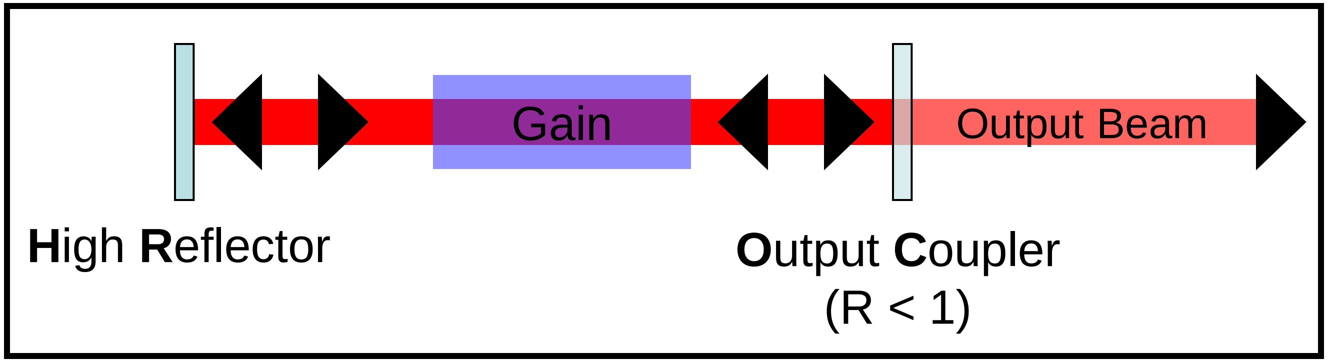
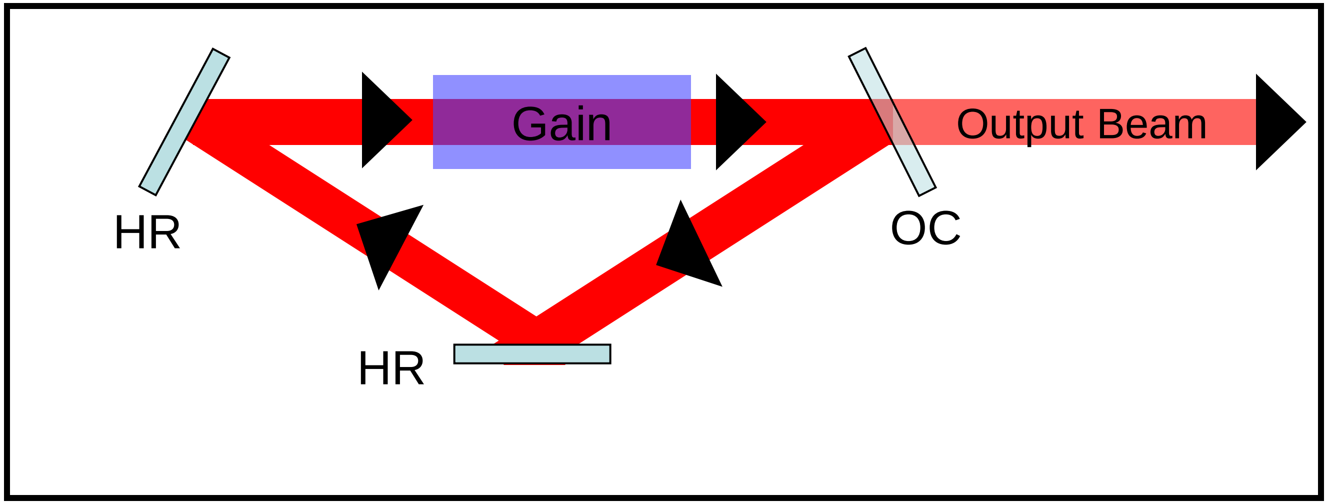
Laser Cavity Geometries

Lasers come in two generic configurations:

1) Linear cavity (also called “standing wave”)



2) Ring cavity



Steady State

Write the electric field inside the cavity as

$$E(z,t) = \mathrm{Re}\left[E_0 e^{-i(\omega t - kz)}\right]$$

And define E_0 to the field just to the right of the HR

Just before the OC, the field is

$$E = \mathrm{Re}\left[E_0 e^{\frac{\omega}{c} n'' l_g} e^{-i\omega t} e^{-i\frac{\omega}{c}(n' l_g + l_a)}\right]$$

where n' and n'' refer to the gain medium, l_g and l_a are the physical lengths of the gain medium and air regions, respectively. $n'' > 0$ yield amplification.

The field back at $z = 0$ is obtained from a reflection off the OC, transit through the gain medium and a reflection off the HR, yielding

$$E = \mathrm{Re}\left[r_1 r_2 E_0 e^{2\frac{\omega}{c} n'' l_g} e^{-i\omega t} e^{-i2\frac{\omega}{c} l}\right]$$

In steady state, these must be the same

with $l = n' l_g + l_a$

Steady state conditions

Steady state (no change after one round trip) yields two conditions:

1) Gain condition

$$r_1 r_2 e^{2\frac{\omega}{c}n'' l_g} = 1$$

(gain exactly balances loss)

2) Phase condition

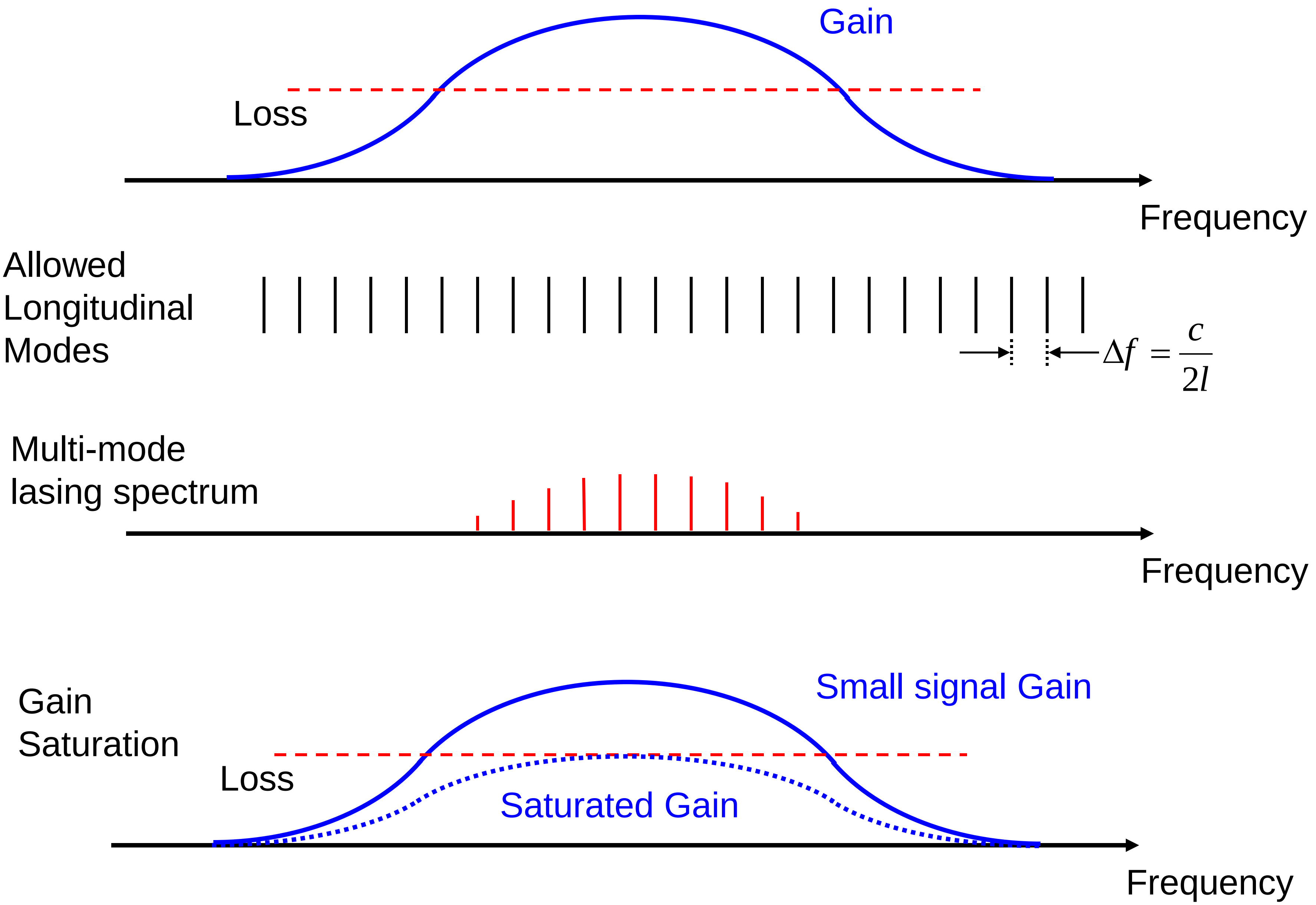
$$\frac{2\omega l}{c} = 2m\pi$$

(round trip phase is an integer multiple of 2π)

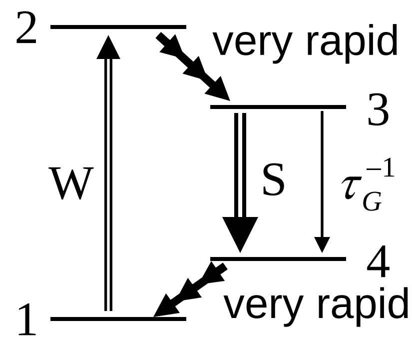
The phase condition determines allowed oscillation frequencies

$$\omega_m = \frac{m\pi c}{l} \quad \text{or} \quad f_m = \frac{mc}{2l}$$

Laser: Frequency Domain



Gain



We will consider a 4-level gain medium and its behavior in a CW laser. A 3-level gain medium can be treated as a special case of a 4 level system.

All of the "atoms" start in the ground state 1.

A "pump" promotes the atoms from state 1 to state 2 at a rate W.

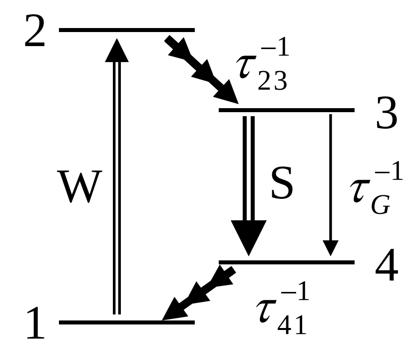
The atoms relax very rapidly from state 2 to state 3.

The transition from 3 to 4 is the "lasing" transition and occurs due to stimulated emission at rate S

They can also "spontaneously" relax at rate τ_G^{-1}

The atoms then relax very rapidly from state 4 back into state 1.

Gain Rate Equations



Conservation law

$$N_1 + N_2 + N_3 + N_4 = N_G$$

(N_G is total density of atoms)

Rate Equations

$$\dot{N}_1 = -W(N_1 - N_2) + \frac{N_4}{\tau_{41}}$$

$$\dot{N}_2 = W(N_1 - N_2) - \frac{N_2}{\tau_{23}}$$

$$\dot{N}_3 = -S(N_3 - N_4) - \frac{N_3}{\tau_G} + \frac{N_2}{\tau_{23}}$$

$$\dot{N}_4 = S(N_3 - N_4) - \frac{N_4}{\tau_{41}} + \frac{N_3}{\tau_G}$$

where $S = \frac{\sigma_{34} I(\omega_{34})}{\hbar\omega_{34}}$

Gain is proportional to population difference on laser level:

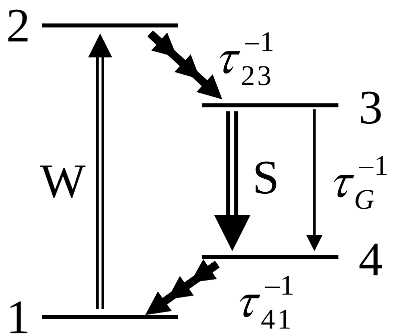
$$g = \frac{\sigma_{34}}{2}(N_3 - N_4) l_g$$

Solving the rate equations yields

$$g = \frac{1}{2}\frac{\sigma_{34} W N_G \tau_G l_g}{1 + (W + S)\tau_G}$$

Small Signal Gain and Saturated Gain

$$g = \frac{1}{2}\frac{\sigma_{34} W N_G \tau_G l_g}{1+(W+S)\tau_G}$$



First consider the small signal gain ($S = 0$) as a function of pump rate

$$g_0 = \frac{1}{2}\frac{\sigma_{34} W N_G \tau_G l_g}{1+W\tau_G}$$

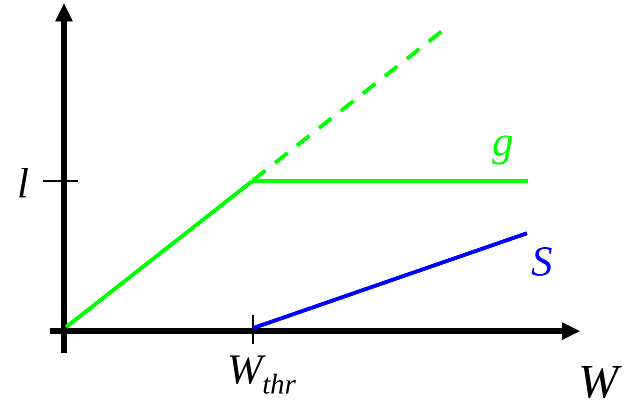
As the pump is turned on, the gain initially increases linearly with pump rate, but saturates once all the atoms are in state 3

Second, at a fixed pump, the saturated gain can be written

$$g = \frac{g_0}{1+\frac{S}{S_{sat}}} \qquad \text{where} \qquad S_{sat} = W + \frac{1}{\tau_G}$$

The intracavity intensity is

$$S = S_{sat}\left(\frac{g_0}{g_{th}} - 1\right)$$



Paraxial wave equation

The propagation of light is given by the wave equation

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial^2 t}$$

Assume we can decompose into a carrier and spatial envelope (more on decomposition in time later) and only treat one polarization

$$E(z,t) = \mathrm{Re}\left[\tilde{E}_0 u(x,y,z) e^{-i(wt-kz)}\right]$$

Plugging this form into the wave equation, we can write

$$\nabla_T^2 u + \frac{\partial^2 u}{\partial z^2} + 2ik\frac{\partial u}{\partial z} = 0$$ where $$\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

If we assume u is slowly varying in space $\left|\partial^2 u/\partial z^2\right| << 2k\left|\partial u/\partial z\right|$

We obtain the paraxial wave equation

$$\nabla_T^2 u + 2ik\frac{\partial u}{\partial z} = 0$$

Gaussian Beam

A solution to the paraxial wave equation is a Gaussian beam

$$u_{00}(x,y,z)=\frac{w_0}{w(z)}e^{-\left(x^2+y^2\right)/w^2(z)}e^{ik\left(x^2+y^2\right)/2R(z)}e^{-i\phi(z)}$$

$$w^2(z)=w_0^2\left[1+\left(\frac{z}{z_0}\right)^2\right]$$

Size of beam as function of distance

$$\frac{1}{R(z)}=\frac{z}{z^2+z_0^2}$$

Radius of curvature of phase fronts

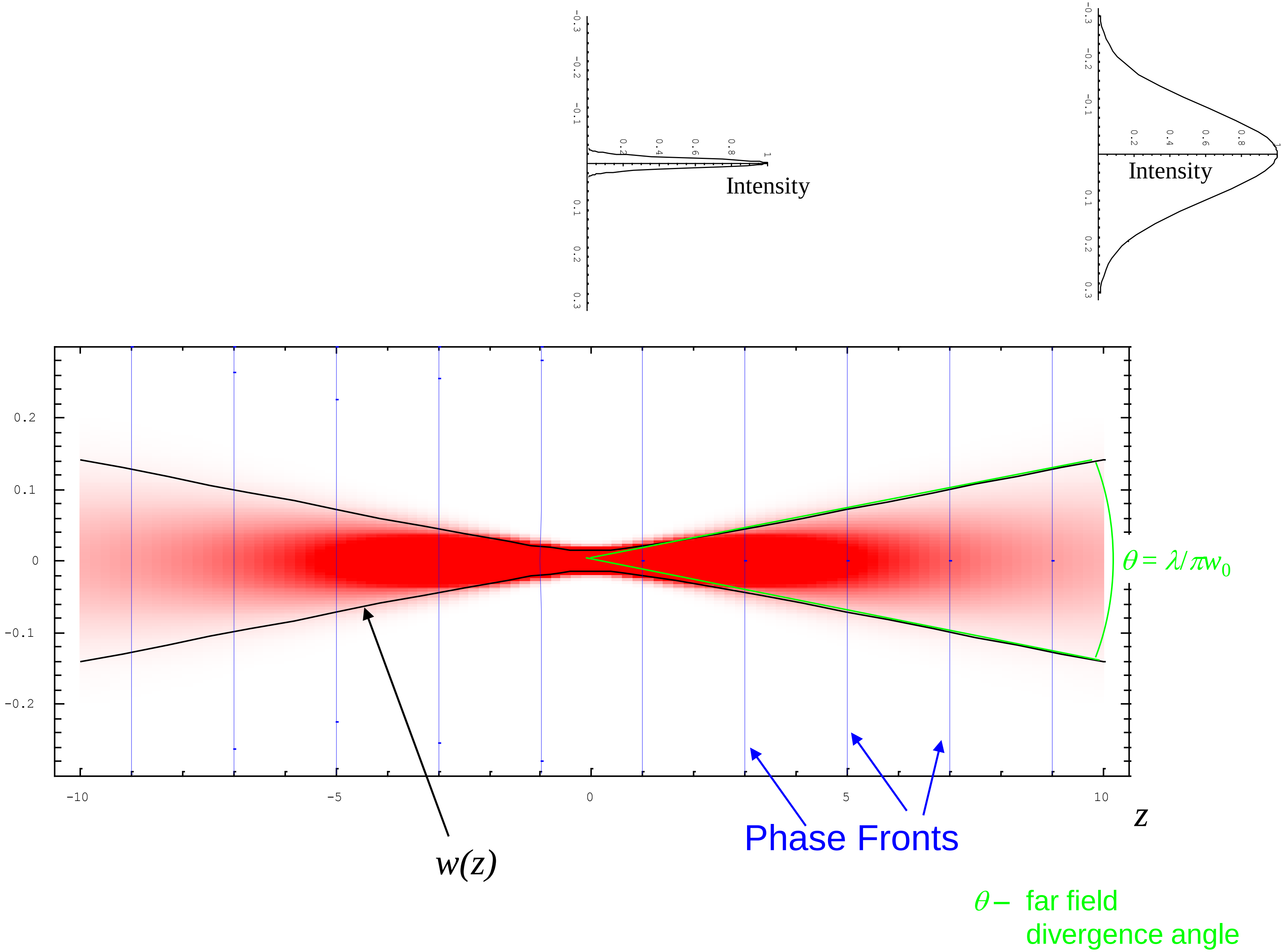
$$\phi(z)=\tan^{-1}\left(\frac{z}{z_0}\right)$$

Overall phase

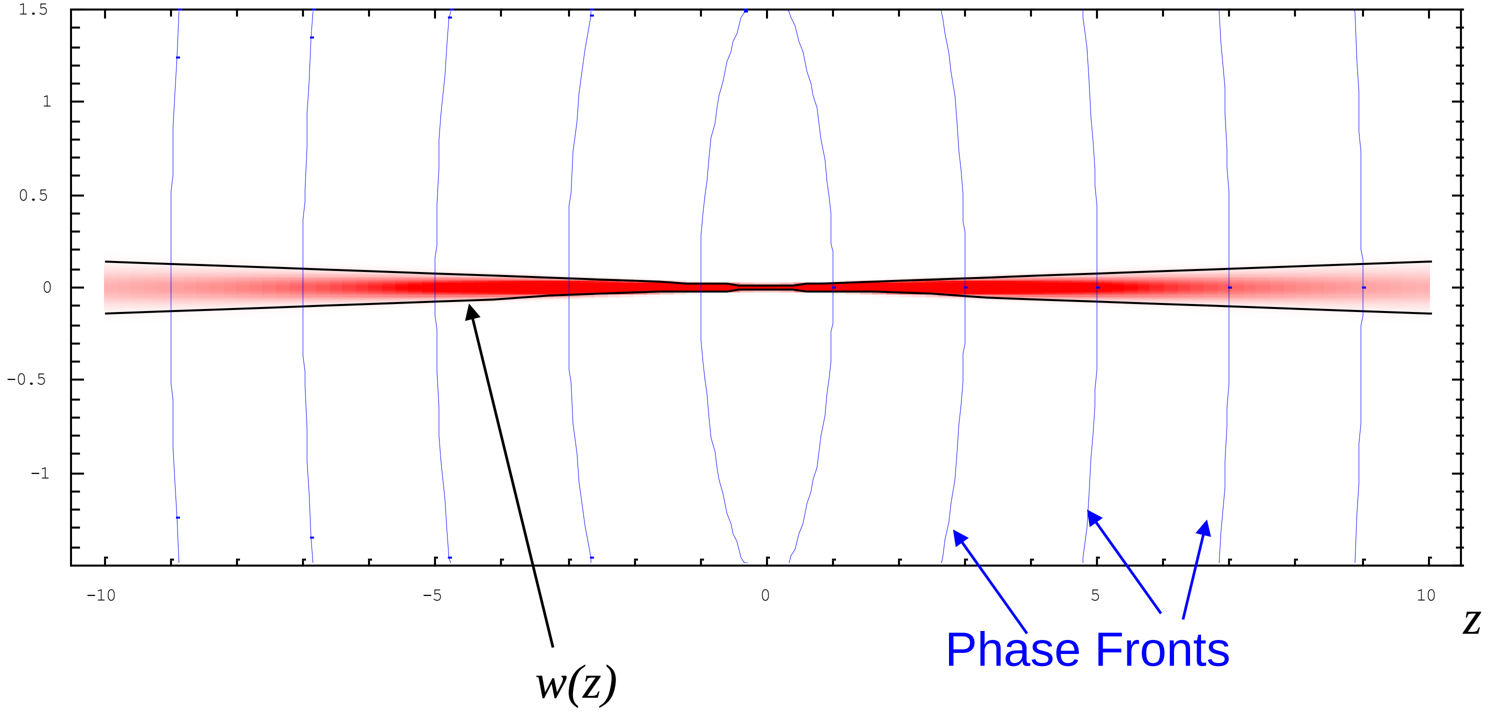
$$z_0=\frac{\pi w_0^2}{\lambda}$$

Rayleigh range

The only free parameter is the Rayleigh range, z_0, or the waist size, w_0.



Intensity
Intensity
$\theta = \lambda / \pi w_0$
z
$w(z)$
Phase Fronts
$\theta -$ far field divergence angle



1.5
1
0.5
0
-0.5
-1
-10
-5
0
5
10
z
w(z)
Phase Fronts

Propagation of Gaussian Beams

To calculate the propagation of a Gaussian beam, use the complex q parameter

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{i\lambda}{\pi w^2(z) n}$$

The effect on an optical element is given by

$$q_{out} = \frac{A q_{in} + B}{C q_{in} + D}$$

Where A, B, C and D are the elements of the ray matrix for the element

The ray matrices for a few common elements

Propagation through a length d of "free space"

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

A lens with focal length f or spherical mirror with radius $2f$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$$

An interface between a medium with index n_1 and a medium with index n_2

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$$

Ray matrices

An important property of ray matrices is that the matrix for a cascaded system is simply given by the product of the matrices for the individual elements.

For example, the matrix for a dielectric slab surrounded by free space is

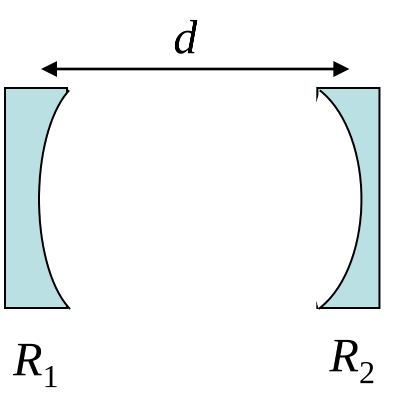
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix} = \begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$$

If we calculate the matrix for one round trip in a resonator, after one round trip the q parameter must be the same, i.e.

$$q = \frac{Aq + B}{Cq + D}$$

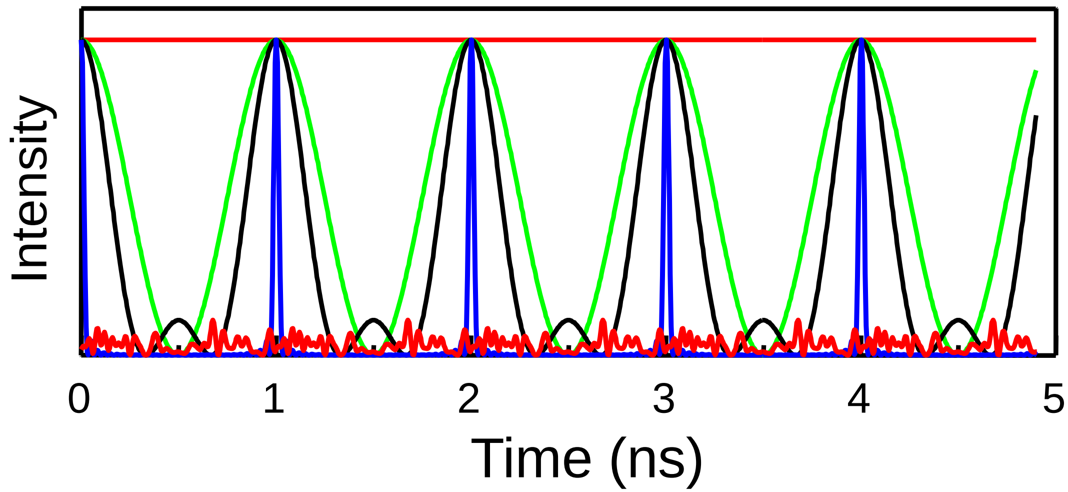
For a simple two mirror cavity, this yields

$$0 < \left(1 - \frac{d}{R_1}\right)\left(1 - \frac{d}{R_2}\right) < 1$$



Mode-locking: The basic idea

Ultrashort pulses are produced by "mode-locked" lasers. In a mode-locked laser, multiple longitudinal modes laser simultaneously and are locked together in phase.



Mathematically:

$$e(z,t) = \mathrm{Re}\left[\sum_m E_m e^{-i(\omega_m t - k_m z + \phi_m)}\right] = \mathrm{Re}\left[e^{-i\omega_0 (t - z/c)} \sum_m E_m e^{-i[m\Delta\omega(t - z/c) + \phi_m]}\right]$$

here $\omega_m \to \omega_0 + m\Delta\omega = \omega_0 + \frac{2m\pi c}{L}$ and $k_m \to \frac{\omega_m}{c}$

assume E_m's constant (flat spectrum) and ϕ_m's = 0 (modelocked) use $\sum_{m=0}^{q-1} a^m = \frac{1-a^q}{1-a}$

$$e(z,t) = \mathrm{Re}\left[E_0 e^{-i\omega_0 (t - z/c)} \frac{\sin \frac{N\Delta\omega}{2}(t - z/c)}{\sin \frac{\Delta\omega}{2}(t - z/c)}\right]$$

for N modes lasing

Title

Text