Absolute measurement of a long, arbitrary distance to less than an optical fringe

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Received November 28, 2003

A scheme is proposed for high-precision, absolute length measurement for an arbitrary optical distance of a few meters to beyond 10^6 m. The approach utilizes a phase-stabilized femtosecond laser to provide both incoherent, time-of-flight information and coherent, fringe-resolved interferometry. Such a combined measurement capability allows an optical wavelength resolution to be achieved for absolute length measurement over a large dynamic range.

OCIS codes: 120.3180, 120.0120, 320.7160.

Traditional length measurement can be categorized by two general methodologies. The first approach is based on using incoherent detection of light pulses to convert time-of-flight information to distance measurement.[1,2] Such a technique is suitable for making absolute length measurements over a large distance. However, it is limited in measurement precision and resolution by the best available electronic instruments. A resolution of 3 ps, corresponding to a distance of ~1 mm, is the nominal limit for this scheme. The second approach utilizes detection of optical coherence, i.e., interference fringes, to achieve enhanced resolution. This is classical wavelength-based interferometry, which is excellent for measurement of incremental displacements but not immediately suitable for determining an absolute distance. Over the years researchers have developed multiwavelength optical interferometry to remove the distance ambiguity inherent in a single wavelength measurement.[3,4] The multiwavelength measurement allows synthetic wavelengths, which are typically on a microwave or millimeter wave scale using heterodyne beatings among several optical frequency components, to be used for absolute distance calibration. For enhanced performance, one desires several different synthetic wavelengths to be available, distributed at various length scales. However, the system becomes complex when multiple cw laser sources are involved. A special case of interferometry known as white-light interferometry uses an incoherent, broadband light source to facilitate determination of zero length difference.[5] It is obviously not convenient to adapt a white-light interferometer for general-purpose length metrology.

In this Letter a scheme is proposed for absolute length metrology at arbitrary distances with a resolution better than an optical fringe. The proposed approach takes advantage of a phase-stabilized optical comb from a mode-locked femtosecond laser. Thinking along this direction arises from a clear motivation. An ultrafast pulse train allows determination of an absolute distance by use of time-of-flight information as in the incoherent measurement approach. However, a phase-stabilized femtosecond pulse train also allows one to build optical interference fringes when pulses traversing different arms of an interferometer are allowed to overlap and interfere, given a proper adjustment of the mode-locked laser. Essentially we create a white-light interferometry condition at any desired length difference between the two arms in the interferometer. Note that pioneering work on using femtosecond lasers for length measurement has been undertaken by Minoshima and Matsumoto.[6]

Although the present scheme does not address the dispersion issue for interferometers immersed in a dispersive medium, it will have a strong effect on spaceborne applications, in which the use of coherent laser sources for precision measurement will be a concentrated effort of future NASA missions. In a number of proposed space-based interferometric configurations, nanometer or even picometer resolution is the desired spatial quantity in maintaining a distance beyond 10^6 m between satellites flying in formation. Such measurement precision can be achieved only through superstable laser-based technologies.[7,8] What is new now is the demonstrated capability of faithful transfer of stability from stabilized cw lasers to mode-locked pulsed laser sources. Furthermore, a stable clock signal in the radio-frequency (rf) domain can be extracted from an optical local oscillator by the phase-coherent optical comb generated by mode-locked laser, creating an optically based atomic clock.[9] One can therefore envisage that a stable mode-locked laser in space will eventually provide both time-keeping and distance-ranging services, truly uniting time and length metrology.

The key point that we need to address is how to smoothly bridge the measurements made by the incoherent, time-of-flight approach and the coherent, optical interference method. The time-delay measurement provides initial and yet unambiguous information about the unknown length in terms of a proper integer multiple of the pulse period, and the interference technique affords enhanced resolution and precision, providing an analog, sub-pulse-period fraction. Clearly some stringent requirements on the stability of the pulse repetition frequency and the pulse-to-pulse carrier envelope phase must be met. Here these conditions are discussed for an interferometer scheme adapted from the commonly used Michelson interferometer (Fig. 1). A mode-locked femtosecond laser sends a pulse train into the interferometer. Pulse repetition frequency f_{rep} is either synchronized to a low phase-noise rf signal or directly derived

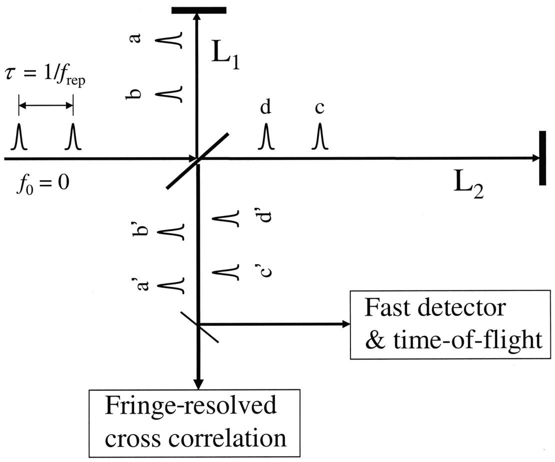

Fig. 1. Interferometer configuration for measuring absolute distances by use of a combination of incoherent time-of-flight and coherent fringe-resolved techniques. L_1 is a known length for the reference arm, and L_2 is the unknown distance to be measured. L_1 can be rather short, for example, on the millimeter or centimeter scale, and an exact-fractions approach can determine it. The pulse train used in the measurement has a repetition frequency $f_{\rm rep}$ and its pulse-to-pulse carrier-envelope phase slippage is stabilized to zero. a′ and b′ represent pulses that have traveled through L_1 (corresponding to pulses a and b), and c′ and d′ are for L_2.

from an ultrastable optical frequency standard. The pulse-to-pulse carrier-envelope phase slippage is eliminated by stabilization of the offset frequency (f_0) of the optical comb to zero. The high stability of the controlled signals of $f_{\rm rep}$ and f_0 establishes a stable interference fringe pattern between a pair of pulses even after they have traveled through long, differential delay lines. The pulse train is coupled into both arms of the interferometer after the beam splitter. The length of one arm, L_1, is the reference standard, and the length of the second arm, L_2, is the quantity to be determined. We denote $\Delta L = L_2 - L_1$. Two example pulses going through $L_1 (L_2)$ and exiting to the detection port are labeled a′ and b′ (c′ and d′), as shown in Fig. 1. The detection method includes a time-of-flight fast photodetector that has the capability of detecting pulse separations of a few picoseconds in time, along with a fringe-resolved optical cross correlator that detects the interference fringes between two pulses when they are adjusted to overlap after emerging from their respective arms.

Monitoring the change in the timing offset between the two pulse trains when scanning $f_{\rm rep}$ determines the absolute length scale of ΔL in a granular manner. Figure 2 illustrates the situation. At the first repetition frequency, $f_{\rm rep1}$ (corresponding to a pulse period of τ_1), a timing offset of Δt_1 exists between the two pulse trains traveling through the different arms. When the repetition frequency increases to $f_{\rm rep2}$, pulses c′ and d′ move closer to a′ and b′, respectively, with a smaller timing offset of Δt_2. Hence we have $2\Delta L/c = n\tau_1 - \Delta t_1$ and $2\Delta L/ = n\tau_2 - \Delta t_2$, where c is the speed of light and n is an integer. From these two equations we can uniquely determine the integer number n by measuring τ_1, τ_2, Δt_1, and Δt_2, leading to a preliminary estimate of ΔL (and thus L_2) to within 10^{-11} s $\times$ 3 $\times$ 10^8 m/s = 3 mm. To reach a higher-precision measurement of ΔL, we rely on a continuous measurement of the mean value of $f_{\rm rep}$ when it is scanned until the two pulses a′ and c′ (and also between b′ and d′) are brought to overlap to produce interference fringes, as shown in Fig. 3. A simple estimate is made of the required changes in $f_{\rm rep}$ to reach the pulse-overlap regime. The maximum change would be required when initially at $f_{\rm rep1}$ pulse c′ from L_2 is halfway between pulses a′ and b′ from L_1. In other words, $\Delta t_1 \approx \tau_1/2$. We then find τ_3 such that $2\Delta L/c = n\tau_3 - \Delta t_3$, with $\Delta t_3 \ll \tau_3$

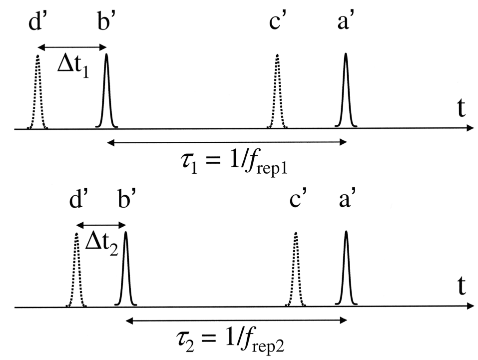

Fig. 2. A controlled change in $f_{\rm rep}$ of the mode-locked laser allows precise scanning of the timing offset between the two pulse trains that have traveled through two different arms of the interferometer. When the $f_{\rm rep}$ increases, pulse c′, which lags behind pulse a′, slowly catches up.

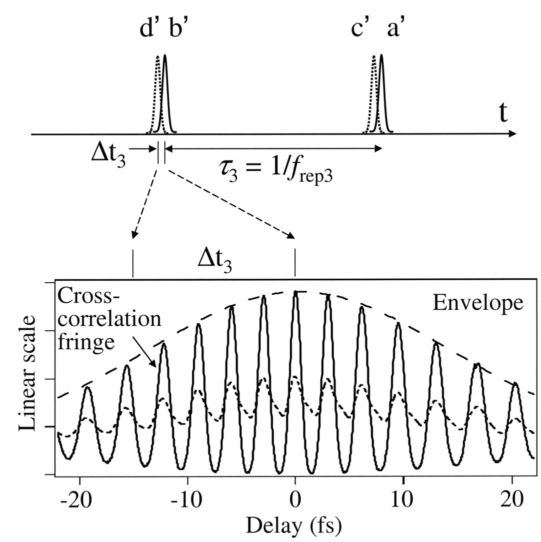

Fig. 3. Transition from the incoherent to the coherent measurement regime, where the two pulse trains are made to overlap partially with a proper choice of $f_{\rm rep}$. When the pulse trains overlap, an interference fringe starts to develop, as illustrated by the representative experimental trace shown as a solid curve at the bottom. Stable optical fringes are made possible by control of the pulse-to-pulse carrier-envelope phase shift. Introduction of a rms timing jitter into $f_{\rm rep}$ reduces the fringe contrast to ~30%, shown as a dashed curve at the bottom. The optical fringe offers a clear definition of the incremental length and the location where the two pulses exactly overlap.

and approaching within the pulse width. Hence, $n\tau_1 - \tau_1/2 \approx n\tau_3$, resulting in a fractional change in the pulse period by the amount $(\tau_1 - \tau_3)/\tau_1 \approx 1/(2n)$. For a mode-locked laser operating at a 1-GHz repetition frequency, $\Delta L \geq 1.5$ m would imply $n \geq 10$, and subsequently a maximum 5% change in the repetition frequency would amount to a maximum cavity length adjustment of ~1.5 cm, within reach of a linear motor under servo control. For $\Delta L \sim 15$ m, $n \sim 100$, then the laser cavity length change is a mere 1.5 mm.

A more significant question is whether one can obtain the distance resolution needed to resolve optical fringes when the pulses overlap, under the condition of the current state-of-the-art stability of a mode-locked laser. It is interesting to note the difference between the precisely measured mean value of f_{rep} for setting the distance versus rms timing jitter associated with f_{rep} when integrated over the entire relevant bandwidth for a time-domain fringe detection. Clearly, when a long distance ΔL is involved, the contrast of the interference fringe (as shown at the bottom of Fig. 3), taken at a small value of ΔL, will be significantly reduced if there is excessive timing jitter in the repetition signal or additional noise in the pulse-to-pulse carrier-envelope phase. One needs to evaluate the stability of f_{rep} and f_0 over the entire bandwidth extending from the upper bound of the Nyquist frequency ($f_{\mathrm{rep}}/2$) down to the lower bound that corresponds to the total measurement time. Synchronization results on f_{rep} have indicated that a rms timing jitter of the order of 1 fs or less over an integration bandwidth extending from 1 Hz to 10 MHz is achievable.[10–12] The mean value of f_{rep}, on the other hand, is determined after a long averaging time for frequency counting, and it has been demonstrated that it can be measured at or below a precision of 1×10^{-14} at 1 s and continues to improve at longer averaging times. It has also been reported that the rms phase noise associated with f_0, and thus the carrier-envelope phase noise, can be controlled to be under 0.2 rad for many minutes.[13]

An example is worked out with ΔL near 1.5×10^6 m. Other values of ΔL can be properly scaled from the following calculations. The round-trip time, $2\Delta L/c$, is ~10 ms. For such a long distance, a lower-repetition-rate laser, such as $f_{\mathrm{rep}} = 100$ MHz, is a better choice. One has approximately $n = 10^6$ pulses within the round-trip distance of $2\Delta L$. Therefore, if initially the two pulse trains coming from the two arms do not overlap, the maximum adjustment that needs to be made on f_{rep} is 5×10^{-7} in fractional terms. To determine the change in f_{rep} that will result in a shift of a single optical fringe in the cross-correlation detection, we note the condition is $n(\tau_3 - \tau_3{}') = \lambda/c$, where λ is the carrier wavelength and $\tau_3{}'$ is a slightly different pulse period from τ_3. This expression leads to $n(\Delta f_{\mathrm{rep}}/f_{\mathrm{rep}}) = f_{\mathrm{rep}}(\lambda/c)$. The fractional change in f_{rep} is, hence, $(\Delta f_{\mathrm{rep}}/f_{\mathrm{rep}}) = \lambda/(n\tau c) \approx \lambda/(2\Delta L) \sim 2.7 \times 10^{-13}$. With the present stability parameters of a mode-locked laser, one can fully expect to reliably scan through optical fringes in the cross correlator with a step size fine enough to divide an optical fringe by more than a factor of 10. The transition from the incoherent measurement to the fringe-resolved coherent measurement is made by precise frequency counting of f_{rep}. However, timing jitter in f_{rep} is expected to reduce the fringe contrast. The dashed curve in the bottom part of Fig. 3 shows such a contrast reduction when a rms timing jitter of 1 fs is introduced into the f_{rep} signal, with the fringe contrast reduced from nearly 100% to ~30%. Nevertheless, the central interference fringe under the envelope still offers a clear definition of when the two pulses from their respective arms finally overlap maximally. Length ΔL can then be read out from the precisely determined mean value of f_{rep} and the appropriate integer number n. Note in passing that the actual pulse width is not of particular importance, although a shorter pulse does help to enhance the detection of the central fringe. It is also worth noting that, if the optical comb spectrum is broadened by some external nonlinear process to cover a broad spectrum, then it is conceivable that a first-order cross-correlation measurement, the type on which a white-light interferometer is based, might bring a sharper fringe contrast with an enhanced signal-to-noise ratio.

Thanks to J. L. Hall and J. E. Faller for their excellent traditions in interferometry. Comments from R. J. Jones are appreciated. This work was supported by NASA, the U.S. Office of Naval Research, the National Institute of Standards and Technology, and the National Science Foundation. The author's e-mail address is ye@jila.colorado.edu.

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