Special relativistic time dilation

*Symmetric* effect (contrast with general relativity later)

Bob sees Alice’s clock moving, observes it runs slow compared to his

Alice sees Bob’s clock moving, observes it runs slow compared to his
Twin paradox

$\nu = 0.99 \, c$

Alice travels to a nearby star and back at near the speed of light: her twin Bob stays on Earth

Twin paradox

On Alice’s return, who is older?

Bob: Alice’s “clock” ran slow, so less time has passed for her than for Bob
**Twin paradox**

BUT... from Alice’s point of view Bob was moving fast relative to her, so isn’t it a symmetric situation? How can Alice age less than Bob?

**Not symmetric** – Alice was accelerated to high speed, slowed down at her destination, accelerated to come home. Bob experiences none of this.

**How strong is time dilation?**

Depends on Lorentz factor \[ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \]

Clock moving at speed \( v \) relative to us runs slow by a factor of \( \gamma \)

If \( v \ll c \), then \( v^2 / c^2 \) is very small and \( \gamma \sim 1 \)
How strong is time dilation?

\[ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \]

<table>
<thead>
<tr>
<th>( v )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 c</td>
<td>1.15</td>
</tr>
<tr>
<td>0.9 c</td>
<td>2.29</td>
</tr>
<tr>
<td>0.99 c</td>
<td>7.09</td>
</tr>
<tr>
<td>0.999 c</td>
<td>22.4</td>
</tr>
</tbody>
</table>

Fastest astronomical jets have \( \gamma \sim 100 \)

Experiments

Impact of cosmic Rays with upper atmosphere creates muons

Unstable: lifetime at rest 2.2 \( \mu s \)

cosmic rays

Earth
Experiments

If they travel at almost \( c \), distance before they decay:

\[
L = ct
= 300,000 \text{ km/s} \times 2.2 \times 10^{-6} \text{ s}
= 0.66 \text{ km}
\]

Muons should decay before reaching surface

Muons are produced at \( v \sim 0.995 \, c \) (\( \gamma \sim 8 \))

Variation of flux with height consistent with time dilation by this factor
Length contraction

Objects are *contracted* by a factor \( \gamma \) in the direction along which they move.

e.g. the muons in the previous experiment “see” the Earth’s atmosphere as being much thinner than we measure it.

Relativistic addition of velocities

What velocity do we measure for baseball?

\[
v_2 = \frac{v + V}{1 + \frac{vV}{c^2}}
\]

As long as \( v \) and \( V \) are < \( c \), we never see \( v_2 \) faster than light.
Special relativity results (so far)

- observers in relative motion disagree about simultaneity of spatially separated events
  - “mixing” of time and space
- moving clocks run slow
- moving rulers are short
- cannot add velocities to exceed c
  - speed of light is an ultimate speed limit

Finite speed of light + mixing of space and time create odd optical effects for \( v \sim c \)

Recommend movies by Ute Kraus

http://www.spacetime-travel.org/
In “reality”, geometric distortions *plus*:

- redshift (light from objects coming toward us shifted to blue, receding to red)
- “beaming” – light from a fast moving object is concentrated to forward direction

Effects needed to understand emission from actual astronomical sources!

- gamma-ray bursts
- disks of gas near black holes

**Time dilation implies distance contraction**

**Imagine a 1-way trip to the Sun at 0.87c**

- Distance to Sun: 93 million mi. = 8 light-minutes
- Travel time (Earth clock): 8min/0.87 = 9.2 min
- Travel time (astronaut clock): 4.6 min due to time dilation
- Distance traveled (according to astronaut): 0.87x4.6= 4 light-minutes= 46.5 million mi.!

**Distance to Sun is contracted according to the astronaut**
“Lorentz” contraction is symmetric ...

Astronauts see:

Earth observer sees:

The Drive-Thru Paradox

Imagine a relativistic ...

... trying to visit a

... without slowing down!
The Drive-Thru Paradox

Imagine a relativistic car... ...trying to visit a

... without slowing down!

Because it’s moving relativistically it can fit under the carport...

...but that’s not how the driver sees it
The Drive-Thru Paradox

Q: How can both be true?
A: Relativity of simultaneity.

Server finds car fits under roof simultaneously.

Driver thinks front end is under roof while back end hasn’t arrived yet.