Detecting extrasolar planetary systems

Nearest star forming region to the Sun is TW Hydrae association at $d \sim 50$ pc

Recall 1 au subtends an angle of 1 arc sec at $d = 1$ pc

Jupiter’s orbit ($a = 5$ au) has an angular radius of $\sim 0.1$ arc sec at $d = 50$ pc

Can’t directly image planets in the process of forming within protoplanetary disks

Consider planet, radius $R_p$, at distance from star $a$. Fraction of starlight intercepted:

$$f = \frac{R_p^2}{4a^2}$$

Earth: $R_p = 6.4 \times 10^8$ cm, $a = 1.5 \times 10^{13}$ cm, $f = 5 \times 10^{-10}$

Jupiter: $R_p = 7.1 \times 10^9$ cm, $a = 7.8 \times 10^{13}$ cm, $f = 2 \times 10^{-9}$
If the planet reflected all the intercepted starlight, then the magnitude difference between the planet and the star would be:

\[ m = 2.5 \log F + \text{constant} \]

\[ m_p - m_* = 2.5 \log F_p + 2.5 \log F_* \]

\[ m_p - m_* = 2.5 \log \left( \frac{F_p}{F_*} \right) \]

Find \( m \sim 22 \) magnitudes for Jupiter, 23 magnitudes for Earth. Implied apparent magnitude is not too bad, e.g. for Earth orbiting a 5th magnitude G star (Solar type) would need to reach \( V \sim 27 \) - requires 5800 s of exposure using HST’s Advanced Camera for Surveys.
Contrast between star and planet is the main problem

e.g. *brightest* stars in this image are around 18\textsuperscript{th} magnitude

No direct detections of extrasolar planets...
Indirect detection of extrasolar planets

Radial velocity method

Star - planet system is a special case of a spectroscopic binary where:
- Only observe the radial velocity of one component
- Very large ratio of masses between components

For circular orbits, as previously:

\[
\frac{v_\ast}{v_p} = \frac{m_p}{M_\ast}
\]

Kepler’s laws give:

\[
v_p = \sqrt{\frac{G(M_\ast + m_p)}{a}} \sqrt{\frac{GM_\ast}{a}}
\]

…for star-planet separation a
Velocity of the star due to presence of orbiting planet is:

\[ v_* = \frac{m_p}{M_*} \sqrt{\frac{GM_*}{a}} \]

How large is this velocity?

**Jupiter:** \( m_p = 10^{-3} \, M_{\text{sun}} \), \( a = 7.8 \times 10^{13} \, \text{cm} \), \( M_{\text{sun}} = 2 \times 10^{33} \, \text{g} \)

\[ v_* \geq 1.3 \times 10^3 \, \text{cm} \, \text{s}^{-1} = 13 \, \text{m} \, \text{s}^{-1} \]

**Earth:** \( m_p = 3 \times 10^{-6} \, M_{\text{sun}} \), \( a = 1.5 \times 10^{13} \, \text{cm} \)

\[ v_* \geq 10 \, \text{cm} \, \text{s}^{-1} \]

Currently the best observations measure the stellar velocity to a precision of \( \sim 3 \, \text{m} \, \text{s}^{-1} \) (ultimate limit is not known, but is though to be \( \sim 1 \, \text{m} \, \text{s}^{-1} \)). Can detect extrasolar `Jupiters’, but not Earths.
Radial velocity observables:

For a planet on a circular orbit, observables are the period and the amplitude of the radial velocity signal of the star:

Amplitude of the signal equals the velocity of the star around its orbit **only** if the inclination is 90 degrees (edge-on). Generally:

$$v_{obs} = v_* \sin i$$

Detect a smaller signal if the system is not edge-on, no signal at all if the system is face-on.
Planet masses from radial velocity measurements:

\[
v_{\text{obs}} = v_* \sin i = \frac{m_p}{M_*} \sqrt{\frac{GM_*}{a}} \sin i
\]

\[
m_p = v_{\text{obs}} \frac{M_* \sqrt{a}}{GM_*} \frac{1}{\sin i}
\]

Inclination factor is unknown unless we see eclipses (transits)

Observed amplitude of stellar `wobble' - measure this directly

Don’t know stellar mass, but can get a good estimate from high signal to noise spectrum

Derive this from \( M_* \) and period \( P \):

\[
P = 2\pi \sqrt{\frac{a^3}{GM_*}}
\]
Radial velocity observables are:

- **Orbital period** (or semi-major axis)
- $m_p \sin (i)$ - i.e. a lower limit to the true planet mass
- **Eccentricity of the orbit**

Selection effects:

$$v_* = \frac{m_p}{M_*} \sqrt{\frac{GM_*}{a}}$$

For fixed observational sensitivity, the minimum detectable planet mass scales as the square root of the orbital radius:

- Detectable
- Undetectable

Also fail to detect very long period planets - won’t (yet) have seen a complete orbit

Method favors detection of massive planets at small radii from the star…
Astrometry

Conceptually identical to radial velocity method - look for motion of star in the plane of the sky due to orbiting planet. Star moves in a circle radius \( a_* \), where:

\[
a_* M_* = a \, m_p
\]

(definition of center of mass)

Angular size of the circle on the sky:

\[
\theta = \frac{a_*}{d}
\]

Units: if the star-planet separation \( a \) is in astronomical units, and \( d \) is in parsecs, then angle \( \theta \) is in arcseconds.
e.g. for Jupiter mass planets:

\[
\begin{array}{c}
0.5 \quad m_p \\
m_{Jupiter} \\
5 \text{ au} \\
10 \text{ pc} \\
\text{mas}
\end{array}
\]

Not achieved yet, but promising as fundamental limits do not preclude detection of even Earth-mass planets.

Sensitivity is complementary to radial velocity searches:

Easiest to find massive planets at large orbital radius, as long as the period isn’t much longer than the duration of the search.
Transits

Look for the periodic dimming of the star as the planet (seen from Earth) transits across the stellar disk.

Requires a favorable inclination (almost edge-on)

Probability of seeing transits: \[ P = \sin \theta \frac{R_\star}{a} \]

At 1 au around Sun: P = 0.5%, at 5 au: P = 0.1%

Need to continuously monitor large number of stars
Giant planets all have very similar radii $\sim 0.1 \, R_{\text{sun}}$

Signal for Jupiter is $\sim 1\%$ drop in stellar flux during transit

For Earth, radius is $\sim 0.01 \, R_{\text{sun}}$

Signal for Earth is drop in flux by 1 part in $10^4$

Photometry accurate to $\sim 0.1\%$ is possible (very difficult) from the ground, accuracy at $10^{-5}$ level thought achievable in space.
NASA has approved a Discovery class mission to search for transits by terrestrial planets:

- monitor $10^5$ stars for 4 years
- 0.95 m telescope
- designed for a photometric precision of $10^{-5}$
- planned launch in Fall 2007

Built by Ball aerospace

If all stars (in some mass range) have on average 2 planets with $R = R_{\text{earth}}$ orbiting between 0.5 au and 1.5 au, ~50 will be detected. Likely to provide first clue as to how common habitable planets are.

Possible that gravitational lensing (even less direct method, discussed next semester) will find Earth-mass planets first.