2. Stellar Physics

Define a **star** as an object that is:

- **Bound by self-gravity**
- **Radiates energy that is *primarily* released by nuclear fusion reactions in the stellar interior**

Other energy sources are dominant during star formation and stellar death:

- **Star formation** - before the interior is hot enough for significant fusion, gravitational potential energy is radiated as the radius of the forming star contracts. *Protostellar* or *pre-main-sequence* evolution.
- **Stellar death** - remnants of stars (white dwarfs and neutron stars) radiate stored thermal energy and slowly cool down. Sometimes refer to these objects as stars but more frequently as *stellar remnants.*

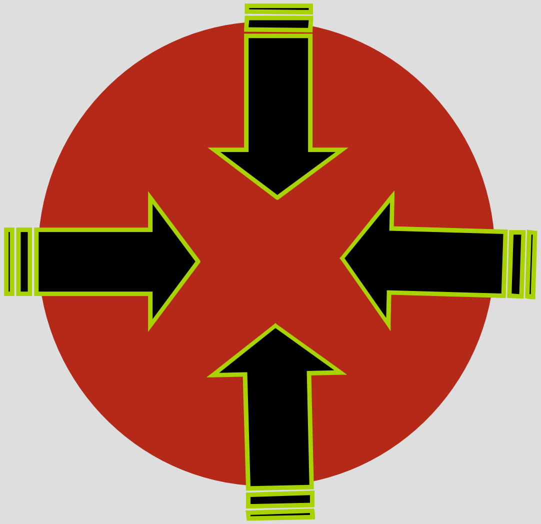
With this definition:

- **planets** are not stars - no nuclear fusion
- objects in which release of gravitational potential energy is always greater than fusion are not stars either - these are called **brown dwarfs**

Distinction between brown dwarfs and planets is less clear, most people reserve `planet' to mean very low mass bodies **in orbit** around a star.

Irrespective of what we call them, physics of stars, planets, stellar remnants is similar. Balance between:

- **Gravity**
- **Pressure**



Basic assumptions

c.f. textbook Section 1.3

Problem of stellar structure is simplified by making several reasonable assumptions, which hold in most (not all) cases.

1) Spherical symmetry

An isolated, non-rotating star which does not contain strong magnetic fields will be spherically symmetric, i.e.:

> **All** quantities (e.g. density, temperature, pressure) depend only on the distance from the center of the star - radius r.

Not a general property of all self-gravitating systems - e.g. an elliptical galaxy remains elliptical because interactions between stars are rare.

Sun is rotating and has sunspots - evidence of magnetic fields. Is it OK to ignore these?

- **Rotation**

Gravitational potential at distance r from a point mass m is:

$$\Phi = -\frac{Gm}{r}$$

`Average' element of gas in a star is about distance R from the center, and has mass M interior to its radius, where R and M are the stellar radius and total mass. Typical potential is thus:

$$\Phi \sim -\frac{GM}{R}$$ ⇒ gravitational binding energy $$E_{grav} \sim M\Phi \sim -\frac{GM^2}{R}$$

This is an order of magnitude estimate only.

Solar rotation period is about P = 27 days. Angular velocity:

$$\Omega = \frac{2\pi}{P} \approx 2.7 \times 10^{-6}\ \mathrm{s}^{-1}$$

Rotation energy is of the order of:

$$E_{rotation} \sim M\Omega^2 R^2$$

Compare magnitude of gravitational and rotational energy:

$$\square = \frac{E_{rotation}}{\left|E_{grav}\right|} = \frac{M\Omega^2 R^2}{GM^2/R} = \frac{\Omega^2 R^3}{GM} \sim 2 \times 10^{-5}$$

↑ Depends upon **square** of rotation velocity

…even rotation rates much faster than the Sun ought to be negligibly small influence on structure.

- **Magnetic fields**

Magnetic fields in sunspots are fairly strong, of the order of kG strength. Suppose same field fills Sun:

$$E_{magnetic} = \text{Volume} \times \text{Energy density}$$

$$= \frac{4}{3}\pi R^3 \times \frac{B^2}{8\pi} = \frac{B^2 R^3}{6}$$

Ratio to gravitational energy is:

$$\frac{E_{magnetic}}{\left|E_{grav}\right|} = \frac{B^2 R^3 / 6}{GM^2 / R} = \frac{B^2 R^4}{6GM^2} \sim 10^{-11}$$

Estimates suggest that unless something really weird is going on (e.g. Sun rotates super-fast on the inside but not at the surface) magnetic fields / rotation are too small to seriously affect assumption of spherical symmetry.

2) Isolation

In the Solar neighborhood, distances between stars are enormous: e.g. Sun's nearest stellar companion is Proxima Centauri at d = 1.3 pc. Ratio of Solar radius to this distance is:

$$\frac{R_{sun}}{d} \approx 2 \times 10^{-8}$$

Two important implications:

- Can ignore the gravitational field and radiation of other stars when considering stellar structure.
- Stars (almost) never collide with each other.

Once star has formed, initial conditions rather than interactions with other stars determine evolution.

Important caveat: **binaries**

Most stars (about 2 / 3) are members of binary systems, and so have stellar companions much closer than the Sun. However, very broad distribution of separations - from stars touching to 1000’s of au. Most binaries have the stars far enough apart that their evolution is effectively independent.

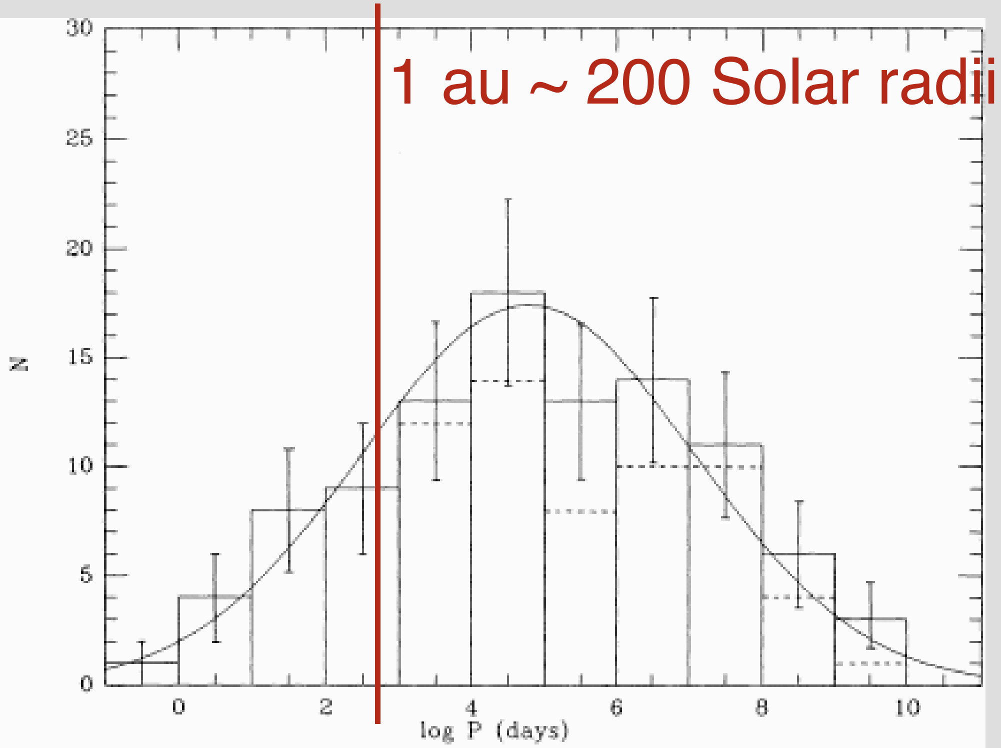

1 au ~ 200 Solar radii

Fig. 7. Period distribution in the complete nearby G-dwarf sample, without (dashed line) and with (continuous line) correction for detection biases. A Gaussian-like curve is represented whose parameters are given in the text

Results of a survey of Solar type stars for binaries by Duquennoy & Mayor

3) Uniform initial composition

Suppose the star forms from a molecular cloud with a composition of:

- **Hydrogen**, fraction of gas by mass X
- **Helium**, fraction of gas by mass Y
- **All other elements (`metals')** Z

$$\left.\begin{array}{l}X\\Y\\Z\end{array}\right\} \quad X+Y+Z=1$$

Reasonable to assume that initially, composition is constant throughout the star - i.e. X(r) = a constant.

Not true once fusion is underway - e.g. core of the Sun is enriched in helium relative to the surface (Y is larger in the core, smaller toward the surface).

4) Newtonian gravity

Newtonian gravity is only an approximation to Einstein's theory of General Relativity. For the Sun:

$$\frac{1}{2}v_{esc}^2 = \frac{GM}{R} \Rightarrow v_{esc} = \sqrt{\frac{2GM}{R}} = 620 \text{ km/s}$$

Ratio to speed of light: $\frac{v}{c} \sim 2 \times 10^{-3}$

Implies gravity in the Sun is very well approximated by ordinary Newtonian formulae.

Not true for neutron stars.

5) Static

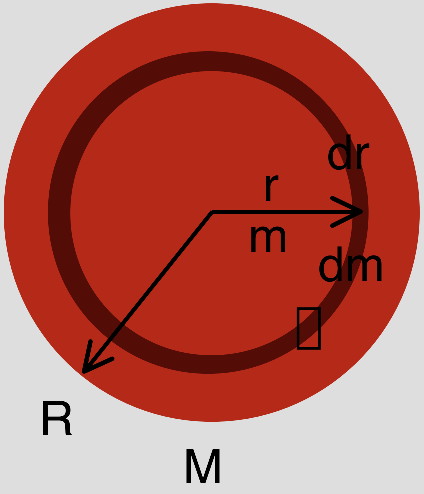
Dynamical time scale for Sun: $t_{dyn} \approx \frac{R}{v_{esc}} \sim 20 \text{ minutes}$

Sun is obviously not collapsing / exploding on this time scale! Implies that pressure and gravitational forces are in very close balance within the Sun, i.e. Sun is very nearly static. Slow changes due to:

- Changing composition (time scales of Gyr)
- Mass loss due to Solar wind (even longer at current mass loss rates)

Note: important classes of stars pulsate - these can’t be assumed to be static.

Description of a star in spherical symmetry



Let r be the distance from the center
Density as function of radius is $\rho(r)$

If m is the mass *interior* to r, then:

$$m(r) = \int_0^r 4\pi r^2 \rho(r)\, dr$$

Differential form of this equation is: $dm = 4\pi r^2 \rho\, dr$

Two **equivalent** ways of describing the star:

- Properties as f(r): e.g. temperature T(r)
- Properties as f(m): e.g. T(m)

Second way often more convenient, because (ignoring mass loss) total mass M of the star is fixed, while radius R evolves with time.