MATHEMATICAL METHODS

This model syllabus defines the core material for Mathematical Methods. Instructors should use their discretion in deciding the ordering of topics, the depth to which each is covered, and additional material to include (example: complex analysis would be a logical additional topic that could be covered with a strong class). It is anticipated that instructors will draw upon a range of examples from astrophysics and planetary science to illustrate the core material.

LINEAR ALGEBRA

Brief review of linear algebra, as necessary

Matrix definition, algebra Vector spaces, Gram-Schmidt orthogonalization Eigenvalues and eigenvectors, diagonalization Matrix inversion Numerical methods for linear algebra

ORDINARY DIFFERENTIAL EQUATIONS

Review of ODEs, as necessary

Classification Basic solution methods: exact differentials, integrating factors Series solutions, singular points, Frobenius method, second solution Green functions Nonlinear ODEs, perturbation analysis

NUMERICAL METHODS FOR ODEs

Goal: to be able to understand the appropriate techniques to use to solve different ODEs numerically, and their limitations

Introduction to approaches, finite difference methods, concepts of order, accuracy, stability Techniques for initial value problems Techniques for boundary value problems

INTEGRAL TRANSFORMS

Understanding the basis of techniques such as convolution, power spectrum estimation, etc

Sturm-Liouville problems, expansion in eigenfunctions

Fourier series and integrals Fourier and Laplace transforms Applications e.g. to convolution, Fast Fourier Transforms Wavelets

PARTIAL DIFFERENTIAL EQUATIONS

Classification of PDEs, boundary conditions Characteristics Separation of variables Solution using integral transforms Green functions

SPECIAL FUNCTIONS

Bessel functions Legendre polynomials Spherical harmonics

NUMERICAL METHODS FOR PDEs

Goal: an understanding of some of the techniques for solving PDEs, and the considerations involved in choosing methods (stability, efficiency, obeying conservation laws, etc).

Introduction to different approaches: finite difference, spectral methods. Von Neumann stability analysis, the CFL condition Examples of numerical schemes for hyperbolic and parabolic equations Relation of the numerical system to the physical PDE Elliptic equations: Solution via direct methods and via relaxation