Stellar masses

Possible ways to measure M:

Stellar spectrum: details of the spectrum depend upon the surface gravity:

$$g = \frac{GM}{R^2}$$

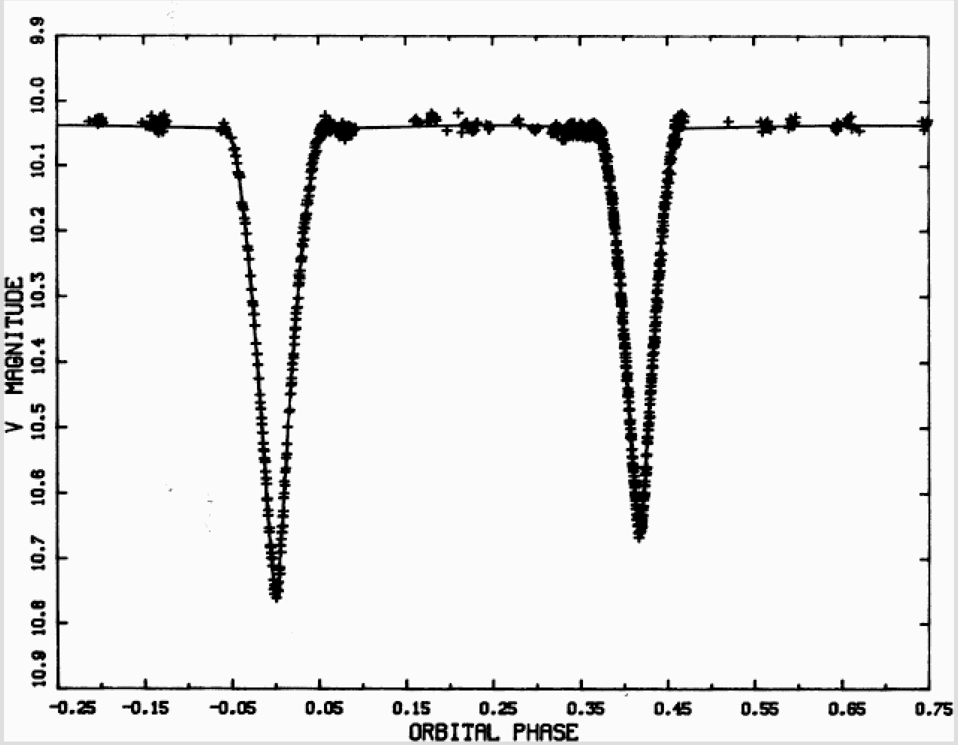
…e.g. at fixed R the density will vary with M, and density determines amount of collisional broadening of the lines. In practice, g and M are not well enough determined for this to work well.

Binary stars: main source of stellar masses. Binary stars are common - most useful binaries for mass determinations are ones where we see motion of both stars.

Types of binary star

c.f. *Carroll*, Chapter 7

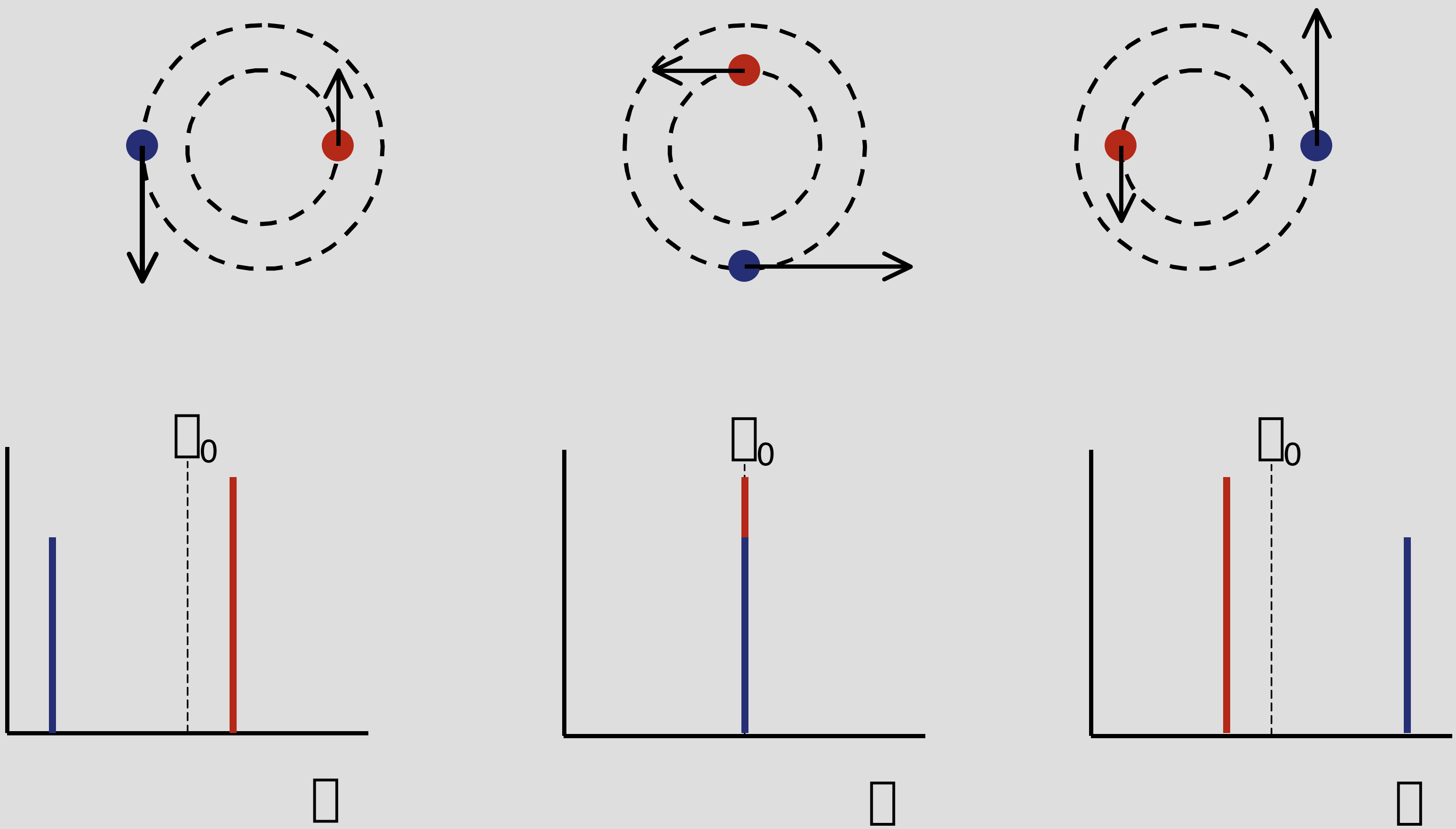
Visual binary: can resolve each of the stars in the binary individually. If close enough, can see motion around the orbits.



Eclipsing binary: binary where the orbital plane is almost perpendicular to the line of sight to the observer.

YY Sagittarii from Lacy (1993)

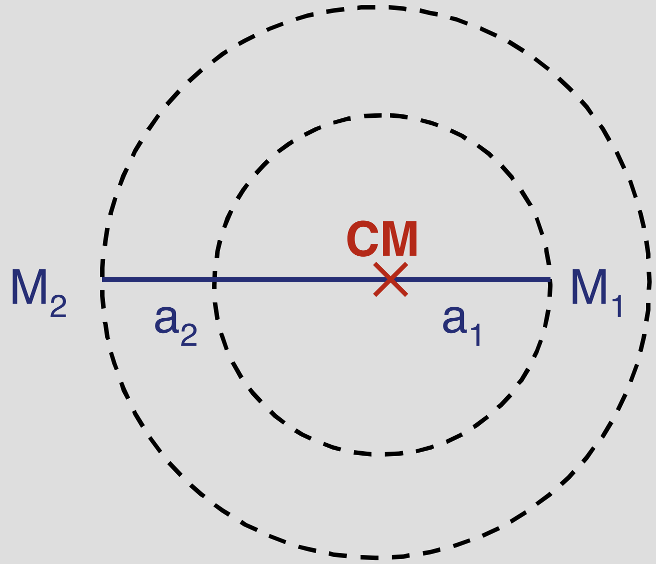
Spectroscopic binary: see periodic Doppler shifts in the positions of spectral lines from one or both components in the binary.



Measure P, line of sight v for each star.

Kepler's Law: consider for simplicity circular orbits

Stars mass: M_1 and M_2, orbital radii a_1 and a_2



In orbit around the center of mass (CM) of the system

From definition of center of mass: $M_1 a_1 = M_2 a_2$

Let total separation: $a = a_1 + a_2$

Then:

$$a_2 = \frac{M_1}{M_1 + M_2} a$$

Apply Newton's law of gravity and condition for circular motion to M_2:

$$\frac{GM_1 M_2}{a^2} = M_2 a_2 \omega^2$$

ω is angular velocity of the binary

Substitute for a_2:

$$\omega = \sqrt{\frac{G(M_1 + M_2)}{a^3}}$$

$$P = \frac{2\pi}{\omega}$$

Visual binary: see each orbit so know immediately a_2 / a_1:

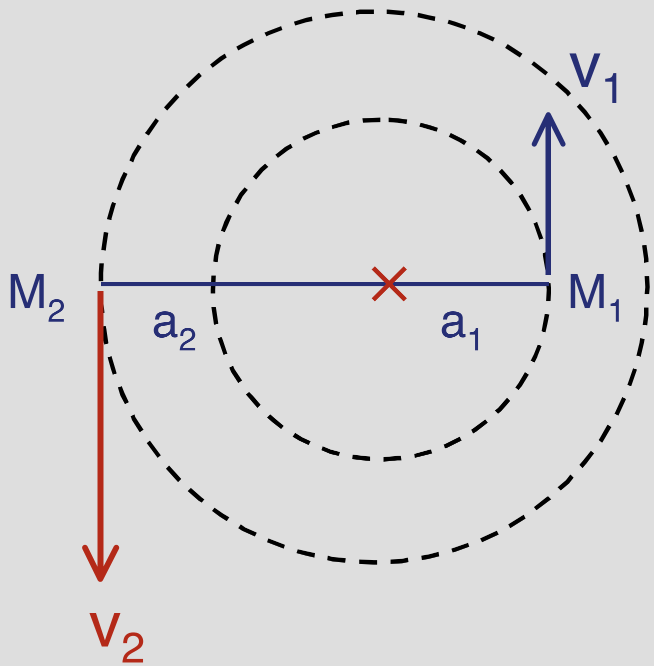
➡ determines *ratio* of masses M_1 / M_2

If we know distance, then angular separation + d gives a, which with period P determines *sum* of masses $M_1 + M_2$

Enough information to get both M_1 and M_2…

Spectroscopic binaries

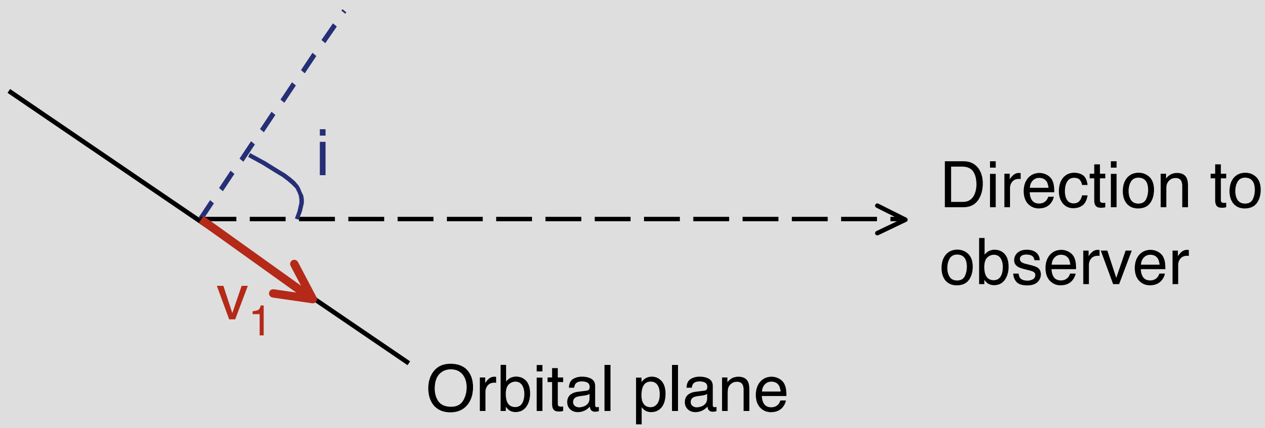
Consider spectroscopic binaries with circular orbits - often a good approximation because tides in close binaries tend to circularize the orbits.



Velocities are constant around the orbit:

$$Pv_1 = 2\pi a_1$$
$$Pv_2 = 2\pi a_2$$

But… don't observe v_1 and v_2 - only the component of those velocities along our line of sight:



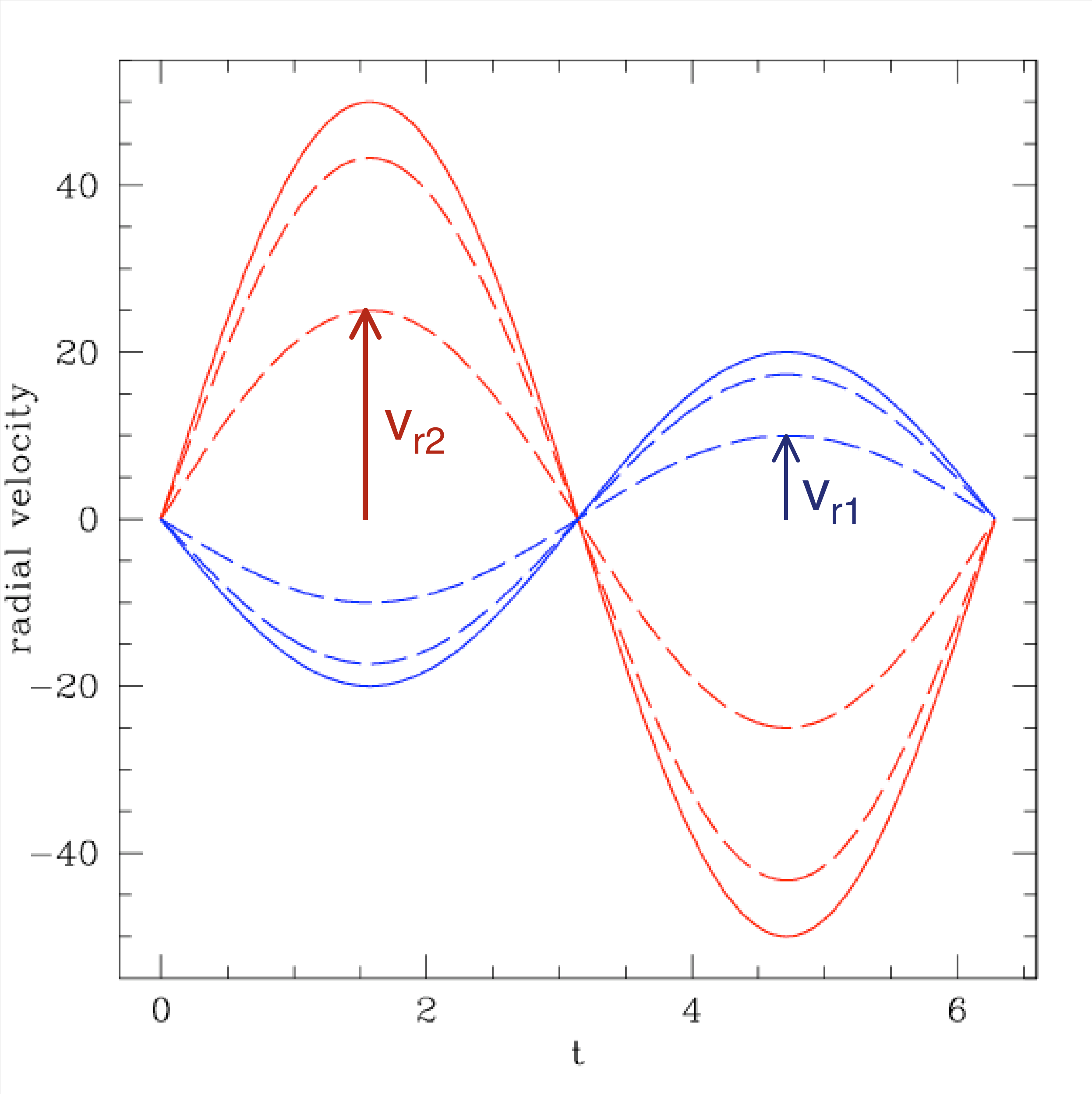
Maximum component of velocity along the line of sight is:

$$v_{r1} = v_1 \cos(90 - i) = v_1 \sin i$$

$$v_{r2} = v_2 \sin i$$

Radial velocities are the observables

i is the inclination angle of the binary system



Example for:

- i = 90°
- i = 60°
- i = 30°

Ratio of maximum observed radial velocities is:

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2 \sin i}{v_1 \sin i} = \frac{2\pi a_2 / P}{2\pi a_1 / P} = \frac{a_2}{a_1} = \frac{M_1}{M_2}$$

Ratio of masses can be found if we see spectral lines from both stars (a `double-lined' spectroscopic binary), without knowing the inclination.

To find the sum of the masses, note: $a = a_1 + a_2 = \frac{P}{2\pi}\left(v_1 + v_2\right)$

Use Kepler's law again: $P^2 = \frac{4\pi^2 a^3}{G\left(M_1 + M_2\right)} = \frac{P^3\left(v_1 + v_2\right)^3}{2\pi G\left(M_1 + M_2\right)}$

$$M_1 + M_2 = \frac{P}{2\pi G}\left(v_1 + v_2\right)^3$$

Replace v_1 and v_2 with the observable radial velocities:

$$M_1 + M_2 = \frac{P}{2\pi G} \frac{(v_{r1} + v_{r2})^3}{\sin^3 i}$$

So… can determine sum of masses (and hence the individual masses M_1 and M_2) **only** if the inclination can be determined.

Requires that the stars are also eclipsing:

- Detailed shape of lightcurve gives i
- Obviously must be close to $i = 90^o$ to see eclipses!

Rare binaries are main source of information on stellar masses…