Stellar Evolution

**Main-sequence evolution**: star burns H in core, core composition slowly changes from H to He. Small changes in the external properties (L, T_e, R)

Main-sequence lifetime is strongly mass-dependent, more massive stars:
- sustain higher core temperatures
- have higher rates of nuclear fusion
- are more luminous and exhaust H fuel more quickly

\[ L \propto M^{3.5} \quad t_{ms} \propto M^{-2.5} \]

Star leaves the main sequence when it stops burning hydrogen in the core. Normally leads to *expansion* of the envelope, and the formation of a giant. Depending upon mass, final outcome is a white dwarf, a neutron star, or a black hole.
Overview

Initial stellar mass in Solar masses

- Brown dwarfs
- Main sequence hydrogen burning
- Deuterium burning
- C, O, Si etc.
- He burning
- Supernova explosions
- Carbon / oxygen white dwarfs
- Helium white dwarfs
- Neutron stars
- Black holes

Time / yr

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Evolution of brown dwarfs

Consider a protostar with thermal energy $U$ and gravitational potential energy $W$. Virial theorem:

$$0 = 2U + W$$

(ideal gas, with $g = 5/3$)

Suppose star slowly contracts by an amount $\Delta R$ (negative):

$$\Delta U = \frac{1}{2} \Delta W$$

Also have:

$$W = k \frac{GM^2}{R} \Rightarrow \Delta W = k \frac{GM^2}{R^2} \Delta R$$

At fixed mass, $\Delta U \frac{\Delta R}{R^2} > 0$ (since $\Delta R$ is negative)

As a star contracts, it heats up.
Note: this property is counter-intuitive - a star contracts because it has *lost* (radiated away) energy, and gets *hotter* as a result. Self-gravitating systems have **negative specific heat capacity**.

Ensures stability of nuclear burning in stars:

Energy generation rate per unit mass:  

\[ q \, a \, T^b \]

…where b - the temperature sensitivity of the nuclear reaction rate, is *at least* +4 and sometimes +20 or more.

Suppose temperature in the core momentarily drops, reducing the nuclear reaction rate:

- pressure drops
- core of the star contracts
- heats up due to release of gravitational potential energy
- restores the reaction rate to equilibrium value
For brown dwarfs below the hydrogen burning limit (0.08 $M_{\text{sun}}$), core becomes degenerate before it gets hot enough for hydrogen fusion. Degeneracy pressure prevents further contraction, so star just cools off as further energy is radiated:

Figure from Burrows et al.
Luminosity evolution of stars, brown dwarfs, and planets

Burrows et al. 2001
Evolution of stars

Look at HR diagrams of clusters where all the stars have (roughly) similar ages:
Theoretical evolutionary tracks

- $20 \ M_{\text{sun}}$
- $7 \ M_{\text{sun}}$
- $1 \ M_{\text{sun}}$

Core H burning
Core He burning
Constant radius
Evolution of the core is controlled by the need for increasingly higher temperatures for nuclear burning of heavier elements:

- Initially, burn hydrogen in the core
- Once hydrogen is exhausted, too cool to burn helium
- Core contracts, heats up
- Helium burning stars
- If star is massive enough, sequence repeats for carbon burning, then oxygen, silicon etc…

Dominant **observational** signature of post-main-sequence evolution is rapid expansion of the envelope to form a red giant star.
Why do stars become red giants?

No simple and fully accepted explanation of this phenomenon. Good plausibility argument:

Suppose the core contraction at the end of hydrogen burning occurs on a timescale shorter than the Kelvin-Helmholtz time of the whole star. Then:

\[
\text{Energy conservation:} \quad \mathcal{E} + U = \text{constant}
\]

\[
\text{Virial theorem:} \quad \mathcal{E} + 2U = \text{constant}
\]

…must both hold. Only possible if \(\mathcal{E}\) and \(U\) are conserved separately.
Structure of the star:
• core, radius $R_c$, mass $M_c$
• envelope, radius $R$ (total stellar radius), mass $M_{env}$

For $M_c >> M_{env}$:

$$0 = \frac{GM_c^2}{R_c^2} \frac{dR_c}{dt} + \frac{GM_c M_{env}}{R} \frac{dR}{dt}$$

Now assume the division between core and envelope is fixed, and differentiate with respect to time:

$$\frac{dR}{dR_c} = \frac{M_c}{M_{env}} \frac{R}{R_c^2}$$

Envelope expands as the core contracts.
For stars with masses between $0.5 \, M_{\text{sun}}$ and $8 \, M_{\text{sun}}$, core burning ends with the core a mixture of carbon and oxygen. Envelope is now very large, and weakly bound to the core. Final stages of burning generate a lot of luminosity, which acts to blow away the envelope:

Exposed core cools to become a degenerate **white dwarf**.