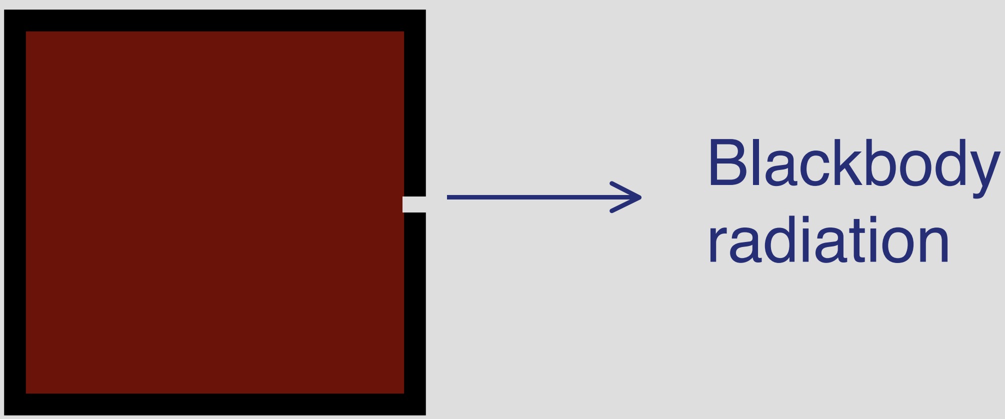
# Sources of radiation

Most important type of radiation is **blackbody radiation**. This is radiation that is in thermal equilibrium with matter at some temperature T.

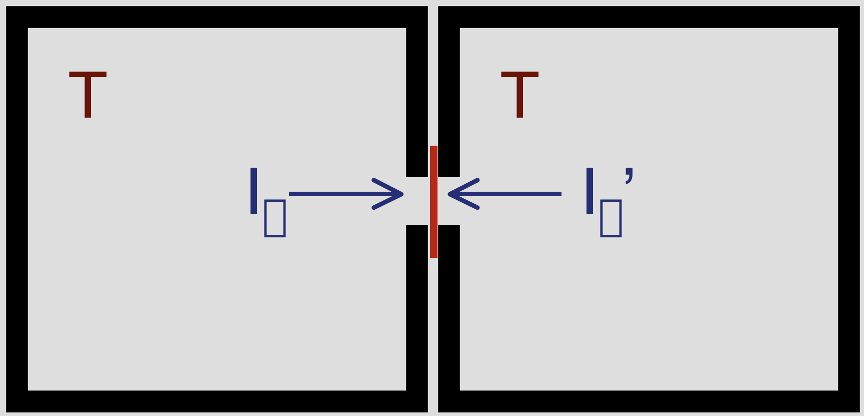
Lab source of blackbody radiation: hot oven with a small hole which does not disturb thermal equilibrium inside:




Important because:

- Interiors of stars (for example) are like this
- Emission from many objects is roughly of this form.

A general principle in physics is that **energy cannot flow spontaneously from a cold body to a hot one** (a statement of the second law of thermodynamics). Implies that the intensity of blackbody radiation can only depend on T:




Imagine two blackbody cavities at the same temperature T, separated by a filter that transmits only radiation with frequency close to $\nu$. If:

$$I_\nu \neq I_\nu'$$

…there would be a net energy flow, in violation of the 2nd law. Hence $I_\nu$ can only depend on T.

## Spectrum of blackbody radiation

The frequency dependence of blackbody radiation is given by the **Planck function**:

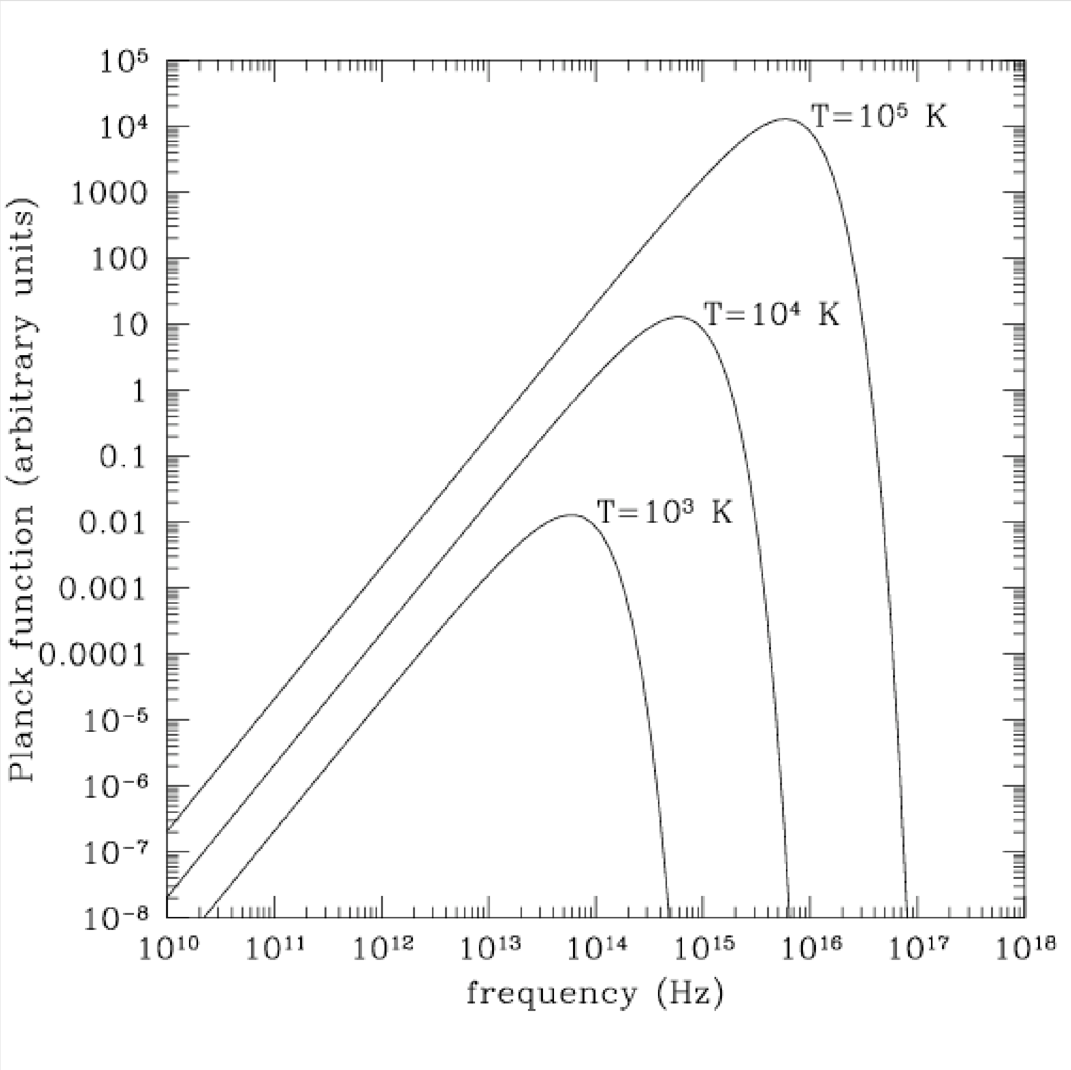
$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

- $h = 6.63 \times 10^{-27}$ erg s is Planck's constant
- $k = 1.38 \times 10^{-16}$ erg $K^{-1}$ is Boltzmann's constant

Same units as specific intensity: erg $s^{-1}$ $cm^{-2}$ $steradian^{-1}$ $Hz^{-1}$

Derivation in *Harwit*, or see different derivation on the web page.

# Properties of blackbody radiation




Plot $B_\nu(T)$:

- Continuous spectrum
- Increasing T increases $B_\nu$ at **all** frequencies
- Higher temperature shifts the peak to higher frequency / shorter wavelength.

**Thermal radiation**:

$$S_\nu = B_\nu$$

**Blackbody radiation**:

$$I_\nu = B_\nu$$

Differentiate $B_\nu(T)$ with respect to frequency and set resulting expression to zero to find where the Planck function peaks.

$$h\nu_{\max} = 2.82kT$$

$$\nu_{\max} = 5.88 \times 10^{10} T \text{ Hz K}^{-1}$$

*Wien displacement law* - peak shifts linearly with increasing temperature to higher frequency.

*Rayleigh-Jeans law*: for low frequencies $h\nu << kT$:

$$B_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

Often valid in the radio part of the spectrum, at frequencies far below the peak of the Planck function.

The energy density of blackbody radiation:

$$u(T) = aT^4$$

…where a = 7.56 x $10^{-15}$ erg $cm^{-3}$ $K^{-4}$ is the radiation constant.

The emergent flux from a surface emitting blackbody radiation is:

$$F = \sigma T^4$$

…where $\sigma$ = 5.67 x $10^{-5}$ erg $cm^{-2}$ $K^{-4}$ $s^{-1}$ is the Stefan-Boltzmann constant.

e.g. star, radius R, temperature T, emitting as a blackbody, has a luminosity:

$$L = 4\pi R^2 \sigma T^4$$

Emission from most astronomical sources is only roughly described by the Planck function (if at all).

Source has emergent flux F (integrated over all frequencies), define the **effective temperature** $T_e$ via:

$$F \equiv \sigma T_e^4$$

Effective temperature is the temperature of a blackbody that emits same flux. e.g. for the Sun:

$$L_{sun} = 4\pi R_{sun}^2 \sigma T_e^4$$

…find $T_e$ = 5770 K.

Note: effective temperature is well-defined even if the spectrum is nothing like a blackbody.

## Which objects have blackbody spectra?

Radiation will be blackbody radiation wherever we have matter in thermal equilibrium with radiation - i.e. at large optical depth. To show this, consider transfer equation with $S_\nu = B_\nu(T)$, and assume T is constant:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T)$$

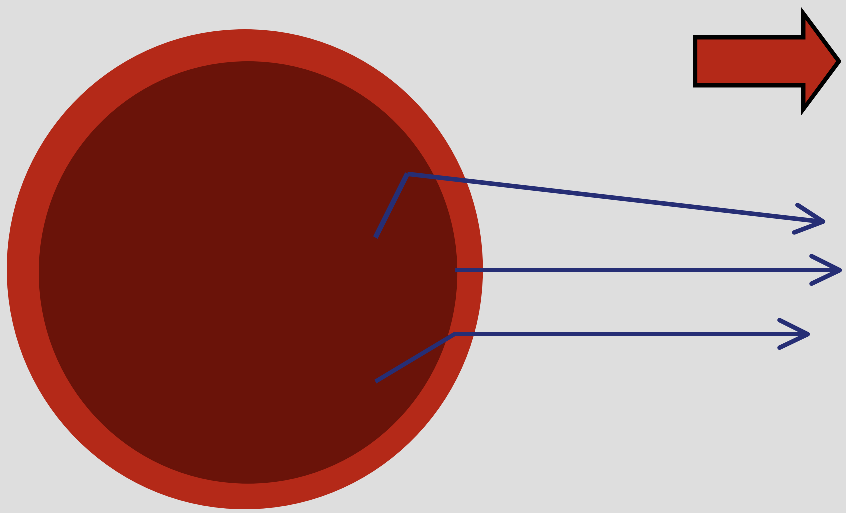
Can integrate this equation, with solution:

$$I_\nu(\tau) = B_\nu + e^{-\tau}\left[I_0 - B_\nu\right]$$

-> 0 as $\tau$ becomes large

where $I_0$ is the value of $I_\nu$ at $\tau = 0$. Conclude $I_\nu = B_\nu$ at high optical depth, e.g. in the center of a star.

Recall interpretation of optical depth: at $\tau = 1$ there is (very roughly speaking) a 50% chance that a photon headed toward us will suffer an absorption or scattering along the way.

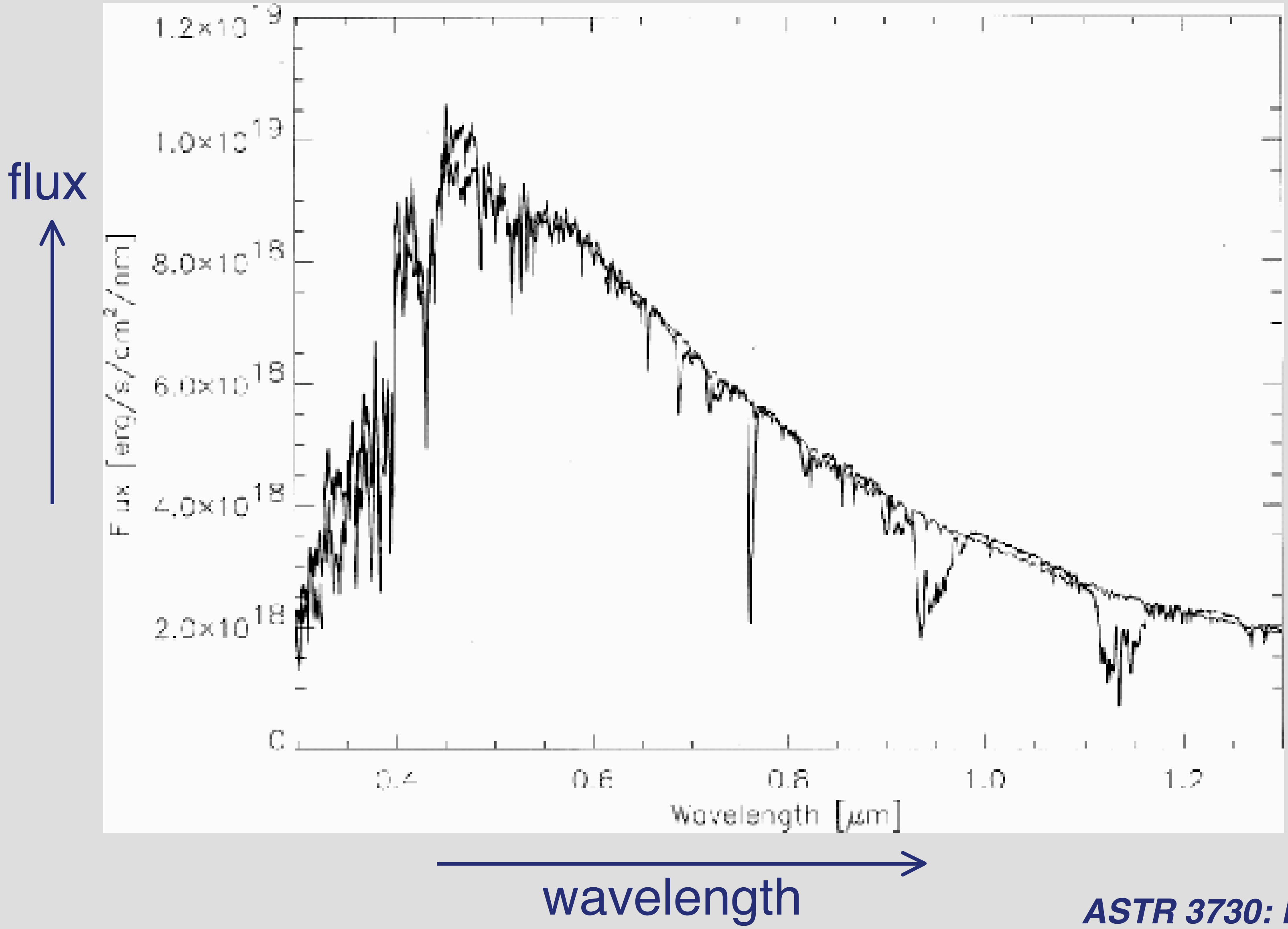


By definition, photons we can observe from an optically thick source come from near the $\tau = 1$ surface.

Since $\tau = 1$ is *not* \`high optical depth', observed radiation from an optically thick source is not necessarily blackbody spectrum.

Deviations will depend upon the frequency dependence of the opacity around the $\tau = 1$ surface (the photosphere).

Solar spectrum - approximately of blackbody form. Very cool stars show larger departures from thermal spectra.



## Giant planets

Two sources of radiation:

- Directly reflected Sun light
- Absorbed Solar radiation, reradiated as a cool blackbody

e.g. Jupiter: $L_{sun} = 3.86 \times 10^{33}\ \mathrm{erg\ s^{-1}}$

$a_J = 7.8 \times 10^{13}\ \mathrm{cm}$ Jupiter orbital radius

$\mathrm{R_J} = 7.1 \times 10^{9}\ \mathrm{cm}$ Jupiter radius

Solar radiation incident on the planet is:

$$L_J = \frac{\pi R_J^2}{4\pi a_J^2} \times L_{sun} \approx 2 \times 10^{-9} L_{sun}$$

Suppose planet directly reflects 10% - in the optical Jupiter is ~$10^{10}$ times fainter than the Sun as seen from another star - about 25 magnitudes.

Absorb and reradiate as a blackbody:

$$L_J = 4\pi R_J^2 \times \sigma T_J^4$$

If all Sun light absorbed, estimate T = 120 K. Use:

$$h\nu_{\max} = 2.82kT$$

Find $\nu_{max}$ = 7 x $10^{12}$ Hz, which corresponds to a wavelength of around 40 microns.