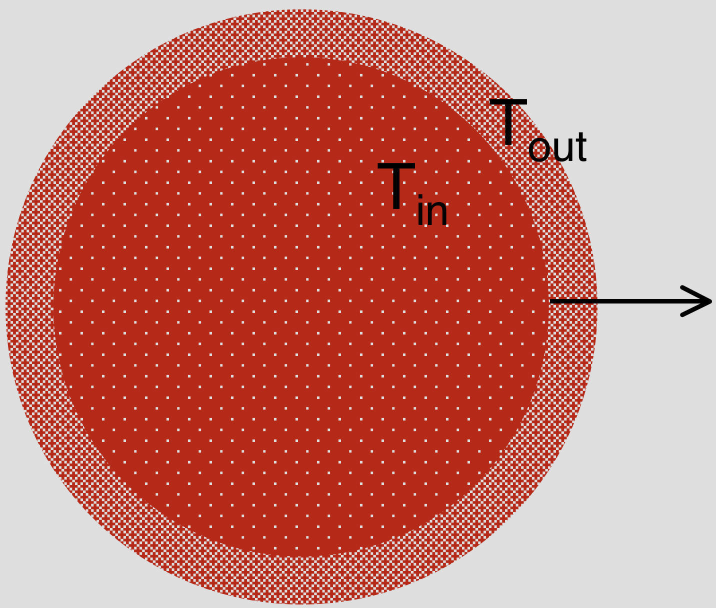
Next, consider an optically thick source:

- Already shown that in the interior, radiation will be described by the Planck function.
- Radiation escaping from the source will be modified because the temperature (and thus the Planck function) **varies** along the path.

Example: model a star using a two layer model:



Radiation starts from the inner layer as blackbody radiation at temperature T_{in}.

Escapes through an atmosphere of optical depth and temperature T_{out}.

Use same solution as before to describe change in intensity of the radiation:

$$I_\nu(\tau_\nu) = I_0 e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu})$$

Escaping radiation → $I_\nu(\tau_\nu)$

$B_\nu(T_{in})$ → I_0

$B_\nu(T_{out})$ → B_ν

Valid provided that all the gas is in thermal equilibrium (LTE).

Assume that optical depth of outer layer is small and use approximate expansion for the exponential as before:

$$I_\nu(\tau_\nu) = B_\nu(T_{in}) e^{-\tau_\nu} + B_\nu(T_{out})\left[1 - e^{-\tau_\nu}\right]$$

$$I_\nu(\tau_\nu) = B_\nu(T_{in})\left[1 - \tau_\nu\right] + B_\nu(T_{out}) \cdot \tau_\nu$$

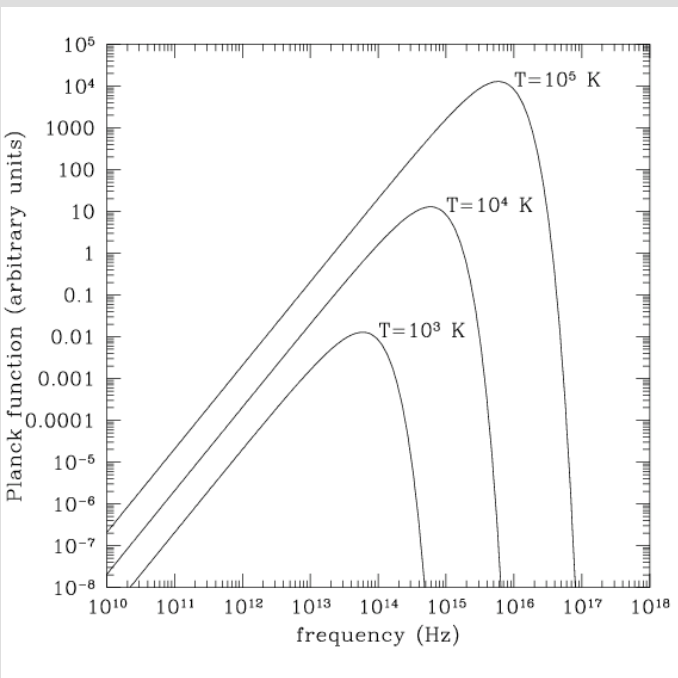
$$I_\nu(\tau_\nu) = B_\nu(T_{in}) + \tau_\nu\left[B_\nu(T_{out}) - B_\nu(T_{in})\right]$$

$$I_\nu(\tau_\nu) = B_\nu(T_{in}) + \tau_\nu\left[B_\nu(T_{out}) - B_\nu(T_{in})\right]$$

Initial radiation intensity

Change in intensity caused by the outer layer. Depends upon frequency.

Recall that intensity of blackbody radiation increases at **all** frequencies as the temperature goes up.



Sign of the second term depends upon whether $B_\nu(T_{out})$ is larger or smaller than $B_\nu(T_{in})$ - i.e. on whether $T_{out} > T_{in}$.

$$I_\nu(\tau_\nu) = B_\nu(T_{in}) + \tau_\nu\left[B_\nu(T_{out}) - B_\nu(T_{in})\right]$$

1) $T_{out} > T_{in}$: second term is positive:

Escaping intensity is **larger** at frequencies where τ_ν is greatest (frequencies corresponding to spectral lines). Expect emission lines on top of the continuum.

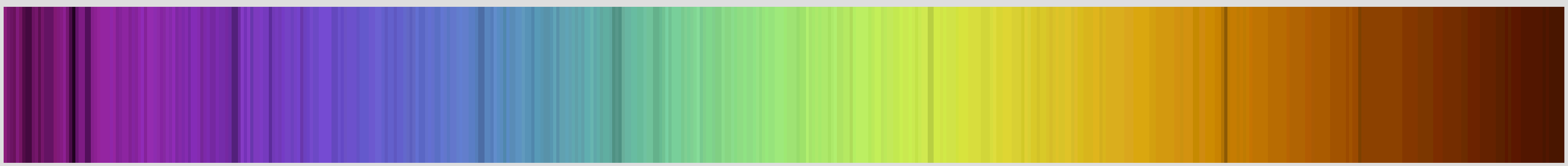
2) $T_{out} < T_{in}$: second term is negative:

Escaping intensity is **reduced** at frequencies where τ_ν is greatest (frequencies corresponding to spectral lines). Expect absorption lines superimposed on the continuum.

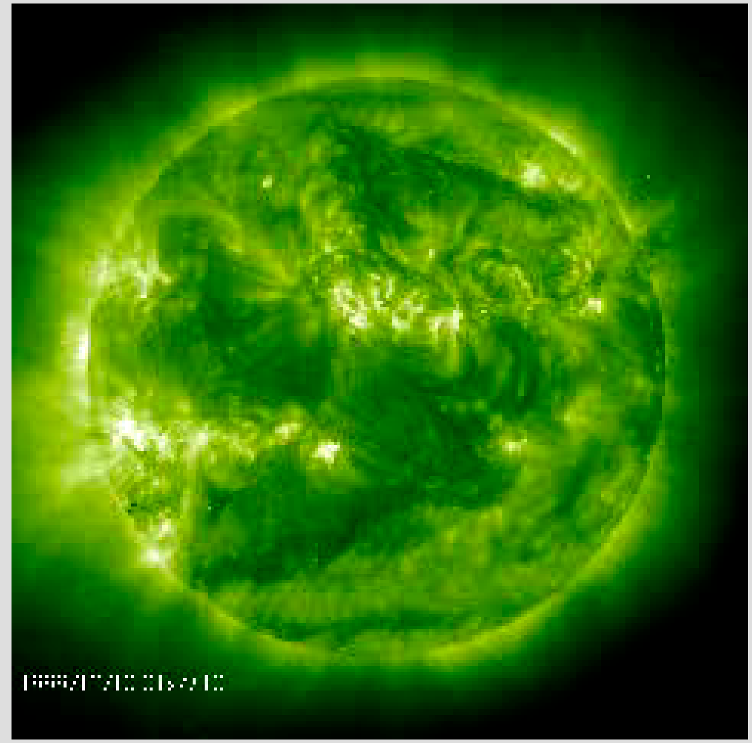
For the Sun, temperature near the optical photosphere **decreases** outward (as it must since energy transport is from the center to the outside).

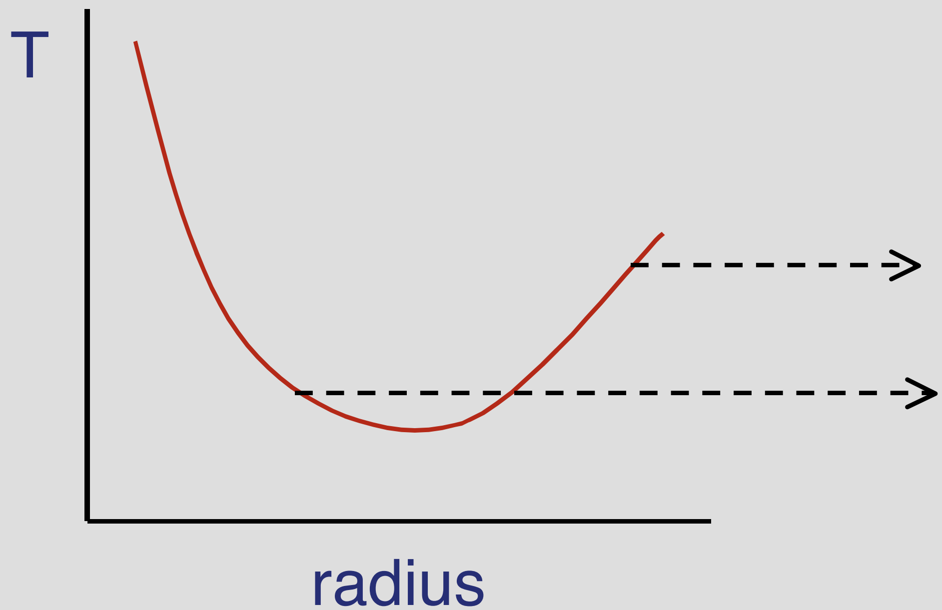
In second regime: $T_{out} < T_{in}$

Expect to see an **absorption spectrum**, as observed:



Note: see strong UV and X-ray emission from the Solar corona, so obviously the temperature there is much hotter than that of the photosphere…





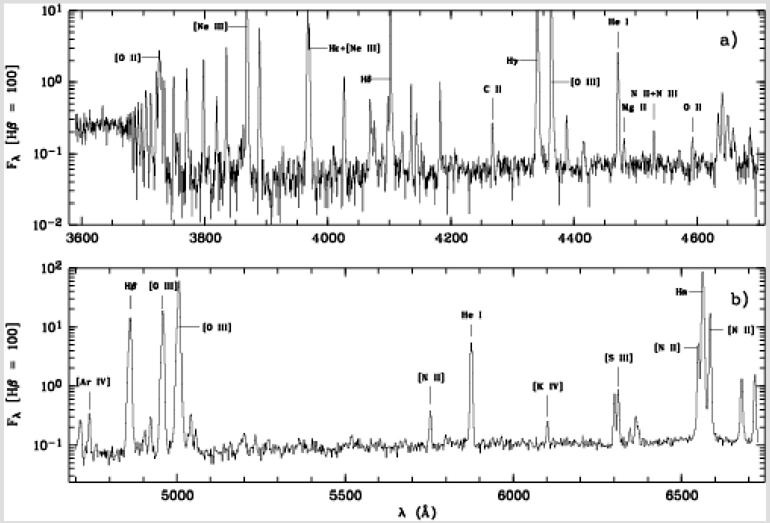
UV radiation comes from region where T increasing, so emission line spectrum

Optical radiation comes from region where T decreasing, so absorption spectrum

Summary:

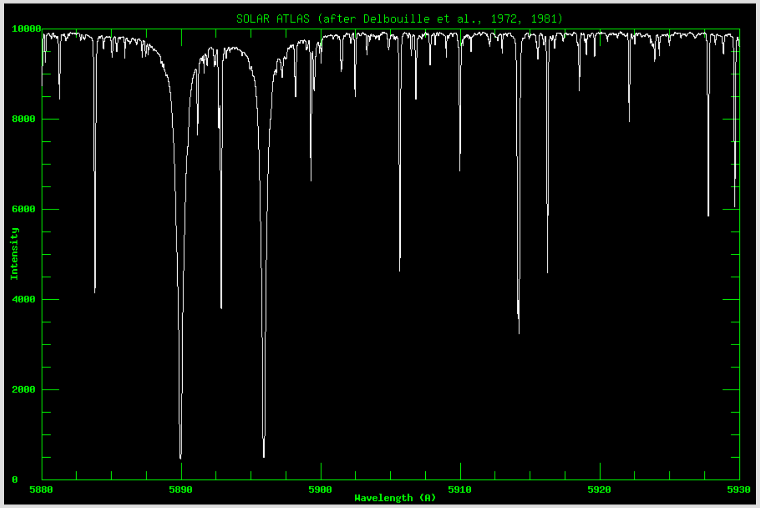
Emission line spectra:

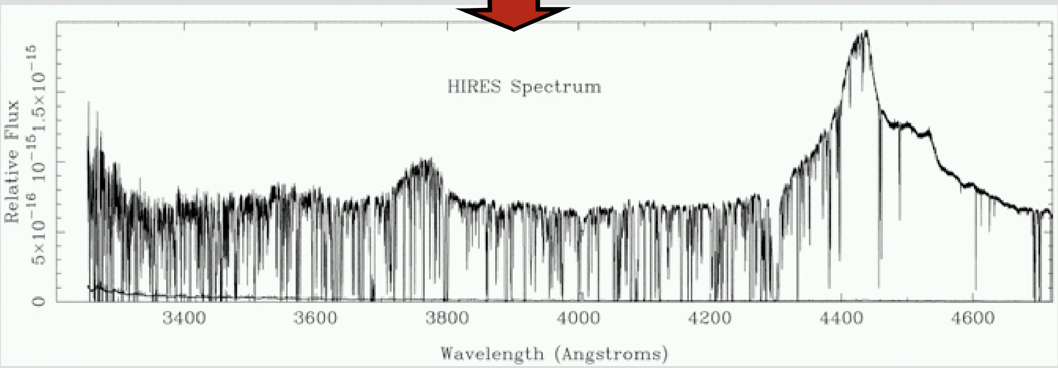
- Optically thin volume of gas with no background light
- Optically thick gas in which T increases outward



Absorption line spectra:

- Cold gas lies in front of a source of radiation at a higher temperature

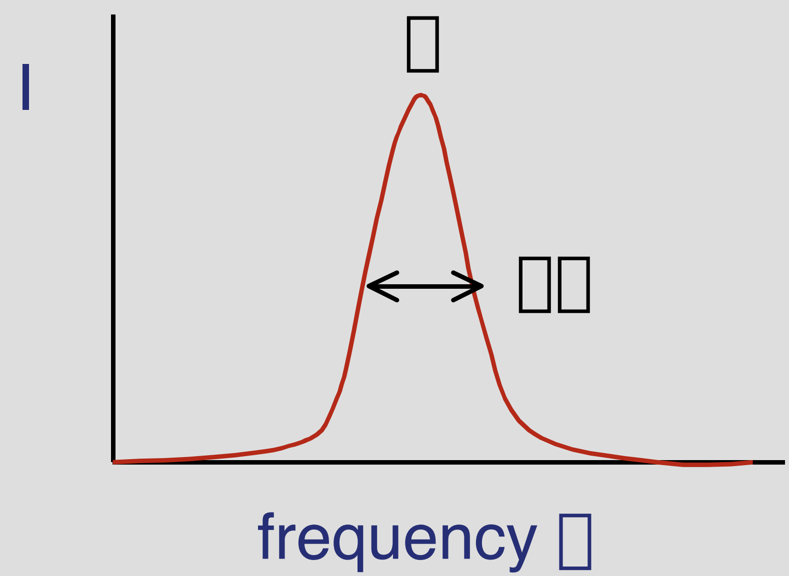




Broadening of spectral lines

An individual atom making a transition between energy levels emits one photon with a well-defined energy / frequency.

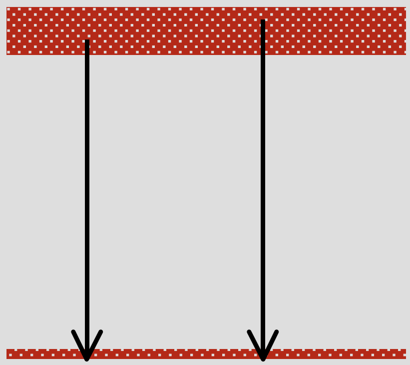
However, profiles of real spectral lines are not infinitely narrow.

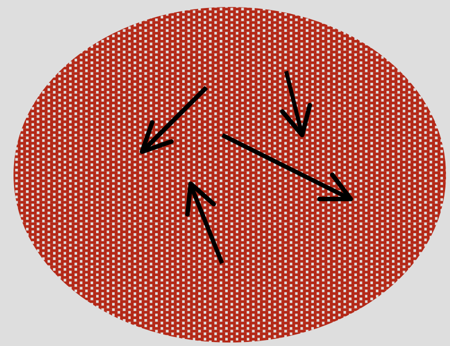


e.g. for an emission line, width of the spectral line $\Delta\nu$ could be defined as the full width at half the maximum intensity of the line.

Details of definition don't matter - important to see what causes lines to have finite width.

Two basic mechanisms:

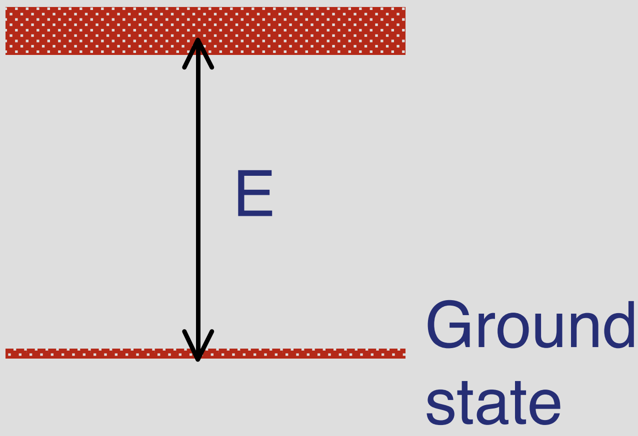




1) Energy levels themselves are not infinitely sharp:
emitted photons have a range of frequencies

2) Atoms in the gas are moving relative to the observer:
observed photons don't have the same frequency as the emitted photons because of the Doppler effect.

Natural linewidth



Consider excited state with energy E above the ground state.

Electrons in excited state remain there for average time Δt before decaying to ground state.

Uncertainty principle: energy difference between states is uncertain by an amount ΔE given by:

$$\Delta E \Delta t \approx \frac{h}{2\pi}$$

But since $E = h\nu$, $\Delta E = h\Delta\nu$ $\Rightarrow$ $\Delta\nu \approx \frac{1}{2\pi\Delta t}$

Broadening due to this effect is called the natural linewidth.

Natural linewidth sets absolute minimum width of spectral lines. However, normally very small - other effects dominate.

e.g. for hydrogen n=2 to n=1 transition (Lyman α transition) the lifetime is of the order of 10^{-9} s.

Natural linewidth is $\sim 10^8$ Hz.

Compare to frequency of transition: $\frac{\Delta\nu}{\nu} \approx 10^{-7}$

In **astrophysical** situations, other processes will often give much larger linewidths than this.

Collisional broadening

In a dense gas, atoms are colliding frequently. This effectively reduces the lifetime of states further, to a value smaller than the quantum mechanical lifetime.

If the frequency of collisions is ν_{col}, then expect to get a collisional linewidth of about $\Delta\nu \sim \nu_{col}$.

Frequency of collisions increases with density - expect to see broader lines in high density regions as compared to low density ones.

e.g. a main sequence star (small radius) has a higher density at the photosphere than a giant of the same surface temperature. Spectral lines in the main sequence star will be broader than in the giant.