Stability of star clusters and galaxies

Already mentioned different ways in which a galaxy evolves:

- Constituent *stars* evolve, with higher mass stars leaving the main sequence and producing stellar remnants. After a burst of star formation, an initially blue galaxy reddens and fades.

→ **`Passive evolution'**

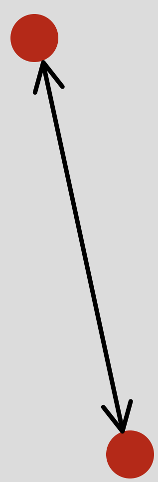
- Mergers - if two galaxies merge, interaction can alter the morphology of the galaxy, and trigger star formation if there is cool gas present.

Third *potential* source of evolution is caused by the purely gravitational interactions between stars. Evaluate how important this is for star clusters and galaxies.

Strong encounters

In a large stellar system, gravitational force at any point due to all the other stars is almost constant. Star traces out an orbit in the smooth potential of the whole cluster.

Orbit will be changed if the star passes very close to another star - define a strong encounter as one that leads to $\Delta \mathbf{v} \sim \mathbf{v}$.



Consider two stars, of mass m. Suppose that typically they have average speed V.

Kinetic energy: $\frac{1}{2} m V^2$

When separated by distance r, gravitational potential energy: $\frac{Gm^2}{r}$

By conservation of energy, we expect a large change in the (direction of) the final velocity if the change in potential energy at closest approach is as large as the initial kinetic energy:

<u>Strong encounter</u>

$$\frac{Gm^2}{r} \square \frac{1}{2} m V^2 \square \quad r_s \equiv \frac{2Gm}{V^2}$$

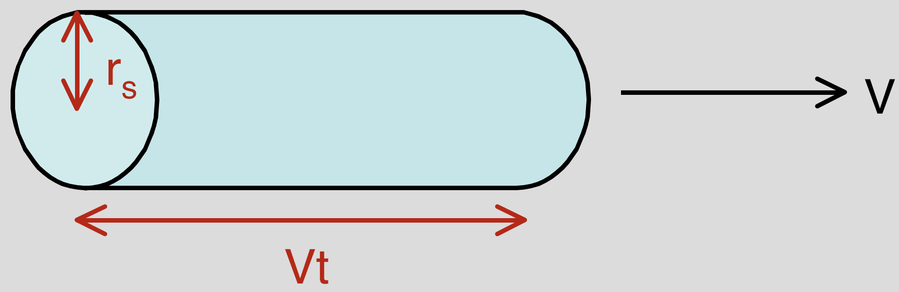
Strong encounter radius

Near the Sun, stars have random velocities V ~ 30 km s^{-1}, which for a typical star of mass 0.5 M_{sun} yields r_s ~ 1 au.

Good thing for the Solar System that strong encounters are very rare…

Time scale for strong encounters:

In time t, a strong encounter will occur if any other star intrudes on a cylinder of radius r_s being swept out along the orbit.



Volume of cylinder: $\pi r_s^2 V t$

For a stellar density n, mean number of encounters: $\pi r_s^2 V t n$

Typical time scale between encounters:

$$t_s = \frac{1}{\pi r_s^2 V n} = \frac{V^3}{4\pi G^2 m^2 n}$$

(substituting for the strong encounter radius r_s)

Note: more important for **small** velocities.

Plug in numbers (being careful to note that n in the previous expression is stars per cubic cm, not cubic parsec!)

$$t_s \approx 4 \times 10^{12} \left(\frac{V}{10 \text{ km s}^{-1}} \right)^3 \left(\frac{m}{M_{sun}} \right)^{-2} \left(\frac{n}{1 \text{ pc}^{-3}} \right)^{-1} \text{ yr}$$

Conclude:

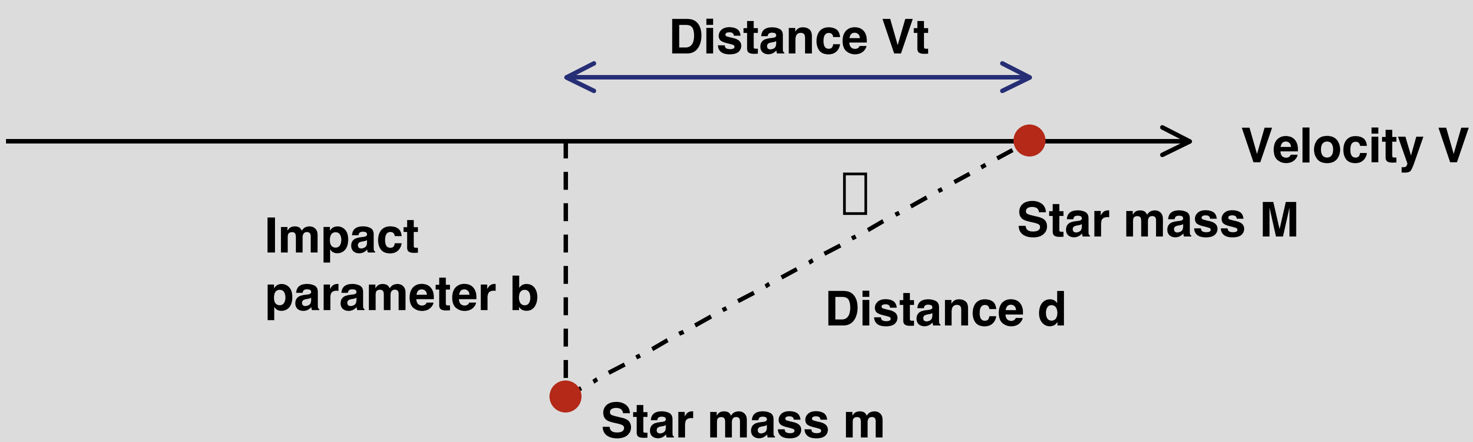
- stars in the disks of galaxies ($V \sim 30$ km s^{-1}, $n \sim 0.1$ pc^{-3} near the Sun), never physically collide, and are extremely unlikely to pass close enough to deflect their orbits substantially.

- in a globular cluster ($V \sim 10$ km s^{-1}, $n \sim 1000$ pc^{-3} or more), strong encounters will be common (i.e. one or more per star in the lifetime of the cluster).

Weak encounters

Stars with impact parameter b >> r_s will also perturb the orbit. Path of the star will be deflected by a very small angle by any one encounter, but cumulative effect can be large.

Because the angle of deflection is small, can approximate situation by assuming that the star follows *unperturbed* trajectory:



Define distance of closest approach to be b; define this moment to be t = 0.

Force on star M due to gravitational attraction of star m is:

$$F = \frac{GMm}{d^2} = \frac{GMm}{b^2 + V^2 t^2}$$ (along line joining two stars)

Component of the force **perpendicular** to the direction of motion of star M is:

$$F_{\perp} = F \sin\theta = F \times \frac{b}{d} = \frac{GMmb}{\left(b^2 + V^2 t^2\right)^{3/2}}$$

Using F = mass x acceleration: $F_{\perp} = M \frac{dV_{\perp}}{dt}$ ← Velocity component perpendicular to the original direction of motion

Integrate this equation with respect to time to get final velocity in the perpendicular direction.

Note: in this approximation, consistent to assume that $V_{||}$ remains unchanged. Whole calculation is OK provided that the perpendicular velocity gain is *small*.

Final perpendicular velocity is:

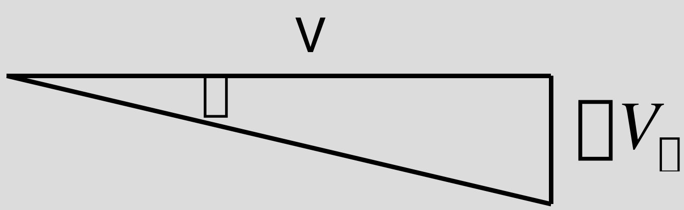
$$\Delta V_{\perp} = \int_{-\infty}^{\infty} \frac{dV_{\perp}}{dt} dt$$

$$= \frac{1}{M} \int_{-\infty}^{\infty} F_{\perp}(t) dt$$

$$= \frac{1}{M} \int_{-\infty}^{\infty} \frac{GMmb}{\left(b^2 + V^2 t^2\right)^{3/2}} dt = \frac{2Gm}{bV}$$

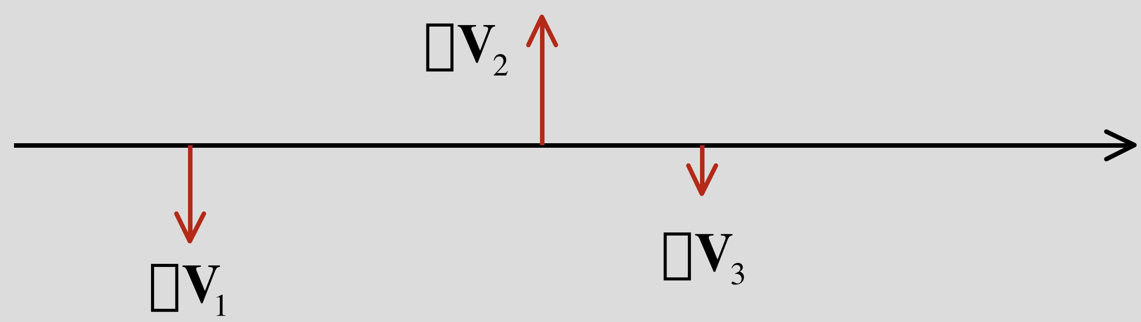
As before, small V leads to larger deflection during the flyby

Deflection *angle* is:



$$\theta \approx \tan\theta = \frac{\Delta V_{\perp}}{V} = \frac{2Gm}{bV^2}$$

Setting V = c, see that this is exactly half the correct relativistic value for massless particles (e.g. photons).



If the star receives many independent deflections, each with a random direction, expected value of the perpendicular velocity after time t is obtained by summing the *squares* of the individual velocity kicks:

$$\langle \Delta V_{\perp}^2 \rangle = \Delta V_1^2 + \Delta V_2^2 + \Delta V_3^2 + \ldots$$

Writing this as an integral (i.e. assuming that there are very many kicks):

$$\langle \Delta V_{\perp}^2 \rangle = \int_{b_{\min}}^{b_{\max}} \left(\frac{2Gm}{bV} \right)^2 dN$$

Where dN is the expected number of encounters that occur in time t between impact parameter b and b + db.

Using identical reasoning as for the strong encounter case:

$$dN = n \times Vt \times 2\pi b\,db$$

Number density of perturbing stars

Distance star travels in time t

Area of the annulus between impact parameter b and b + db

This gives for the expected velocity:

$$\left\langle \Delta V_{\perp}^{2} \right\rangle = \int_{b_{\min}}^{b_{\max}} nVt \left(\frac{2Gm}{bV} \right)^{2} 2\pi b\,db$$

$$= \frac{8\pi G^{2} m^{2} nt}{V} \int_{b_{\min}}^{b_{\max}} \frac{db}{b}$$

$$= \frac{8\pi G^{2} m^{2} nt}{V} \ln\left(\frac{b_{\max}}{b_{\min}} \right)$$

Logarithm means in a uniform density stellar system, `encounters' with stars at distances (b -> 10b) and (10b -> 100b) etc contribute equally to the deflection.

Relaxation time

After a long enough time, the star's perpendicular speed will (on average) grow to equal its original speed. Define this as the *relaxation time* - time required for the star to lose all memory of its initial orbit.

Set: $$V^2 = \langle \Delta V_\perp^2 \rangle = \frac{8\pi G^2 m^2 n t_{\text{relax}}}{V} \ln\left[\frac{b_{\max}}{b_{\min}}\right]$$

…and solve for t_{relax}: $$t_{\text{relax}} = \frac{V^3}{8\pi G^2 m^2 n \ln[b_{\max}/b_{\min}]}$$

Recall that the **strong** encounter time scale was: $$t_s = \frac{V^3}{4\pi G^2 m^2 n}$$

$$\Rightarrow \quad t_{\text{relax}} = \frac{t_s}{2\ln[b_{\max}/b_{\min}]}$$

Frequent distant interactions are more effective at changing the orbit than rare close encounters…