Next, consider an optically thick source:
- Already shown that in the interior, radiation will be described by the Planck function.
- Radiation escaping from the source will be modified because the temperature (and thus the Planck function) varies along the path.

Example: model a star using a two layer model:

Radiation starts from the inner layer as blackbody radiation at temperature $T_{in}$.

Escapes through an atmosphere of optical depth $\tau$ and temperature $T_{out}$. 
Use same solution as before to describe change in intensity of the radiation:

\[ I_{\text{in}}(t_{\text{in}}) = I_0 e^{t_{\text{in}}} + B_{\text{in}}(1 - e^{-t_{\text{in}}}) + B_{\text{out}}(T_{\text{in}}) e^{-t_{\text{in}}} + B_{\text{out}}(T_{\text{out}}) (1 - e^{-t_{\text{in}}}) \]

Valid provided that all the gas is in thermal equilibrium (LTE).

Assume that optical depth of outer layer is small and use approximate expansion for the exponential as before:

\[ I_{\text{in}}(t_{\text{in}}) = B_{\text{in}}(T_{\text{in}}) e^{t_{\text{in}}} + B_{\text{out}}(T_{\text{out}}) B_{\text{in}}(T_{\text{in}}) e^{-t_{\text{in}}} \]
\[ I_\square(\square) = B_\square(T_{in}) + \square\square\left[ B_\square(T_{out}) \square B_\square(T_{in}) \right] \]

Initial radiation intensity

Change in intensity caused by the outer layer. Depends upon frequency.

Recall that intensity of blackbody radiation increases at all frequencies as the temperature goes up.

Sign of the second term depends upon whether \( B_\square(T_{out}) \) is larger or smaller than \( B_\square(T_{in}) \) - i.e. on whether \( T_{out} > T_{in} \).
\[ I(\nu) = B(\nu, T_{\text{in}}) + [B(\nu, T_{\text{out}}) - B(\nu, T_{\text{in}})] \]

1) \( T_{\text{out}} > T_{\text{in}} \): second term is positive:

Escaping intensity is **larger** at frequencies where \( \nu \) is greatest (frequencies corresponding to spectral lines). Expect emission lines on top of the continuum.

2) \( T_{\text{out}} < T_{\text{in}} \): second term is negative:

Escaping intensity is **reduced** at frequencies where \( \nu \) is greatest (frequencies corresponding to spectral lines). Expect absorption lines superimposed on the continuum.
For the Sun, temperature near the optical photosphere decreases outward (as it must since energy transport is from the center to the outside).

In second regime: $T_{\text{out}} < T_{\text{in}}$

Expect to see an absorption spectrum, as observed:
Note: see strong UV and X-ray emission from the Solar corona, so obviously the temperature there is much hotter than that of the photosphere...

UV radiation comes from region where $T$ increasing, so emission line spectrum

Optical radiation comes from region where $T$ decreasing, so absorption spectrum
Summary:

**Emission line spectra:**
- Optically thin volume of gas with no background light
- Optically thick gas in which $T$ increases outward

**Absorption line spectra:**
- Cold gas lies in front of a source of radiation at a higher temperature
Broadening of spectral lines

An individual atom making a transition between energy levels emits one photon with a well-defined energy / frequency.

However, profiles of real spectral lines are not infinitely narrow.

e.g. for an emission line, width of the spectral line \( \Delta \) could be defined as the full width at half the maximum intensity of the line.

Details of definition don’t matter - important to see what causes lines to have finite width.
Two basic mechanisms:

1) Energy levels themselves are not infinitely sharp: **emitted** photons have a range of frequencies.

2) Atoms in the gas are moving relative to the observer: **observed** photons don’t have the same frequency as the emitted photons because of the Doppler effect.
Consider excited state with energy $E$ above the ground state. Electrons in excited state remain there for average time $\bar{t}$ before decaying to ground state.

Uncertainty principle: energy difference between states is uncertain by an amount $\Delta E$ given by:

$$\Delta E \bar{t} \geq \frac{\hbar}{2\hbar}$$

But since $E = \hbar\bar{n}$, $\Delta E = \hbar\Delta n$

Broadening due to this effect is called the natural linewidth.
Natural linewidth sets absolute minimum width of spectral lines. However, normally very small - other effects dominate.

e.g. for hydrogen n=2 to n=1 transition (Lyman α transition) the lifetime is of the order of $10^{-9}$ s.

Natural linewidth is $\sim 10^8$ Hz.

Compare to frequency of transition: $\frac{\text{E}_n}{\text{E}_1} \sim 10^{17}$

In astrophysical situations, other processes will often give much larger linewidths than this.
Collisional broadening

In a dense gas, atoms are colliding frequently. This effectively reduces the lifetime of states further, to a value smaller than the quantum mechanical lifetime.

If the frequency of collisions is $n_{\text{col}}$, then expect to get a collisional linewidth of about $\Delta \nu \approx n_{\text{col}}$.

Frequency of collisions increases with density - expect to see broader lines in high density regions as compared to low density ones.

e.g. a main sequence star (small radius) has a higher density at the photosphere than a giant of the same surface temperature. Spectral lines in the main sequence star will be broader than in the giant.