Amplification

Typical output of a mode-locked oscillator:

1 watt at a repetition rate of 100 MHz $\Rightarrow$ 10 nJ per pulse

For a 10 fs pulse $\Rightarrow$ ~ 1 MW peak power

Focused to 10 micron spot $\Rightarrow$ 10$^{14}$ Watts/cm$^2$ $\Rightarrow$ 2 x 10$^8$ V/cm

Not to shabby, but what if you need more? Compare: inner atomic field is about 1 GV/cm

Amplify:

The average power that can be extracted is basically constant

Lower repetition rate to achieve higher pulse energy

1 watt at 1 kHz $\Rightarrow$ 1 mJ per pulse

50 fs pulse $\Rightarrow$ 20 GW peak power

Focused to 20 micron spot $\Rightarrow$ 5 x 10$^{17}$ Watts/cm$^2$ $\Rightarrow$ 15 GV/cm

These are “typical” numbers, higher are achievable
Gain and Saturation I

Analyze the following geometry:

- Pump from the side a region of width $a$
- How deep ($b$) is the pump region?
- Need to analyze saturation

Assume a 3 level system

Need to describe saturation.

The cross section of a transition is

$$\sigma(\omega) = \frac{\sigma^{(0)}}{1 + T_2^2 (\omega - \omega_0)^2}$$

where $T_2$ is the dephasing time, $\omega_0$ is the resonance frequency and the on resonance cross-section for a transition with dipole moment $\mu$ is

$$\sigma^{(0)} = \frac{\mu T_2 \omega_0}{\epsilon_0 c n \hbar}$$

The cross-section has units of cm$^2$, think of it as absorption(gain) coefficient per atom ($\sigma N$ has units cm$^{-1}$)
Gain and Saturation II

The energy density at position $z$ and time $t$ within the pulse is

$$W(z,t) = \int_{-\infty}^{t} I(z,t')dt'$$

The saturation energy density is

$$W_s = h\omega/2\sigma$$

In a 2-level amplifying medium, an initial $W_0(t)$ becomes

$$W(z,t) = W_s \ln\left[1 - e^a\left(1 - e^{W_0(t)/W_s}\right)\right]$$

Where the small signal gain is

$$a = \sigma\Delta n^{(e)}z$$

For an equilibrium population inversion $\Delta n^{(e)}$, $\Delta n = n_1 - n_0$

Define saturation ratio as

$$s = \frac{W_0}{W_s} = 2\sigma \frac{W_0}{h\omega}$$

which is product of cross-section and number of photons in incident pulse.
Return to the 3-level system and consider absorption of pump.

Write rate equations for number of pump photons, \( F_p \) and population of lower state, \( n_0 \) and middle state, \( n_1 \)

\[
\dot{n}_0(y,t) = -\sigma_{02} n_0(y,t) F_p(y,t)
\]

\[
\frac{\partial}{\partial y} F_p(y,t) = -\sigma_{02} n_0(y,t) F_p(y,t)
\]

\[
n_1(y,t) = N - n_0(y,t)
\]

For initial condition of all atoms in ground state, \( n_0(y,0) = N \)

The inversion density is

\[
\Delta n_{10} = n_1 - n_0 = N \left\{ 1 - \frac{2}{1 - e^{-\sigma_{02} N y (1 - \exp(s_p) )}} \right\}
\]

From which we see it is possible to obtain a region of reasonably constant inversion (gain) if the pump saturates the gain medium (curves for varying \( s_p \))
The small signal intensity gain for a pass through the entire medium is

\[ a = \sigma_{10}(n_1 - n_0)L \]

The energy gain, including saturation, is

\[ G = \frac{W(L)}{W_0} = \frac{\hbar \omega}{2\sigma_{10}W_0} \ln\left[1 - e^a\left(1 - e^{2\sigma_{10}W_0}\right)\right] \]

\[ = \frac{1}{s} \ln\left[1 - e^a\left(1 - e^s\right)\right] \]

Working in saturation is bad for chirped pulse amplifiers (more later)
But advantageous in terms maximizing energy extraction and reduced sensitivity to pulse fluctuations

\[ \frac{\Delta W(L)/W(L)}{\Delta W_0/W_0} = \frac{e^a e^s}{1 - e^a \left(1 - e^s\right)} \frac{1}{G} \]
## Gain Media

<table>
<thead>
<tr>
<th>Medium</th>
<th>(\lambda) ((\mu)m)</th>
<th>(\Delta\lambda) (nm)</th>
<th>(\sigma) (cm(^2))</th>
<th>Life time (s)</th>
<th>Typical Pump</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dyes</td>
<td>0.3…1.0</td>
<td>50</td>
<td>&gt;10(^{-16})</td>
<td>10(^{-8})...10(^{-12})</td>
<td>laser</td>
</tr>
<tr>
<td>Color centers</td>
<td>1…4</td>
<td>200</td>
<td>&gt;10(^{-16})</td>
<td>10(^{-6})</td>
<td>laser</td>
</tr>
<tr>
<td>KrF</td>
<td>0.249</td>
<td>2</td>
<td>3x10(^{-16})</td>
<td>10(^{-8})</td>
<td>discharge</td>
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<tr>
<td>Alexandrite</td>
<td>0.75</td>
<td>100</td>
<td>7x10(^{-21})</td>
<td>3x10(^{-4})</td>
<td>Flashlamp</td>
</tr>
<tr>
<td>Cr:LiSAF</td>
<td>0.83</td>
<td>250</td>
<td>5x10(^{-20})</td>
<td>6x10(^{-5})</td>
<td>Flashlamp Diode</td>
</tr>
<tr>
<td>Ti:sapphire</td>
<td>0.78</td>
<td>400</td>
<td>3x10(^{-19})</td>
<td>3x10(^{-6})</td>
<td>Laser</td>
</tr>
<tr>
<td>Nd:glass</td>
<td>1.05</td>
<td>21</td>
<td>3x10(^{-20})</td>
<td>3x10(^{-4})</td>
<td>Flashlamp</td>
</tr>
</tbody>
</table>

### Generally prefer

Smaller \(\sigma\) (higher saturation energy)

Longer lifetime (pump pulse can be longer, less loss to fluorescence)
Changes in the pulse shape during amplification are generally detrimental

Occur due to:

Saturation
Dispersion

Discussed dispersion extensively already
As long as the amplifier is not strongly saturated, can be pre- or post-compensated

Gain Narrowing
Nonlinearity
Pulse Shaping in Amplifiers: Saturation

Saturation of the gain favors the leading edge of the pulse

Shifts the pulse earlier in time

Steepens the leading edge

The steeper the edge to start with, the less distortion

Dramatically changes “wings” on the pulse

⇒ insert saturable absorbers between amplification stages
Gain Narrowing

The finite bandwidth of the gain medium acts as a spectral filter. Narrows pulse spectrum $\Rightarrow$ lengthens duration.

Same analysis as when we discussed pulse evolution inside the cavity.

Assume Gaussian small signal gain spectrum

$$a = a_0 \exp\left[-\left(\Omega T_g\right)^2 / 2\right]$$

$2.36/T_g$ is spectral width.

Parabolic expansion

$$a = a_0 \left[1 - \left(\Omega T_g\right)^2 / 2\right]$$

Initial Gaussian pulse with temporal width $1.18 \tau_G$ has spectrum

$$\hat{E}(\Omega) = A_0 \exp\left[-\left(\Omega \tau_G / 2\right)^2\right]$$

Which after amplification becomes

$$\hat{E}(\Omega) = A_0 \exp\left[a_0 / 2 - \Omega^2 \left(\tau_G^2 + a_0 T_g^2\right) / 4\right]$$

Duration after amplification is

$$\tau_p' \approx 1.18 \sqrt{\frac{\tau_G^2}{a_0 T_g^2}}$$

In limit of strong amplification, output pulse duration is just due to gain bandwidth, not input pulse duration.
Gain narrowing can also shift the center of the pulse,
However this can be used to compensate for narrowing

Amplified Spontaneous Emission (ASE)

Amplification of spontaneously emitted photons is a severe problem in amplification of ultrashort pulses

Due to pump pulse duration being long compared to signal pulse
Robs available gain from signal pulse
Clamps inversion

Depends on geometry of gain medium

Only spontaneous photons that travel down the gain medium matter

Plot at right shows small signal gain with ASE as a function of it without

For varying ratio of pump photon flux to ASE photon flux, $F_p/F_{ASE}$

$$F_{ASE} = \frac{\eta_F \Delta \Omega \hbar \omega_{ASE}}{4 \sigma_{ASE} T_{10}}$$

Vary small signal gain without changing $F_p$ by length or concentration
Nonlinear index of refraction arises due to

- saturation of off-resonant amplification (or absorption)
- and/or
- nonlinear index of refraction of host materials

Results in

- Self-phase modulation (change in spectrum)
  - Can be exploited to increase spectral width (shorter pulse after compression)
- Self-focusing (change in spatial profile)
  - Generally deleterious, leads to filamentation and possibly damage
The change in phase of a pulse propagating through a resonant material is

\[ \frac{\partial}{\partial z} \varphi = -\frac{1}{2} \sigma \left| \frac{1}{i(\omega - \omega_0)T_2 + 1} \right|^2 (\omega - \omega_0)T_2 \Delta n \]

At position \( z \), the time dependent change due to saturation is

\[ \delta \omega(t) = \frac{\partial \varphi}{\partial t} = -\frac{1}{2} \sigma^{(0)} \left| \frac{1}{i(\omega - \omega_0)T_2 + 1} \right|^2 (\omega - \omega_0)T_2 \int_0^z \frac{\partial}{\partial t} \Delta ndz = -\frac{1}{2} (\omega - \omega_0)T_2 \frac{\partial}{\partial t} \ln \frac{F(z,t)}{F_0(t)} \]

The sign depends on
- Tuning above or below resonance
- Absorption or Gain (\( F \) larger or smaller than \( F_0 \), \( F \)'s are photon fluxes)

In limit of short signal pulse

\[ \delta \omega(t) = -\frac{(\omega - \omega_0)T_2}{2} \frac{e^{-a} - 1}{e^{-a} - 1 + e^{W(t)/W_s}} \frac{I(t)}{W_s} \]

We see that the frequency roughly tracks the intensity
Effect of SPM

Net SPM due to saturation of gain and Kerr effect

Kerr effect frequency shift is

\[ \delta \omega(t) \sim -n_2 \int_0^z \frac{\partial}{\partial t} I(z', t) dz' \]

Tracks derivative of intensity

In saturated amplifiers, former dominates

Plot at right shows typical examples as function of saturation ratio
For a beam with a Gaussian transverse profile:

the Kerr effect will result in a Gaussian transverse index profile
This in turn results in a Gaussian transverse phase profile
Corresponds to a focusing beam

Self focusing will occur when the focusing due to nonlinearity is stronger than the diverging due to diffraction
Nonlinear phase is

$$\phi_{sf}(r) = -n_2 \frac{2\pi}{\lambda} z I_0 e^{-\left(\frac{2r^2}{w_0^2}\right)} \approx -n_2 \frac{2\pi}{\lambda} z I_0 \left(1 - 2 \frac{r^2}{w_0^2}\right)$$

Where in the last step we've made our standard parabolic approximation

We need to dust off a few Gaussian beam formulas….
Self-focusing: Gaussian Beam Refresher

\[ E(x, y, z) = E_0 \left[ \frac{w_0}{w(z)} \exp \left( -\frac{r^2}{w^2(z)} \right) \right] \exp \left[ -i \left( k z - \tan^{-1} \left( \frac{z}{z_0} \right) \right) \right] \exp \left[ -i \frac{k r^2}{2 R(z)} \right] \]

where

- **amplitude**
- **longitudinal phase**
- **radial phase**

\[ w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_0} \right)^2} \]
\[ R(z) = \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right] \]
\[ z_0 = \frac{\pi n w_0^2}{\lambda} \]

Determines radius as function of \( z \)

Radius of curvature of phase fronts as function of \( z \)

- **Phase Fronts**

\[ \theta = \frac{\lambda}{\pi w_0} \]

Far field divergence angle
Compare the phase of a diffracting Gaussian beam

\[
\phi_{\text{diff}} = -\frac{k}{2R(z)} r^2 \approx -\frac{kz}{2z_0^2} r^2
\]

Where in the second expression we've assume \( z \) small so

\[
R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right] \approx \frac{z_0^2}{z}
\]

To the nonlinear phase

\[
\phi_{\text{sf}}(r) \approx n_2 \frac{4\pi}{\lambda} z I_0 \frac{r^2}{w_0^2}
\]

They are equal magnitude when

\[
I_0 \frac{\pi w_0^2}{2} = \frac{\lambda^2}{8\pi m n_2}
\]

Where we have used

\[
z_0 = \frac{\pi m w_0^2}{\lambda}
\]

A tighter focus means higher intensity and more nonlinearity, but also less curvature to the phase front to be overcome, these cancel

Note that the left side is a power, we define this as the critical power \( P_{\text{cr}} \)

Above the critical power, nonlinearity wins and self-focusing occurs
An alternative derivation based on assuming a top-hat intensity profile and calculating the condition for total-internal reflection yields

\[ P_{cr} = \left(1.22\right)^2 \frac{\pi \lambda^2}{32n_n^2} \]

Note this has same dependence on \( \lambda, n \) and \( n_2 \), as our previous result, just differ by a constant (numerical simulations give different constant)

Key point:
It is a critical power, not intensity
Because nonlinear focusing and diffraction scale identically with beam diameter

For a power > \( P_{cr} \), beam comes to a focus in distance

\[ z_{SF} = \frac{0.5 z_0}{\sqrt{P/P_{CR} - 1}} \]

Assuming a waist at the entrance to the nonlinear medium

Once it reaches a focus, higher order linear and nonlinear effects come into play – either the beam totally diffracts away or forms a stable waveguide (filament)
Self-focusing in amplifiers

Check numbers: $n_2$ for sapphire is $8 \times 10^{-16}$ cm$^2$/W

$$P_{cr} = \frac{\lambda^2}{8\pi n n_2} = \frac{(0.8 \times 10^{-4})^2}{8\pi \cdot 1.7 \cdot 8 \times 10^{-16}} = 200 \text{ kW}$$

Easily achievable peak power

Directly from oscillator for 10 fs pulse, but 10 fs usually disperses

Furthermore, ripples in the transverse profile can get “amplified”

Self-focusing can be tolerated if length of amplifying medium $< z_{SF}$

How do we avoid this?
Thermal effects

The pump pulse deposits significant energy into the gain medium. At best fraction of energy extracted by light is $h\nu_{\text{sig}}/h\nu_{\text{pump}} \sim 0.6$ for Ti:sapphire, remainder goes into heat. Temperature dependence of index of refraction at room temperature

$$\frac{dn}{dT} \sim 10^{-5} \left( ^{\circ}\text{C} \right)^{-1}$$

This can cause significant distortion of signal beam.

If the pump is cylindrically symmetric, looks like a simple lens

Design optics to compensate

Work at temperature where $dn/dT \sim 0$ (possible for aqueous dye lasers at 4 C)

Maximize thermal conductivity

Sapphire is high, at liquid nitrogen temperatures is 10 times higher yet
Chirped Pulse Amplification

How can the deleterious effects of high peak power be avoided?

➔ Chirped pulse amplification

Chirp the pulse to stretch it in time (~1000 times) and lower peak power

Amplify

Recompress it

Concept originally developed for radar
First demonstration in optics

We need the stretcher and compressor to have opposite signs of GVD.

Generally the stretcher has normal dispersion.

In our discussion of dispersion, we concentrated on generating anomalous dispersion, how can we generate large amounts of normal dispersion?

1) Large amounts of material

2) Gratings?

Recall grating generated much more anomalous dispersion than prisms…promising but we need to change the sign.

Standard grating pair has second order phase

\[ \frac{d^2 \Psi}{d\Omega^2} = - \frac{\lambda_0}{2\pi c^2} \left( \frac{\lambda}{d} \right)^2 \frac{b}{\sqrt{r}} \frac{1}{r} \]

How can we change the sign?

⇒ Make \( b \) negative?
Use telescope to image first grating “behind” second grating $\rightarrow$ effective negative distance

The effective grating separation is $-2\Delta x$

Neglecting non-ideality of lenses, this has exactly opposite dispersion (to all orders) of standard grating pair

Of course the lenses introduce problems:

- Chromatic dispersion
- Spherical Aberation

These are eliminated by using an all reflective design. For example:
The compressor is typically the standard 2-grating pair.

This can perfectly compensate the grating stretcher, but material dispersion in gain section must be included:

- Gain medium
- Pockel’s cell

These will have different higher-order coefficients → for shortest pulses may need additional dispersion control, e.g. prisms as well.
How much stretching?

Usually the first effect of high intensity is nonlinear phase shift

\[ \varphi_{NL}(t) = \int \frac{2\pi}{\lambda} n_2 I(t, l)dl \]

The peak value of this expression is known as the “B integral”

Estimate of nonlinear group delay

\[ \frac{d\varphi}{d\omega} = \frac{B}{\Delta\omega} \]

Where \( \Delta\omega \) is the half-width of the amplified spectrum

If \( B = 1 \) rad and \( \Delta\omega = 20 \) nm, the nonlinear group delay variation is 17 fs

This can to some extent be compensated by the compressor, but keeping \( B \) less than 1 rad is a good goal
Gain Section

Two primary types:

1) Regenerative

2) Multi-pass

**Pockels Cell**

Electrooptic phase modulator

- Index of refraction depends on applied field
  - Tensorial relationship, typically
    - index increases for light polarized along field direction
    - decreases for light polarized orthogonal to field

Can act as a voltage controlled waveplate

- Typical “quarter-wave voltage” ~ 5 kV
  - switching 5 kV in ~10 ns is challenging, but doable
  - Only ¼ wave needed for double passed cell at end of cavity

Common materials:

- Lithium Niobate
- KDP (potassium dihydrogen phosphate)
Regenerative Amplifiers

Advantages
- Good signal/pump overlap in cavity $\rightarrow$ high efficiency
- Relatively compact and simple to align (one pass through gain medium)
- Easily obtain large number of passes through gain medium

Disadvantages
- Need to keep gain low to prevent ASE or even lasing
- Many passes through dispersive & nonlinear Pockels cell crystal

Note Regen’s can be pumped with either
- Q-switched lasers (10 ns pulses, kHz repetition rates) yielding mJ pulses
- Or
- CW yielding $\mu$J pulses at 100’s of kHz repetition rate
100 kHz Regenerative Amplifier

Multipass Amplifiers

Advantages

- Single pass through Pockels cell on input, less dispersion and nonlinearity
  - No need to switch out, obtained by spatial separation
- Higher gain per pass (no danger of lasing, just ASE)
  - Fewer passes needed → less dispersion and nonlinearity from gain crystal

Disadvantages

- Lower efficiency, pump-signal overlap changes every pass to allow spatial separation between passes
- Larger and more difficult to align