Nuclear processes in stars

Mass of nuclei with several protons and / or neutrons does not exactly equal mass of the constituents - slightly smaller because of the **binding energy** of the nucleus.

Since binding energy differs for different nuclei, can release or absorb energy when nuclei either fuse or fission.

**Example:**

\[ ^4_1 \text{H} + ^4_2 \text{He} \]

4 protons, each of mass 1.0081 atomic mass units: 4.0324 amu

mass of helium nucleus: 4.0039 amu

**Mass difference:** 0.0285 amu = 4.7 x 10^{-26} g

\[ \boxed{E} = \boxed{Mc^2} = 4.3 \boxed{10^{15}} \text{ erg} = 27 \text{ MeV} \]
General calculation: define the binding energy of a nucleus as the energy required to break it up into constituent protons and neutrons.

Suppose nucleus has:
- Proton number $Z$
- Atomic mass number $A$ (number of protons + neutrons)
- Mass $M_{nuc}$

**Binding energy:** $E_B = \left[ (A - Z)m_n + Zm_p - M_{nuc} \right]c^2$

Most useful quantity for considering which nuclear reactions yield energy is the binding energy *per nucleon* - defined as:

$$f = \frac{E_B}{A}$$
Plot of $f$ vs $A$ largely determines which elements can be formed in different stars/

Most bound nucleus is iron 56: $A < 56$ fusion releases energy, $A > 56$ fusion requires energy.

Yield for fusion of hydrogen to $^{56}\text{Fe}$: $\sim 8.5$ MeV per nucleon
Most of this is already obtained in forming helium (6.6 MeV)

Drawn curve as smooth - actually fluctuates for small $A$ - He is more tightly bound than `expected’.
Energetics of fusion reactions

Nuclei are positively charged - repel each other.
If charges on the nuclei are $Z_1 e$ and $Z_2 e$, then at distance $d$ the electrostatic energy is:

$$E = \frac{Z_1 Z_2 e^2}{d}$$

If the nuclei approach sufficiently closely, short range nuclear forces (attractive) dominate and allow fusion to take place.

Nuclear material has roughly constant density, so `close enough’ means within a distance:

$$r_0 \leq 1.44 \times 10^{-13} A^{1/3} \text{ cm}$$

atomic mass number
Schematically:

At $r = r_0$, height of the Coulomb barrier is:

$$E = \frac{Z_1Z_2e^2}{r_0} \sim Z_1Z_2 \text{ MeV}$$

i.e. of the order of 1 MeV for two protons…
For Solar core conditions $T = 1.5 \times 10^7$ K
Thermal energy of particles $= kT = 1300$ eV $= 10^{-3}$ MeV

Classically, there are zero particles in a thermal distribution with enough energy to surmount the Coulomb barrier and fuse.

Quantum mechanically, lower energy particles have a very small but non-zero probability of tunnelling through the barrier:

Probability of finding particle $\sim |Y|^2$ - if barrier is not too wide then non-zero wavefunction allows some probability of tunnelling...
Probability of tunnelling depends upon the energy of the particles, their mass, and the charge:

\[ P \propto E^{1/2} e^{-Z_1 Z_2 e^2 / \hbar E^{1/2}} \]

- \( P \) increases rapidly with \( E \)
- \( P \) decreases with \( Z_1 Z_2 \) - lightest nuclei can fuse more easily than heavy ones
- Higher energies / temperatures needed to fuse heavier nuclei, so different nuclei burn in well-separated phases during stellar evolution.
Competition: most energetic nuclei most likely to fuse, but very few of them in a thermal distribution of particle speeds:

Narrow range of energies around the Gamov peak where significant numbers of particles in the plasma are able to fuse. Energy is $\gg$ typical thermal energy, so fusion is slow.
Nuclear reactions in the Sun

Almost all reactions involve collisions of only two nuclei. So making helium from four protons involves a sequence of steps. In the Sun, this sequence is called the proton-proton chain:

Step 1  \[ ^{1}\text{H} + ^{1}\text{H} \rightarrow ^{2}\text{H} + \text{e}^+ + \nu_e \]

deuteron - one proton + one neutron  electron neutrino  positron

This is the critical reaction in the proton-proton chain. It is slow because forming a deuteron from two protons requires transforming a proton into a neutron - this involves the weak nuclear force so it is slow…
Beyond this point, several possibilities. Simplest:

**Step 2** \[ ^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \square \]

**Step 3** \[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H} \]

Results of this chain of reactions:
- Form one \(^4\text{He}\) nucleus from 4 protons
- Inject energy into the gas via energetic particles: one positron, one photon, two protons
- Produce one electron neutrino, which will escape the star without being absorbed.

Energy yield is \(~10^{-5}\) erg per proton, so \(~4 \times 10^{38}\) reactions per second needed to yield \(L_{\text{sun}}\). About 0.65 billion tonnes of hydrogen fusing per second.