Convection

When the radial flux of energy is carried by radiation, we derived an expression for the temperature gradient:

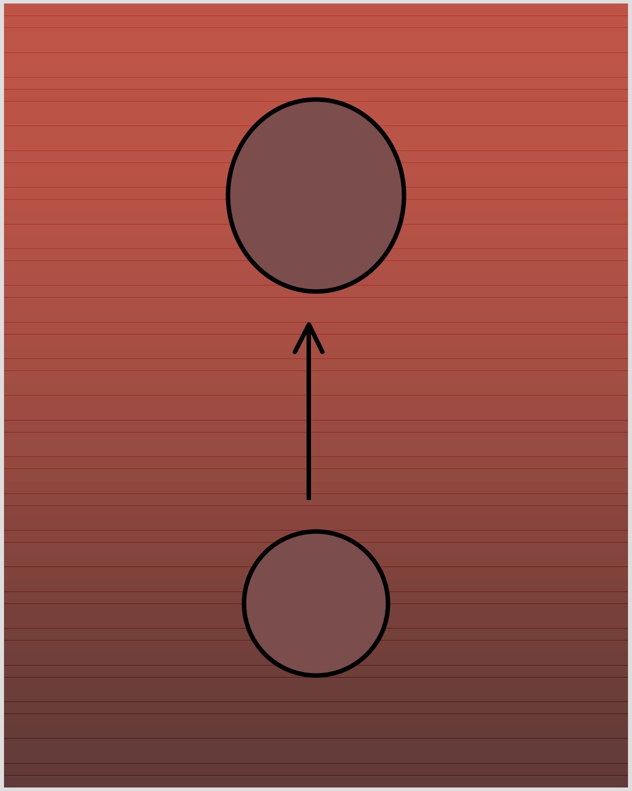
$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}$$

Large luminosity and / or a large opacity κ implies a large (negative) value of dT / dr.

For an ideal gas, the energy density (energy per unit volume) is given by: $\frac{3}{2}nkT$ …with n the number density of particles.

Hot gas near the center of the star has higher energy density than cooler gas above - if we could `swap' the gas over we could transport energy outward… especially if dT / dr is large.

Schwarzschild criteria for convective instability



Imagine displacing a small mass element vertically upward by a distance dr. Assume that **no heat** is exchanged with the surrounding, i.e. the process is **adiabatic**:

- Element expands to stay in pressure balance with the new environment
- New density will *not* generally equal the ambient density at the new location

	<u>Initially</u>	<u>After displacement</u>
Surroundings	$\rho(r)$	$\rho(r + dr)$
Element	$\rho(r)$	ρ^*

If the gas in the element behaves adiabatically, then for an ideal gas:

$$\frac{dP}{P} = \gamma \frac{d\rho}{\rho} \qquad \text{...where } \gamma = 5/3$$

New density of the element is given by:

$$\rho^* = \rho(r) + d\rho = \rho(r) + \frac{1}{\gamma}\frac{\rho}{P}dP = \rho(r) + \frac{1}{\gamma}\frac{\rho}{P}\left(\frac{dP}{dr}\right)dr$$

If:

- $\rho^* > \rho$(r+dr) then the displaced element will be denser than the surroundings and will settle back down. This is the **stable case**.
- $\rho^* < \rho$(r+dr) buoyancy will cause the element to rise even further. **Convective instability**.

Mathematically, the **stable** case is:

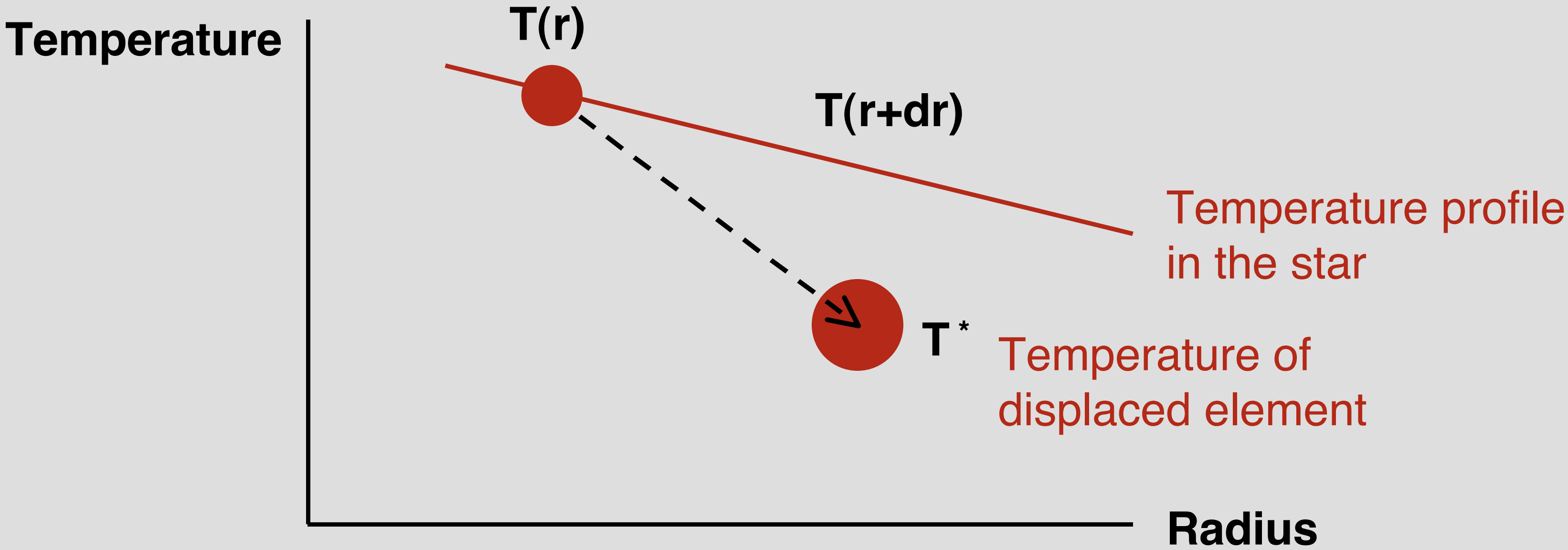
$$\rho^{*} > \rho(r + dr)$$

$$\rho(r) + \frac{1}{\gamma}\frac{\rho}{P}\left(\frac{dP}{dr}\right)dr > \rho(r) + \left(\frac{d\rho}{dr}\right)dr$$

$$\frac{1}{\gamma}\frac{\rho}{P}\left(\frac{dP}{dr}\right) > \left(\frac{d\rho}{dr}\right)$$

This is all the physics that goes into deriving the criteria for convective stability or instability.

Can express same physics in terms of the temperature gradient instead. Graphically, the stable case looks like:



Recall for an ideal gas: $P = \frac{R}{\mu} \rho T$

If the temperature of the displaced element is lower, then the density must be **higher** if the element is in pressure balance. A denser element will fall back - i.e. stability.

Stability condition is therefore:

Temperature gradient in the star

$$\left| \left(\frac{dT}{dr} \right)_{star} \right| < \left| \left(\frac{dT}{dr} \right)_{adiabatic} \right|$$

Temperature gradient when an element is moved adiabatically

Can write this in various ways (see Section 6.5 of the textbook), but important physical point is:

Too steep a temperature gradient leads to the onset of convection in stars

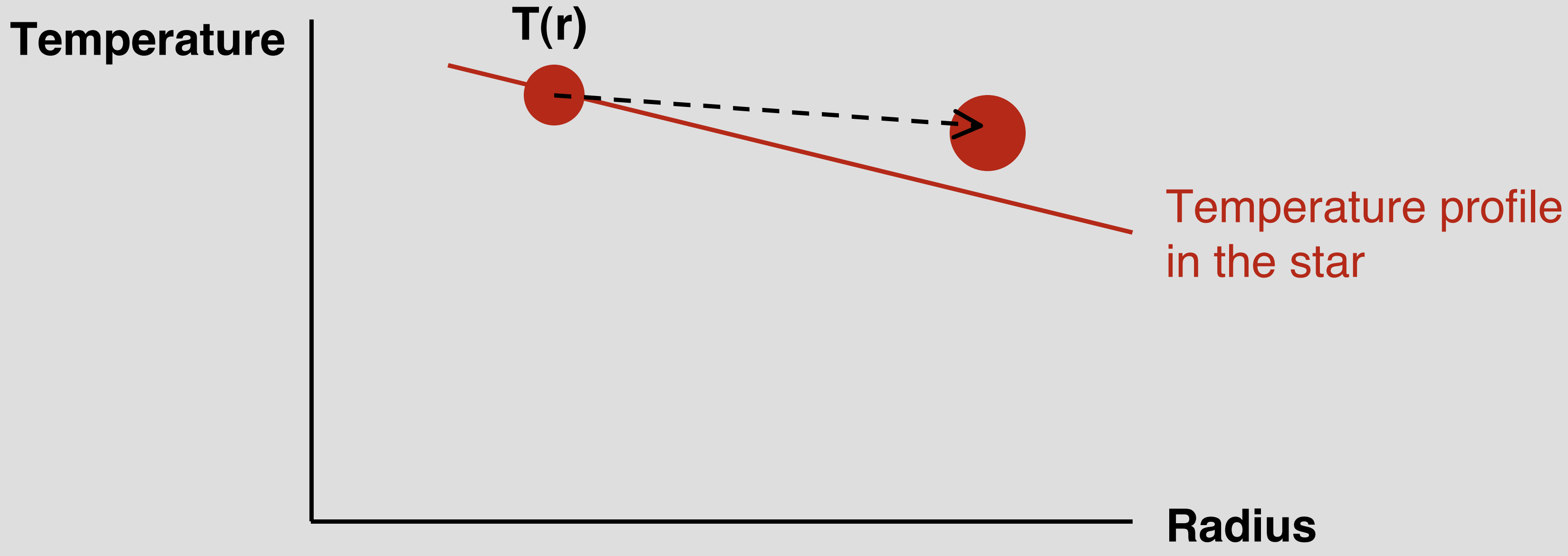
Since a steep gradient is caused by a large luminosity, can convert this into an expression for the **maximum** luminosity that can be transported radiatively:

$$L_{max} \propto \frac{1}{\kappa} \left(1 - \frac{1}{\gamma} \right)$$

omitting lots of factors but keeping the important dependencies on opacity and adiabatic exponent

Larger luminosities lead to convection.

Once convection sets in, motion of the fluid will act as an additional source of energy transport:



In unstable case, displaced element is **hotter** than its surroundings:

- Relax the assumption of adiabaticity
- Radiation will leak out of hot rising bubble of fluid
- Energy flux from hot to cool regions in the star

Recall that the Kelvin-Helmholtz time for a star was very long (of the order of Myr), whereas the dynamical time was short (less than an hour).

Means that even if the convective motions are very slow compared to dynamical (as they are) - convection is still a very efficient way to move energy.

→ **In stars where convection occurs, the temperature gradient is driven to very close to the neutrally stable value - the adiabatic temperature gradient.**

Which stars are convectively unstable?

Low mass stars

$$L_{max} \propto \left(\frac{1}{\kappa}\right)\left(1 - \frac{1}{\gamma}\right)$$

Near the surface, opacity is large (atomic processes) and $\gamma < 5/3$ due to ionization. Leads to **surface convection zones**.

High mass stars

Luminosity of stars increases very rapidly with increasing stellar mass: $L \sim M^4$ for stars of around a Solar mass.

All this energy is generated very close to the core of the star. Can exceed the critical value - **core convection**.

Pre-main-sequence stars

Fully convective due to the large dissipation of gravitational potential energy as they contract.

Regions of convection in main sequence stars