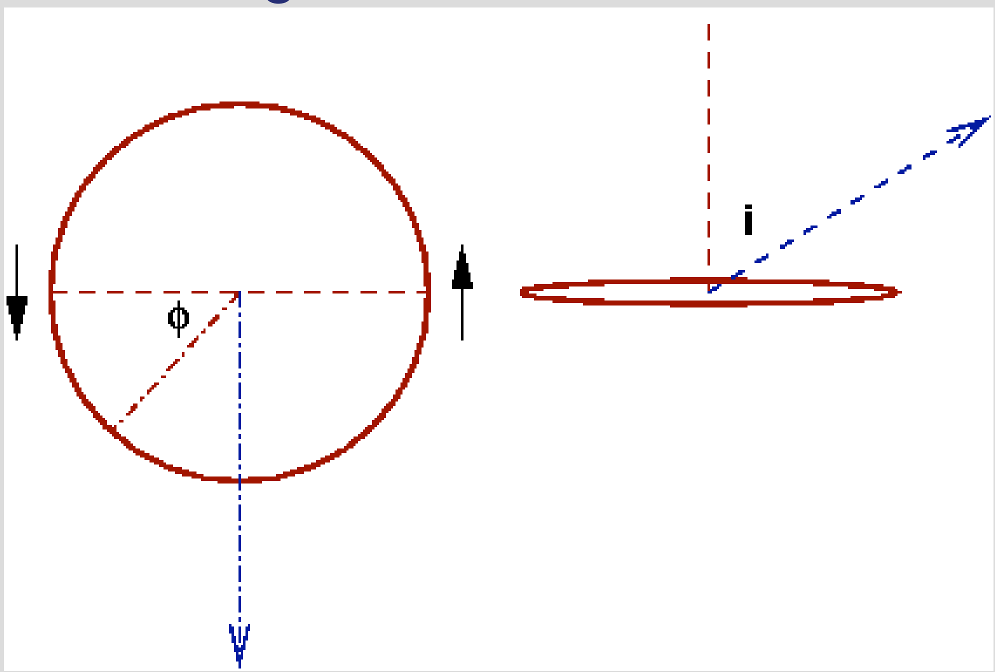
Measuring galaxy rotation curves

Consider a galaxy in pure circular rotation, with rotation velocity V(R). Axis of rotation of the galaxy makes an angle i to our line of sight.



If we measure the apparent velocity in the disk at an angle ϕ, measured **in the disk**, then line of sight (radial) velocity is:

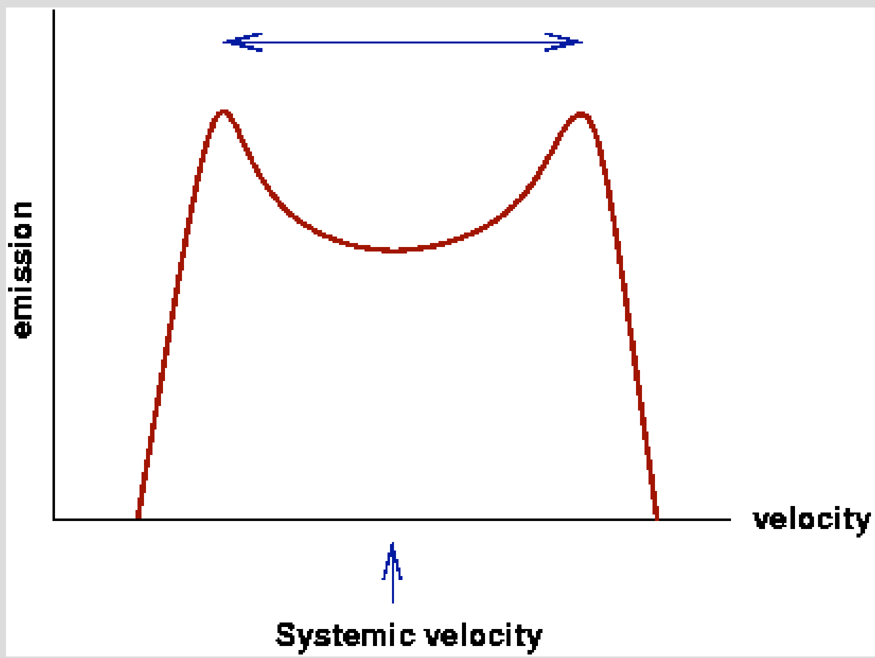
$$V_r(R,i) = V_{sys} + V(R)\sin i \cos \phi$$

where V_{sys} is the systemic velocity of the galaxy.

If we measure V_r across the galaxy, and can infer the inclination i, then obtain the full rotation curve V(R).

Even if the galaxy is not resolved, measuring the amount of emission as a function of the line of sight velocity gives a measure of the peak rotation speed in the galaxy V_{max}:

$$W \approx 2V_{max} \sin i$$



See textbook 5.2-5.3 for more details of observations.

Can use the Doppler shift of any convenient spectral line to measure the line of sight velocity:

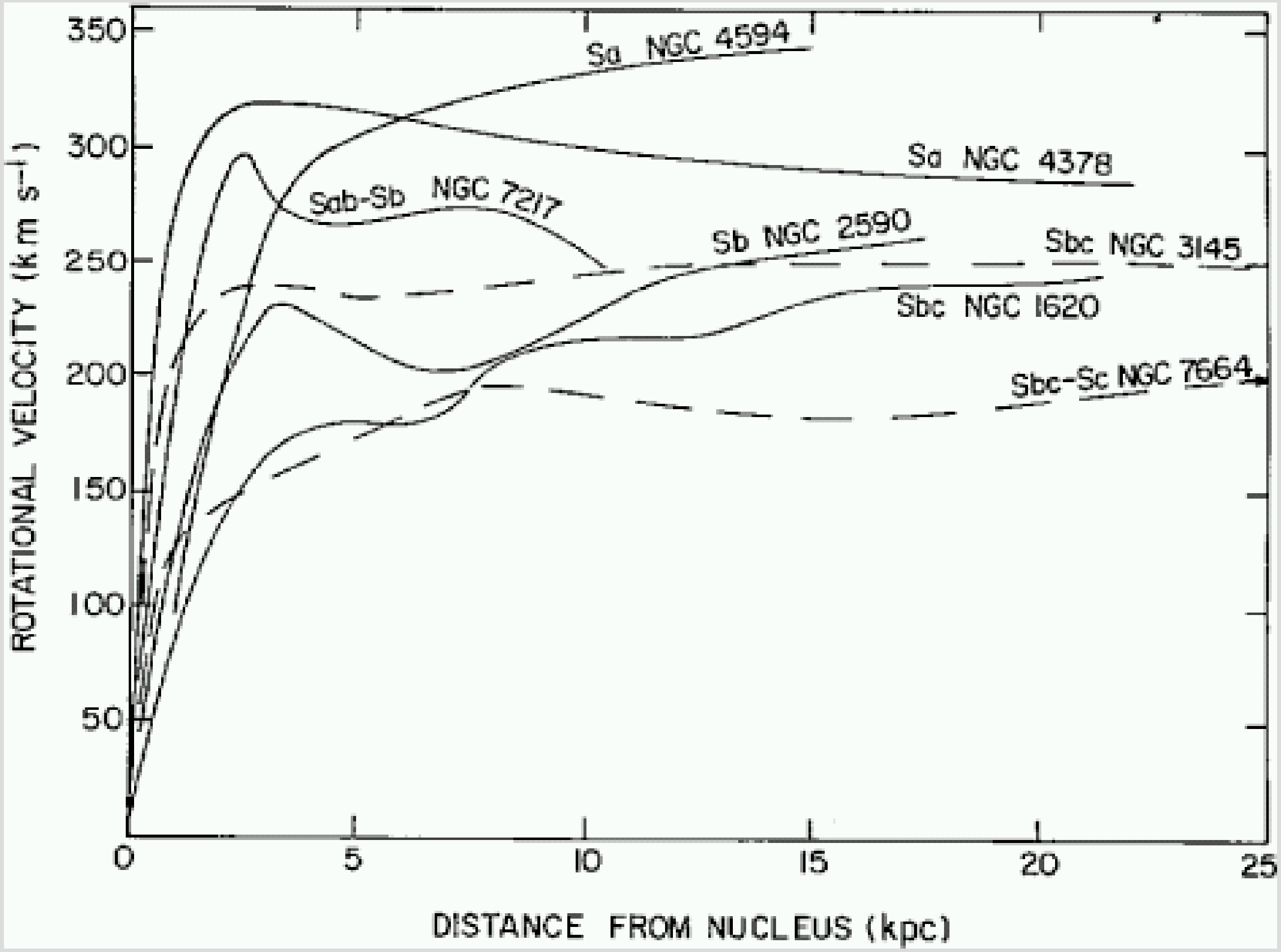
$$\frac{\lambda_{obs}}{\lambda_{emit}} = 1 + \frac{V_r}{c}$$

Optical: for nearby galaxies use Hα spectral line to measure rotation curves. Distant galaxies use spectral line of oxygen. Measures rotation of the stars.

Radio: traditional measure of rotation curves. Use 21cm line of hydrogen. Measures rotation of the neutral hydrogen gas disk.

Major advantage: detectable gas disk extends further out than detectable stellar disk.

Examples of spiral galaxy rotation curves



Typically flat or even rising out to many scale lengths of the exponential disk.

If all the mass in these galaxies was provided by stars and gas, expect that V(R) would drop as $R^{-1/2}$ at R > few x h_R.

Existence of flat (or even rising) rotation curves at these radii imply additional unseen mass - **dark matter**.

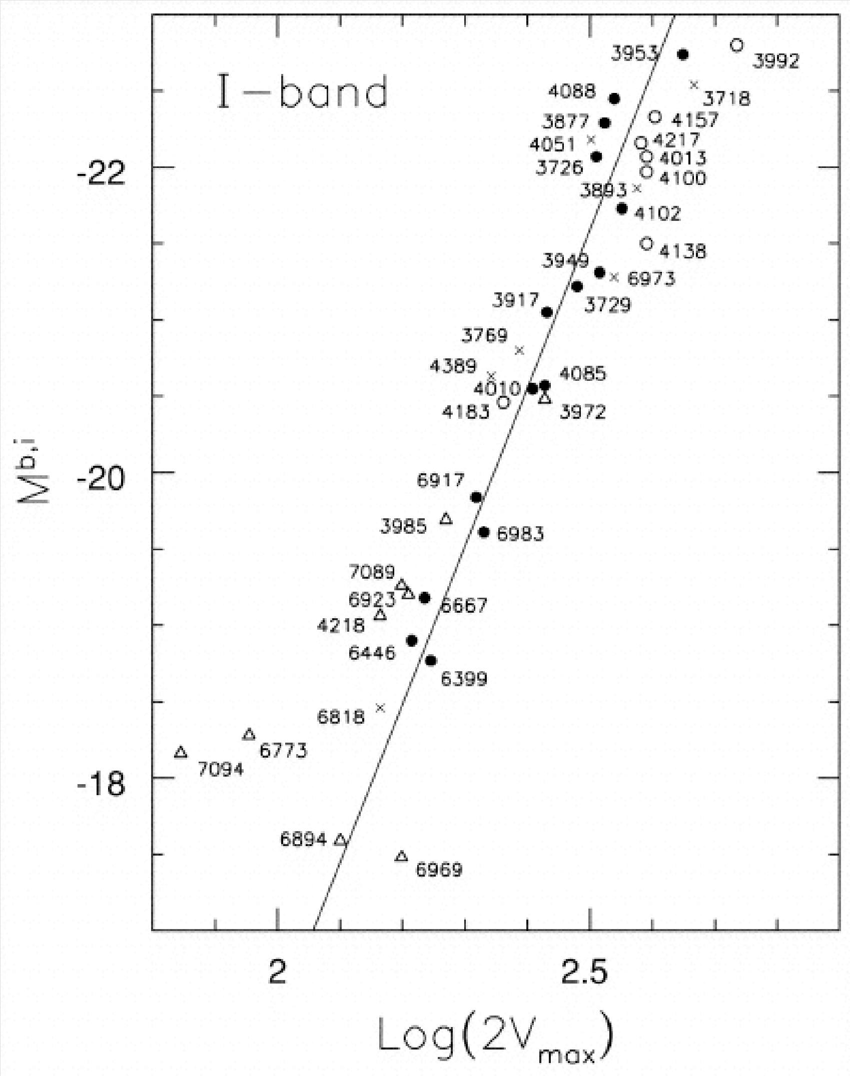
Rotation curve measurements on their own only indicate that the dark matter must be:

- Dynamically dominant at large radii (required proportion of dark matter ~50% in Sa / Sb galaxies, 80-90% in Sd galaxies).
- Have a more extended distribution than either the stars or the gas.

Note: no evidence for dark matter on the scale of the Solar System, or in the nearby Galactic disk.

Tully-Fisher relation

Plot the maximum circular velocity of spiral galaxies against their luminosity in a given band:



Find that L and V_{max} are closely correlated

<u>Tully-Fisher relation</u>

Smallest scatter when L is measured in the red or the near-infrared wavebands e.g. in the H band centered at 1.65 μm:

$$L_H \approx 3 \times 10^{10} \left(\frac{V_{\max}}{196 \text{ km/s}} \right)^{3.8} L_{H,\text{solar}}$$

Roughly, $L \propto V_{\max}^4$

Important use as extragalactic distance indicator:

- Measure V_{max}, eg from radio observations of HI
- Infer L in a given band from the Tully-Fisher relation, and convert to absolute magnitude
- Measure the apparent magnitude
- Use:

$$m - M = 5 \log \left(\frac{d}{10 \text{ pc}} \right)$$

…to estimate distance

Origin of the Tully-Fisher relation

In part, Tully-Fisher relation reflects simple gravitational dynamics of a disk galaxy (see problem 5.10 in textbook). Estimate the luminosity and maximum circular velocity of an exponential disk of stars.

<u>Luminosity</u>

Empirically, disk galaxies have an exponential surface brightness profile:

$$I(R) = I(0)\ \mathrm{e}^{-R/h_R}$$

…with central surface brightness I(0) a constant. Integrate this across annuli to get the total luminosity:

$$L = \int_0^{\infty} 2\pi R I(0) e^{-R/h_R}\, dR$$

Can integrate this expression by parts, finding:

$$L \quad I(0) h_R^2$$

i.e. for constant central surface brightness, luminosity scales with the square of the scale length.

<u>Circular velocity</u>

If the mass in the stars of the exponential disk dominates the rotation curve, then the enclosed mass within radius R will be proportional to the enclosed luminosity:

$$M(R) \quad L(R) \quad \int_0^R 2\pi R' I(0) e^{-R'/h_R} dR'$$

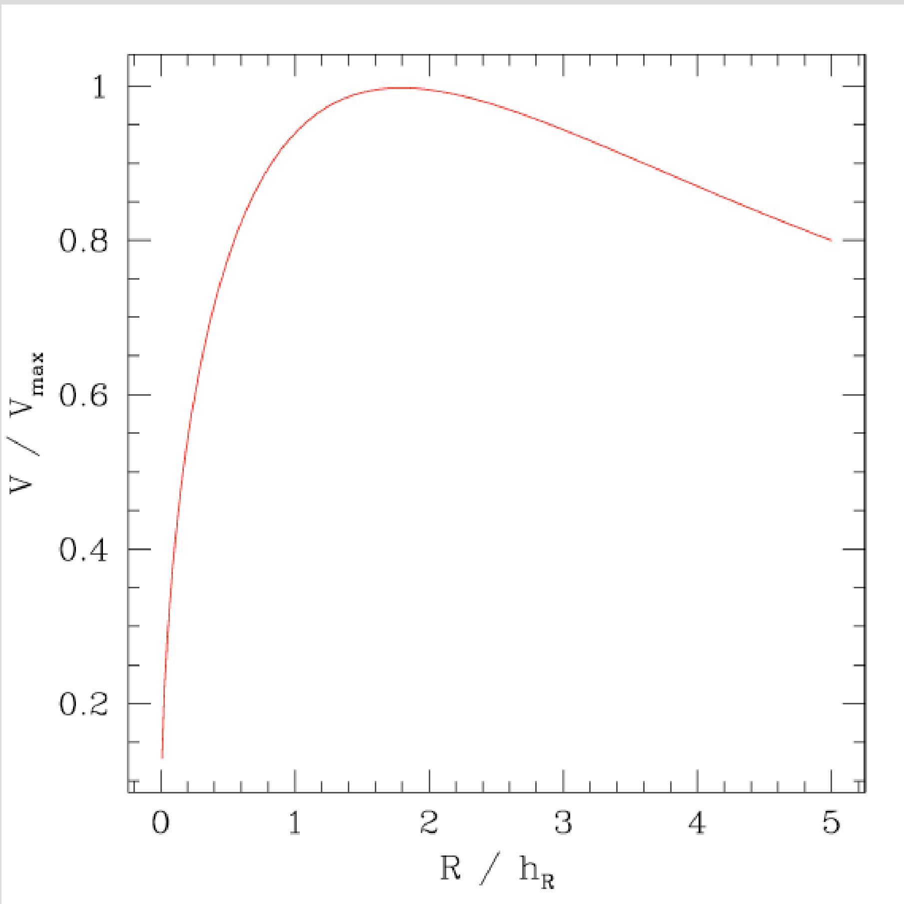
Approximately, use formula for spherical mass distribution to get V(R):

$$\frac{V^2(R)}{R} = \frac{GM(R)}{R^2}$$

This gives,

$$V^2(R) \propto \left[\frac{h_R}{R} - \frac{h_R}{R} e^{-R/h_R} - e^{-R/h_R} \right] h_R$$

Dependence on R always occurs via the combination R / h_R



Function in square brackets peaks at $R \sim 1.8\ h_R$

Conclude that:

$$V_{max} \propto \sqrt{h_R}$$

Eliminate h_R using previous result:

$$L \propto V_{max}^4$$

…the Tully-Fisher relation!