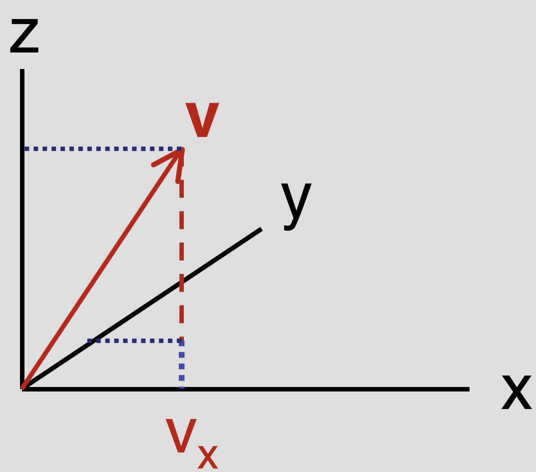
## Doppler or thermal broadening

Atoms in a gas have random motions that depend upon the temperature. For atoms of mass m, at temperature T, the typical speed is obtained by equating kinetic and thermal energy:

$$\frac{1}{2}mv^2 = kT$$

k = Boltzmann's const

Number of atoms with given speed or velocity is given by **Maxwell's law**. Need to distinguish between forms of this law for speed and for any one velocity component:



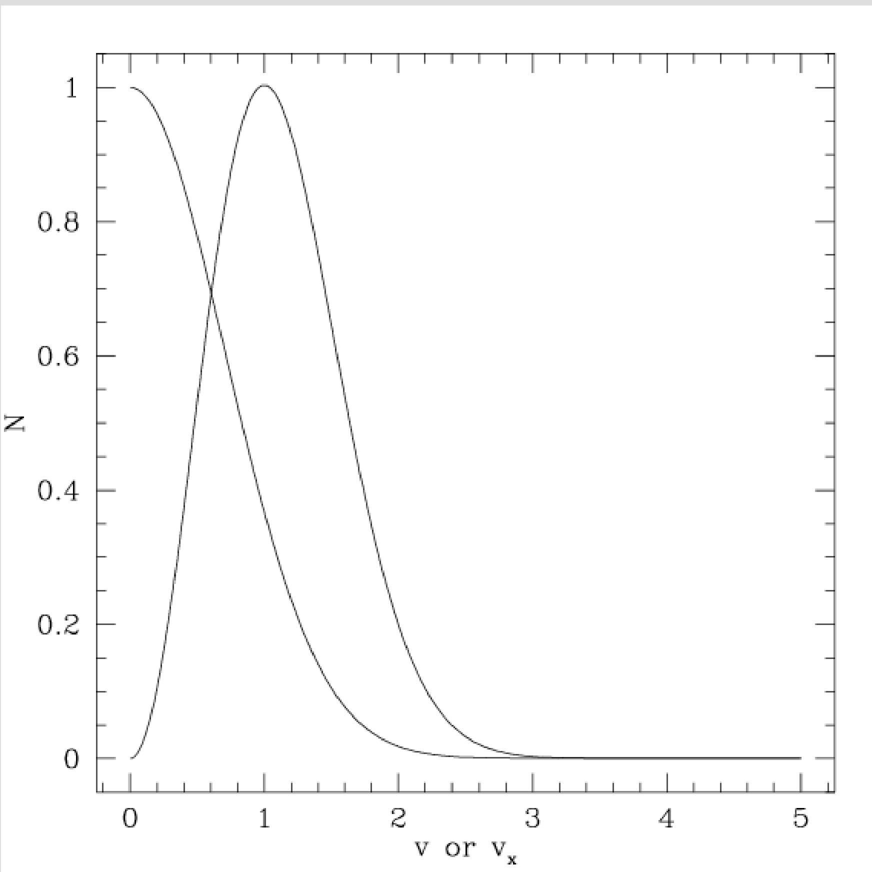

$$|\mathbf{v}|^2 = v_x^2 + v_y^2 + v_z^2$$

Distribution of *one component* of the velocity, say $v_x$, is relevant for thermal broadening - only care about motion along line of sight.

For one component, number of atoms dN within velocity interval $dv_x$ is given by:

$$dN(v_x) \propto \exp\left(-\frac{mv_x^2}{2kT}\right) dv_x$$

Distribution law for speeds has extra factor of $v^2$:




$$dN(v) \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv$$

Most probable speed:

$$v_{peak} = \sqrt{\frac{2kT}{m}}$$

Average speed:

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

$v_x$

Observed at frequency $\nu$

Emits at frequency $\nu_0$

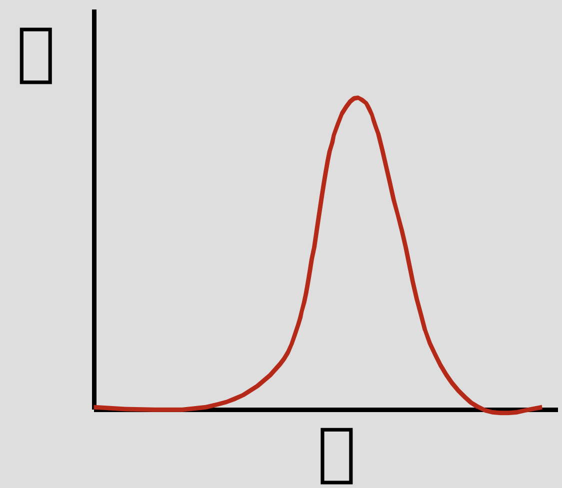
Consider atom moving with velocity $v_x$ along the line of sight to the observer.

**Doppler shift** formula:

$$\frac{\nu - \nu_0}{\nu_0} = \frac{v_x}{c}$$

Combine this with the thermal distribution of velocities:

$$\phi(\nu) = \frac{1}{\Delta\nu_D \sqrt{\pi}} \exp\left[ - \frac{(\nu - \nu_0)^2}{(\Delta\nu_D)^2} \right]$$



…where the **Doppler width** of the line:

$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

If the gas also has large-scale (i.e. not microscopic) motions due to turbulence, those add to the width:

$$\Delta \nu_D = \frac{\nu_0}{c} \left( \frac{2kT}{m} + v_{turb}^2 \right)^{1/2}$$

$v_{turb}$ is a measure of the typical turbulent velocity (note: really need same velocity distribution for this to be strictly valid).

Some numbers for hydrogen:

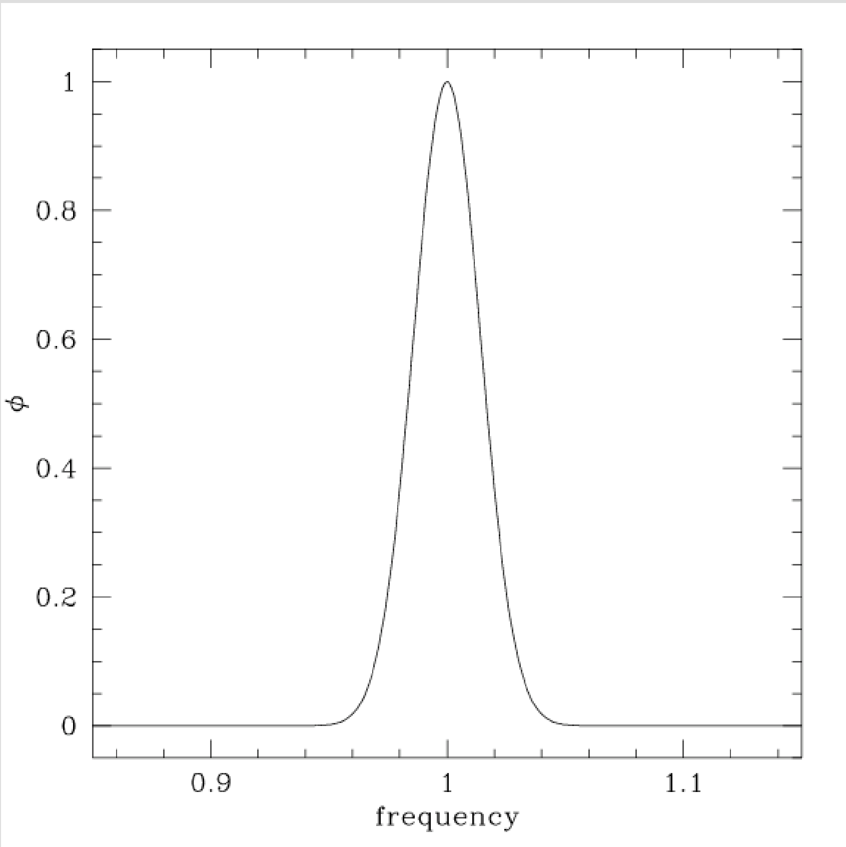
$$\frac{\Delta \nu_D}{\nu_0} \approx 4.3 \times 10^{-5} \left( \frac{T}{10^4 \text{ K}} \right)^{1/2}$$

larger than natural linewidth

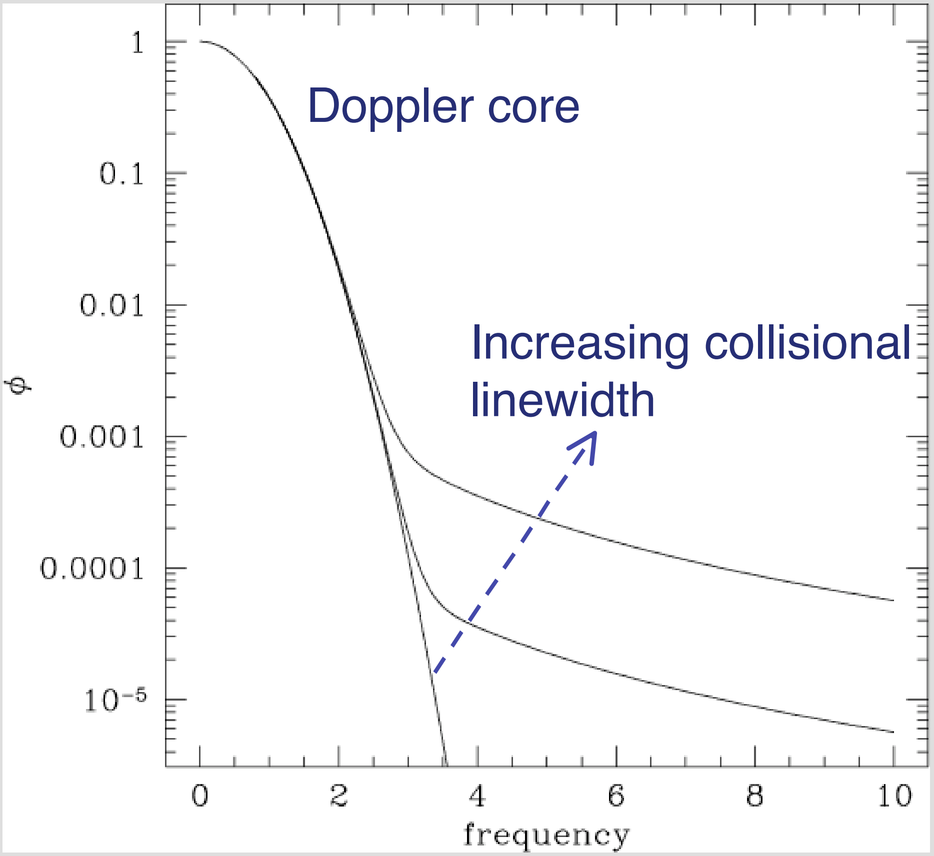
$$\frac{\Delta \nu_D c}{\nu_0} \approx 13 \left( \frac{\mathrm{T}}{10^4 \text{ K}} \right)^{1/2} \text{ km s}^{-1}$$

measured in velocity units, comparable to the sound speed in the gas

## Thermal line profile




## Voigt profile: combination of thermal and natural (or collisional) broadening




**Gaussian**: falls off very rapidly away from line center

Natural line profile falls off more slowly - dominates wings of strong lines

**Summary:**

- **Strength** of different spectral lines depends upon the abundance of different elements, and on the excitation / ionization state (described in part by the Boltzmann formula).
- **Width** of spectral lines depends upon:
  - *Natural linewidth* (small)
  - *Collisional linewidth* (larger at high density)
  - *Thermal linewidth* (larger at higher temperature)

➡ High quality spectrum gives information on composition, temperature and density of the gas.

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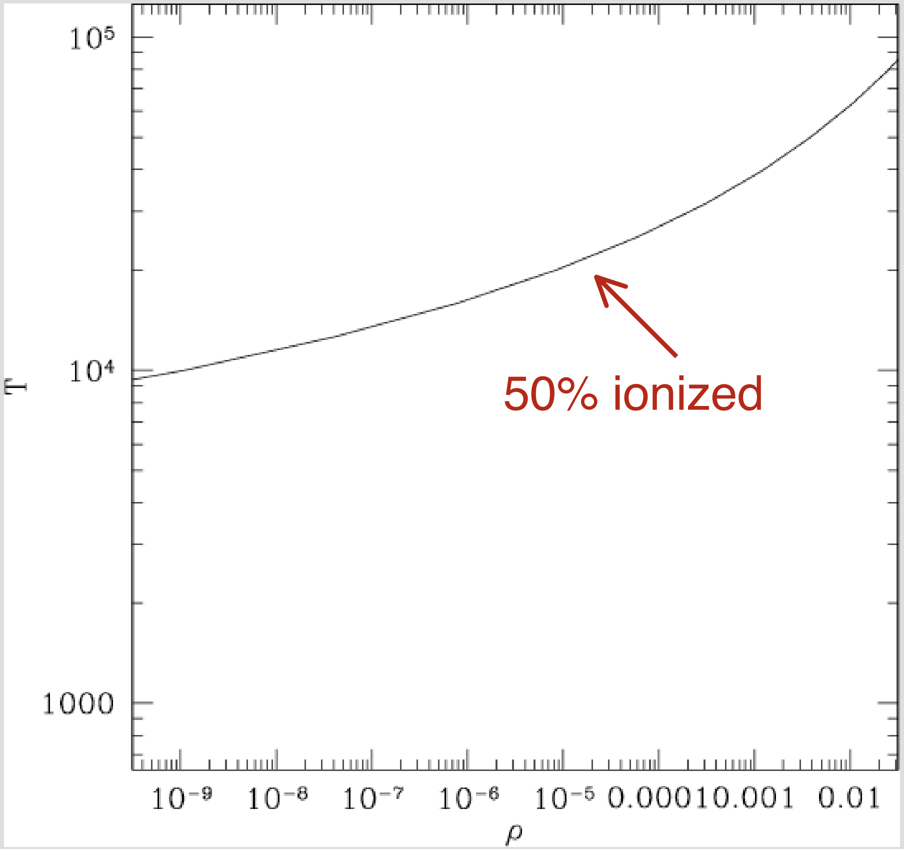
c.f. `Modern Astrophysics' section 8.1: more on thermal broadening, Boltzmann law, and Saha equation (version of Boltzmann law for ionization).

## Free-free radiation: Bremsstrahlung

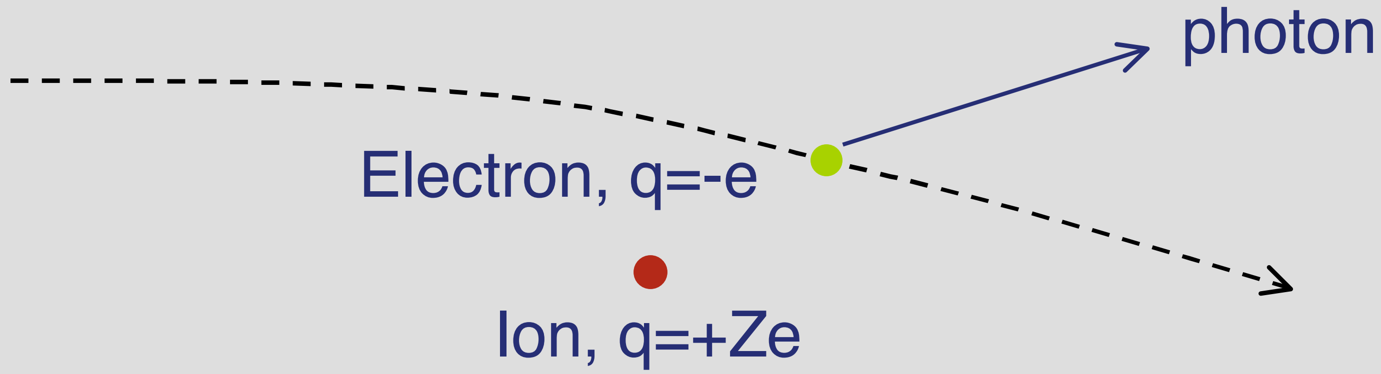
Hydrogen is ionized at T ~ $10^4$ K at low density.

For the same mixture of chemical elements as the Sun, maximum radiation due to spectral lines occurs at T ~ $10^5$ K.




At higher T, radiation due to acceleration of unbound electrons becomes most important.

**Free-free** radiation or **bremsstrahlung**.




`Collisions' between electrons and ions accelerate the electrons. Power radiated by a single electron is given by **Larmor's formula:**

$$P = \frac{2q^2}{3c^3}|\mathbf{a}|^2$$

c.g.s. units: q is the charge, where electron charge = 4.80 x $10^{-10}$ esu. **a** is the acceleration, c is speed of light.

Prefer to work in SI? Larmor's formula: …with q in Coulombs, $\epsilon_0$ is the permittivity of the vacuum [$10^7$ / ($4\pi c^2$) $C^2$ $N^{-1}$ $m^{-2}$]

$$P = \frac{q^2}{6\pi\epsilon_0 c^3}|\mathbf{a}|^2$$

Power is proportional to the square of the charge and the square of the magnitude of the acceleration.

To derive spectrum of bremsstrahlung, and total energy loss rate of the plasma, need to:

- Calculate acceleration and energy loss for one electron of speed v, passing ion at impact parameter b.
- Integrate over all collisions, assuming a distribution of encounter speeds (normally a thermal / Maxwellian distribution).

## Total energy loss rate from Bremsstrahlung

Plasma at:

- Temperature T
- Electron number density $n_e$ (units: $cm^{-3}$)
- Ions, charge Ze, number density $n_i$

Rate of energy loss due to bremsstrahlung is:

$$\epsilon^{ff} = 1.4 \times 10^{-27} T^{1/2} n_e n_i Z^2 \text{ erg s}^{-1} \text{ cm}^{-3}$$

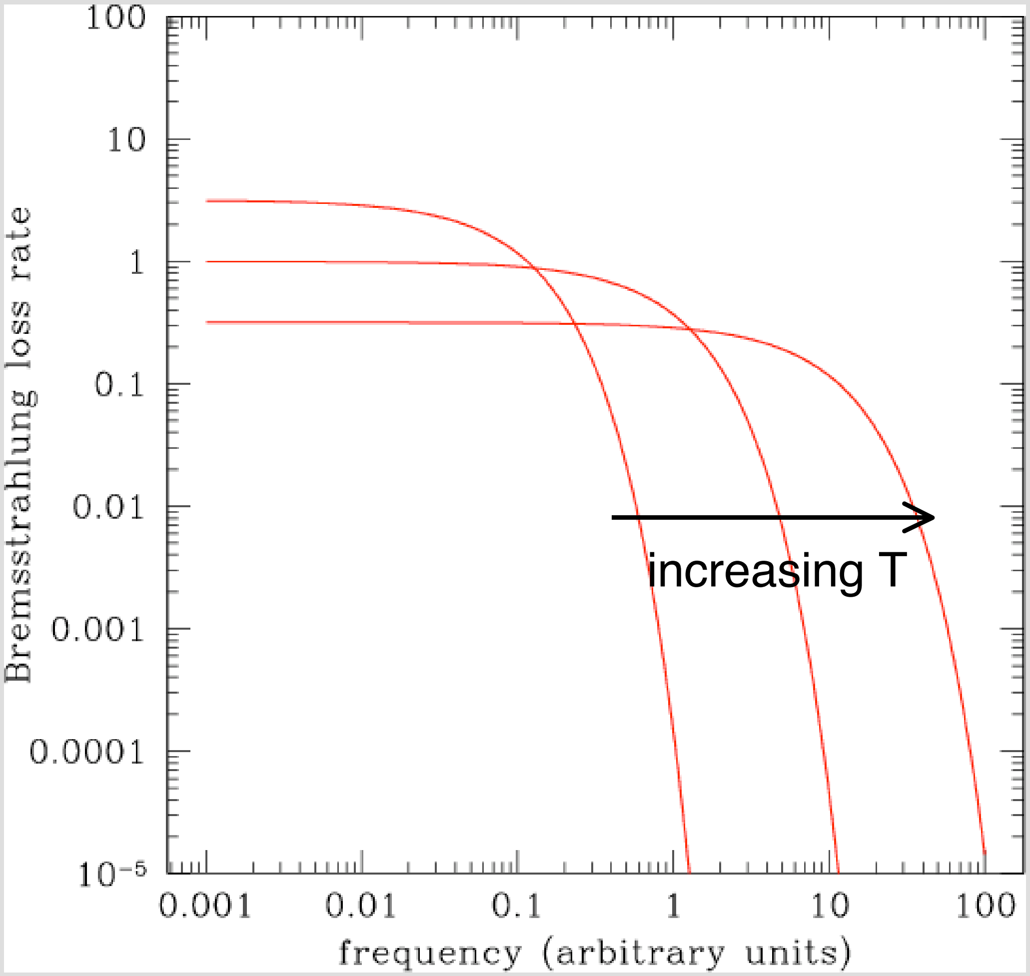
For pure hydrogen, Z=1 and $n_e = n_i$:

$$\epsilon^{ff} = 1.4 \times 10^{-27} T^{1/2} n_e^2 \text{ erg s}^{-1} \text{ cm}^{-3}$$

Note: this is the energy loss rate per unit volume (1 $cm^3$) of the gas.

# Spectrum of bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \text{ erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1}$$




Shape of bremsstrahlung spectrum

Flat spectrum up to an exponential cut off, at $h\nu = kT$.

Energy loss rate (overall and per Hz) depends on the **square** of the density.

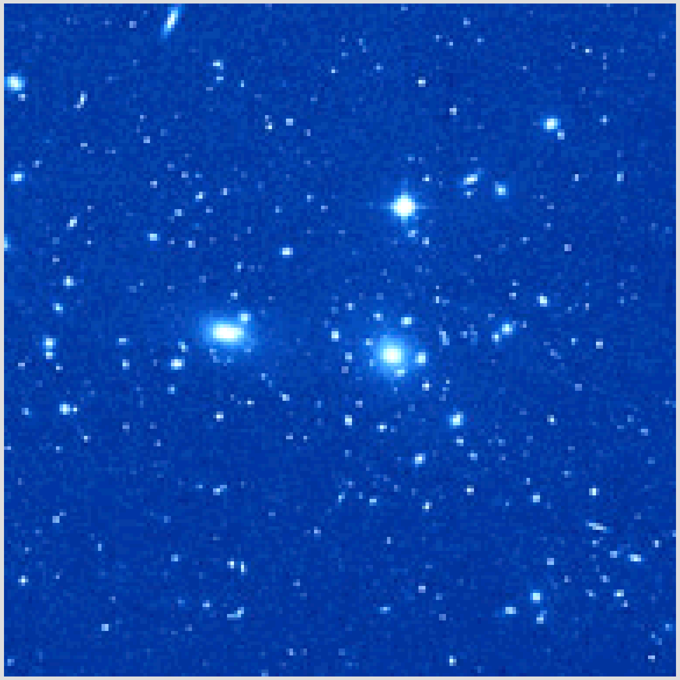
Continuous spectrum.

# When is bremsstrahlung important?

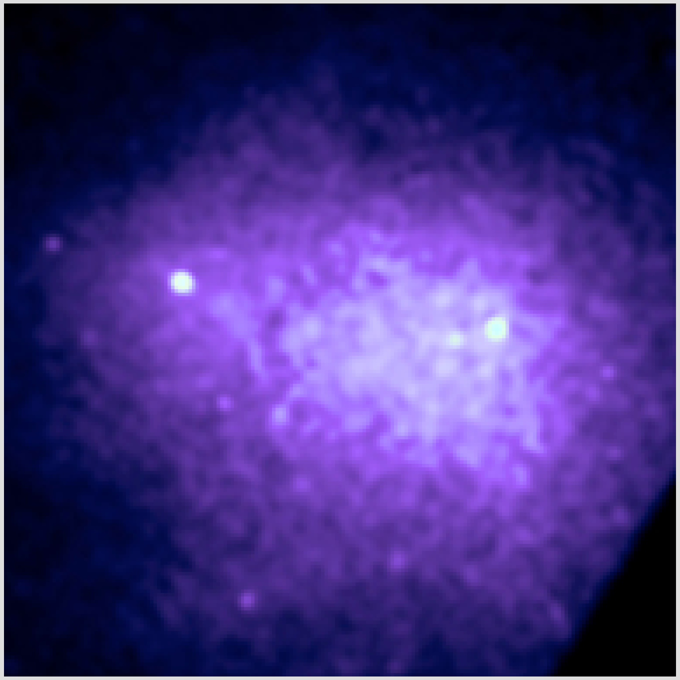
Bremsstrahlung loss rate increases with temperature
Atomic processes become less important as the gas becomes fully ionized

} high T

Example: gas in the Coma cluster of galaxies

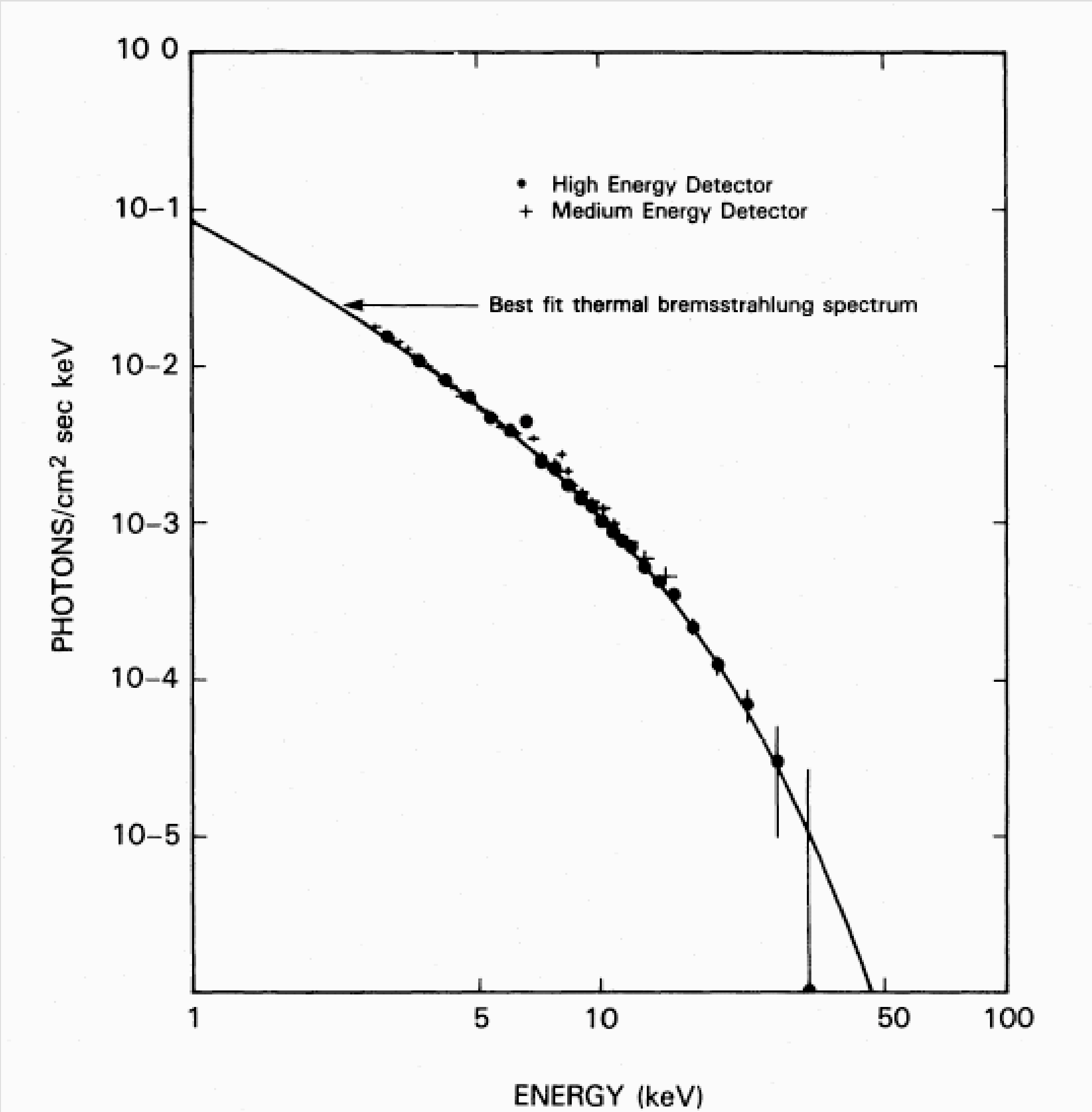


Optical



X-ray

<u>X-ray spectrum of Coma</u>




Shape of spectrum gives the temperature.

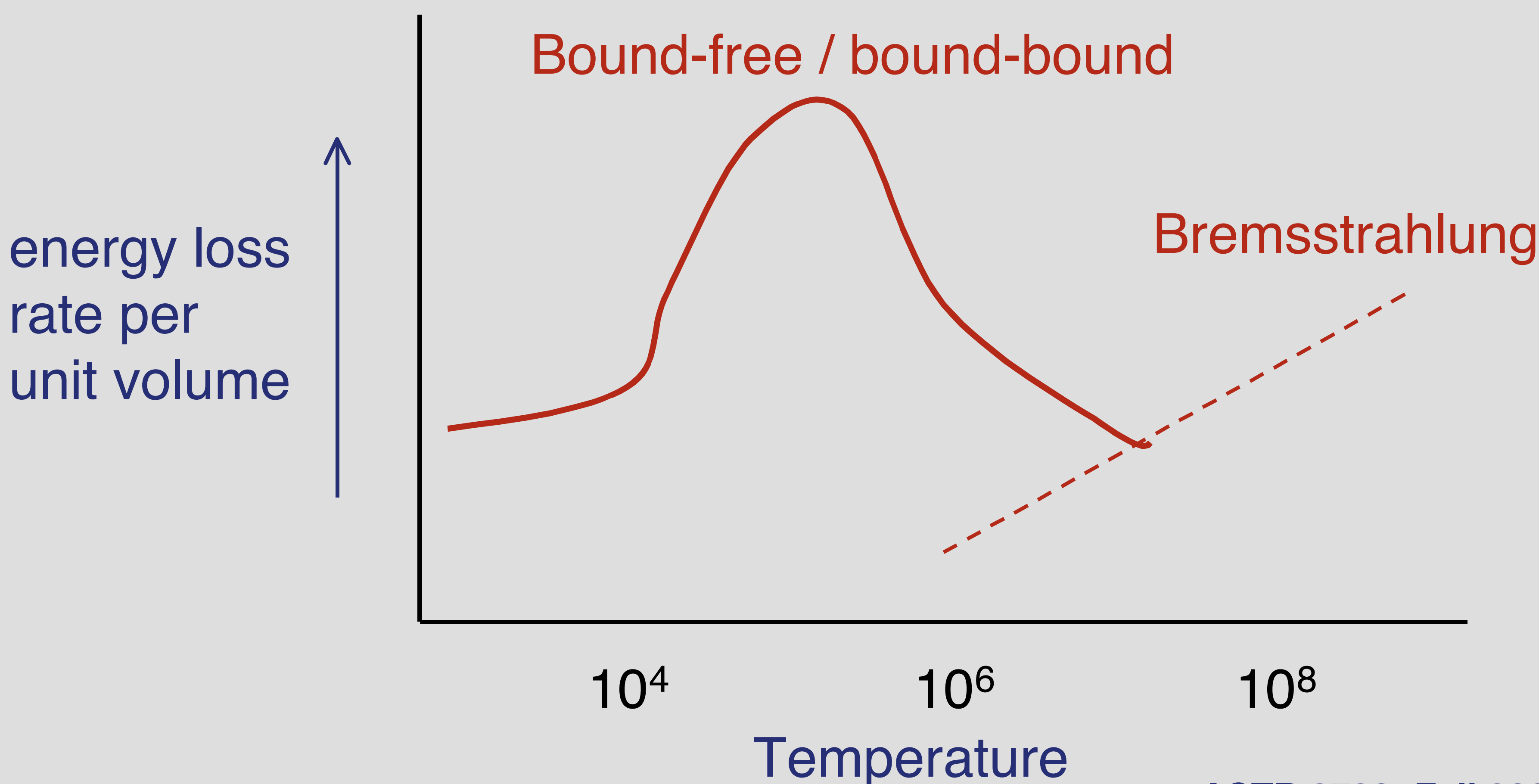
Intensity (for a known distance) gives the density of the gas.

Galaxy cluster: find T = 10 - 100 million K.

## Overall energy loss rate from a gas

Sum up various processes: bound-bound, bound-free, free-free.
Depend upon the square of the density.

Results of a detailed calculation:



Conclude:

- gas of Solar composition cools most efficiently at temperatures ~$10^5$ K - lots of atomic coolants.
- cold gas cools further inefficiently - have to rely on molecules at very low T
- gas at T ~ $10^7$ K also cools slowly - all atoms are ionized but bremsstrahlung not yet very effective.

Use this plot when we consider why gas in the galaxy comes in different phases.