

ASTR 3730: Astrophysics 1 – Problem Set #1 Solutions

- 1) (a) Neutral hydrogen emits radio waves with a wavelength of 21cm. What is the frequency of this radiation?

Use formula: $c = \lambda \nu$. This gives $\nu = 1.43 \times 10^9 \text{ Hz} = 1430 \text{ MHz}$.

- (b) What is the wavelength (in nm) of an X-ray photon with an energy of 6.4 keV?

Use $E = h\nu = hc / \lambda$. Photon energy is $6400 \text{ eV} = 6400 \times 1.6 \times 10^{-12} \text{ erg} = 1.02 \times 10^{-8} \text{ erg}$. This gives $\lambda = 1.9 \times 10^{-8} \text{ cm}$ or 0.19 nm .

- 2) (a) A binary star system in the star forming region of Taurus (at a distance of 150 pc) has a separation between the two stars of 50 au. Can we resolve the binary (i.e. detect the two stars individually) using a ground-based telescope with a resolution of 1 arcsecond?

No. The angular separation of the binary is $\theta = (50 / 150) \text{ arcsec} = 0.333 \text{ arcsec}$ (note that from the definition of the parsec a binary with a separation of $x \text{ au}$ has an angular separation of 1 arcsecond when it is at a distance of $x \text{ pc}$).

- (b) Improved observations show that what was previously thought to be a single 16th magnitude star is actually a close binary consisting of two identical stars. What is the magnitude of each individual star?

Let the flux of the binary be F_{binary} , magnitude m_{binary} . Relationship between flux and magnitude is: $m_{\text{binary}} = -2.5 \log F_{\text{binary}} + \text{constant}$. Let the flux of each star on its own be F_* , with magnitude m_* . If the stars are identical, then $F_* = 0.5 F_{\text{binary}}$. Then:

$$m_{\text{binary}} = -2.5 \log F_{\text{binary}} + \text{constant}$$

$$m_* = -2.5 \log 0.5 F_{\text{binary}} + \text{constant}$$

...note constants are same in both equations

$$\begin{aligned} m_* - m_{\text{binary}} &= -2.5 \log 0.5 F_{\text{binary}} + 2.5 \log F_{\text{binary}} \\ &= -2.5 \log 0.5 - 2.5 \log F_{\text{binary}} + 2.5 \log F_{\text{binary}} \\ &= -2.5 \log 0.5 \end{aligned}$$

$$m_* = m_{\text{binary}} - 2.5 \log 0.5 = 16 + 0.75 = 16.75$$

(recall here $\log (AB) = \log A + \log B$).

- 3) An Active Galaxy (i.e. a supermassive black hole accreting gas from a surrounding galaxy) at a distance of 1 Gpc produces a luminosity in X-rays of $L_x = 10^{41} \text{ erg s}^{-1}$.

(a) What is the X-ray flux at Earth from this source?

L_x is the *total luminosity* of the source across the whole X-ray waveband (i.e. same units as bolometric luminosity). Therefore can use the simple formula for relation between the luminosity and the flux:

$$F_x = \frac{L_x}{4\pi d^2} = \frac{10^{41} \text{ erg s}^{-1}}{4\pi (10^9 \times 3.086 \times 10^{18} \text{ cm})^2} = 8.36 \times 10^{-16} \text{ erg s}^{-1} \text{ cm}^{-2}$$

The *Chandra* X-ray observatory observed the source for 10^6 s (one of the longest exposures made with *Chandra*). Assuming for simplicity that **all** the X-ray photons from the source have an energy of 5 keV, how many photons does *Chandra* collect during the exposure? (You will need to know that the effective area of the *Chandra* telescope for detecting 5 keV photons is 400 cm^2 .)

F_x is the amount of energy that passes through an area of 1 cm^2 at Earth in 1 second. *Chandra* has a collecting area of 400 cm^2 , and collects photons for 10^6 s. The amount of energy collected by the telescope is therefore:

$$\Delta E_x = F_x \times 400 \text{ cm}^2 \times 10^6 \text{ s} = 3.34 \times 10^7 \text{ erg.}$$

One 5 keV photon has an energy (cf question 1) of $8.0 \times 10^{-9} \text{ erg}$. Therefore the number of X-ray photons collected will be:

$$N_x = \frac{\Delta E_x}{E_{5 \text{ keV}}} = 42.$$

Note: to be more precise, this is the **average** number of photons one would expect to collect. With such a small number, the random fluctuations (given by the square root of N_x) are quite large – one exposure would typically give between 35 and 49 photons. Another way to look at this is to note that you can't measure the flux of a very faint source very well – in this case to better than about 15%.